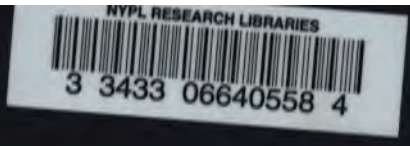

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ARITHMETIC FOR BUSINESS

WITH

SHORT METHODS AND RAPID CALCULATIONS

IN ALL FUNDAMENTAL PROCESSES.

DESIGNED FOR

BUSINESS COLLEGES, ACADEMIES, PUBLIC SCHOOLS, ETC.

AND FOR

ACCOUNTANTS, BUSINESS MEN, CONTRACTORS,

ARTISANS, AND PRIVATE STUDENTS.

BY

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A TEACHER OF WIDE EXPERIENCE IN ALL GRADES OF PUBLIC SCHOOL WORK, AND IN THE TRAINING OF TEACHERS IN PRIVATE SCHOOLS,

AND

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(HAMPDEN SIDNEY COLLEGE AND UNIVERSITY OF VIRGINIA)

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PREFACE.

For a number of years the authors of this work have felt the need of a Business Arithmetic that would meet the requirements of commercial schools and practical men, and, at the same time, be so complete as to embrace all divisions of the subject necessary to a text-book for grammar and high schools. The book was begun several years ago, and much time has been spent in original research, and in the compilation of material. The aims have been to make the work not only simple but eminently practical both as to subject matter and method of treatment. In its preparation, the needs of the business man, the farmer, the mechanic, and the clerk, have been no less considered than those of the teacher and his class-room. Nor has the private student been forgotten. Every process is presented in detail, and yet with such brevity, clearness, and simplicity that those deficient in the fundamental operations may become expert, and master the subject without the aid of a teacher.

The arrangement of the contents of the book is based, as nearly as may be, upon a logical order of sequence. The simpler and more practical subjects are presented first. Involution and Evolution are introduced earlier than in most arithmetics, because of the necessary applications of Square Root and Cube Root in such a comprehensive treatment of Practical Measurements as is found in this work.

The definitions are brief, clear, and exact; the explanations and solutions are simple and comprehensible; and the reasoning is logical, accurate, and conclusive. The authors have endeavored without a multiplicity of words to make every process so plain that any one who will study, not only may understand, but cannot fail to assimilate the method presented.

In the examples the development of the thinking faculty has not been overlooked. It is recognized that mental power is not less important than acquisition of knowledge. To attain both of these results much time and labor have been devoted to the explanations and solutions on which the rules given are founded and to the

statement of the principles. While the authors have endeavored to lead the learner to see the reason for the different processes, thus enabling him to derive his own methods for similar operations, they nevertheless believe that a clear and concise statement of the method to be pursued, embraced in a rule, is always helpful to the student. For this reason rules are generally given in connection with the various operations.

Under every rule such exercises have been prepared as will enable the diligent student to become quick and accurate in all computations. The variety of subjects presented furnish a thorough knowledge of approved business calculations. Special attention is invited to the practical expedients for rapid work given under each of the four fundamental rules and to the great number and variety of newly prepared and carefully graded problems found throughout the book. The searching review exercises afford necessary tests of the student's knowledge. They also fix in the memory the principles already studied, train the mind to habits of independent thought, and thus develop the power of correct reasoning.

To students deficient in a knowledge of Fractions the clear treatment of Common Fractions and Decimal Fractions will prove very helpful. To others the Metric System or Practical Measurements may be more needed; to still others Bills and Accounts, Percentage, and its applications, as Marking Goods, Trade Discounts, Commission, Interest, Discount, Exchange, Stocks and Bonds, etc., may prove most important. The extensive Drill Exercises in Rapid Addition, the Short Processes, and the section devoted to Commercial Paper, will be found of inestimable value to all persons engaged in business.

The authors earnestly believe that the book will prove to be all they have labored to make it, and that it will meet with general favor because it answers a public need.

POUGHKEEPSIE, N. Y.,
June 1, 1904.

CONTENTS.

DEFINITIONS.

1. Arithmetic is the science of numbers and the art of computing with them.

2. A **Unit** is a single thing, or one.

3. A **Number** is a unit or collection of units.

4. A **Concrete Number** is one in which the kind of unit is named; as, 2 balls, 3 dogs.

5. An **Abstract Number** is one in which the kind of unit is not named; as, 2, 3, etc.

6. Similar Numbers are those which have like units; as, 2 books, 5 books.

7. Dissimilar Numbers are those which have unlike units; as 2 books, 5 apples.

8. A **Problem** is a question to be solved.

9. A **Solution** is the process of solving a problem.

10. A **Rule** is a statement of the method of solving a problem.

NOTATION AND NUMERATION.

11. Notation is the method of expressing numbers in writing.

12. Numeration is the method of reading numbers in words.

13. Figures are certain marks or characters used to express numbers.

THE ARABIC SYSTEM.

14. The **Arabic System** of notation expresses numbers by figures.

15. There are ten different figures used as follows:

FIGURES:	1	2	3	4	5	6	7	8	9	0
NAMES AND VALUES:	One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Naught Cipher or Zero.

No number higher than nine can be expressed by a single figure, but by combining them in groups all other numbers may be denoted.

By general agreement the system of grouping *by tens*, called the *decimal* system, has been adopted.

16. Combination.—These figures are combined according to the following principles:

1. *A figure standing alone, or in the first place at the right of other figures, expresses* UNITS *or* ONES.

2. *A figure standing in the second place, counting from the right, expresses* TENS; *in the third,* HUNDREDS; *in the fourth,* THOUSANDS, *etc.*, thus,

10 is 1 ten, or ten.	100 is 1 hundred.
20 " 2 tens, or twenty.	200 " 2 hundred.
30 " 3 tens, or thirty.	500 " 5 hundred.
40 " 4 tens, or forty.	1000 " 1 thousand.
50 " 5 tens, or fifty.	2000 " 2 thousand.
90 " 9 tens, or ninety.	5000 " 5 thousand.

17. The name of each of the first twenty-one places is represented by the following

NUMERATION TABLE.

NAMES.	Hundred-quintillions.	Ten-quintillions.	Quintillions.	Hundred-quadrillions.	Ten-quadrillions.	Quadrillions.	Hundred trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
	3	4	5,	2	7	6,	3	8	2,	7	4	0,	3	0	9,	0	6	5,	7	3	4.
PLACES.	21st.	20th.	19th.	18th.	17th.	16th.	15th.	14th.	13th.	12th.	11th.	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.
PERIODS.	7th.			6th.			5th.			4th.			3d.			2d.			1st.		

18. The number which the figures express in the above table is read, three hundred forty-five *quintillion*, two hundred seventy-six *quadrillion*, three hundred eighty-two *trillion*, seven hundred

forty *billion*, three hundred nine *million*, sixty-five *thousand*, seven hundred thirty-four.

1. For convenience in reading numbers, the figures are separated by commas into periods of three places each, as shown in the table. The first three places constitute the first, or units period; the second three places, the second, or thousands period; the third three places, the third, or millions period, etc.

2. Each period, except the one at the left, must contain three figures.

3. In reading numbers the name of units period is omitted.

TO READ NUMBERS EXPRESSED BY FIGURES.

19. 1. Let it be required to read 8463736.

EXPLANATION.—Beginning at units and pointing off the figures into periods of three places each, we have 8,463,736. The third period is 8 millions, the second period is 463 thousands, the first period is 736 units; therefore the whole is read, eight million, four hundred sixty-three thousand, seven hundred thirty-six.

RULE FOR NUMERATION.—*Begin at the right and separate the numbers into periods of three figures each.*

Beginning at the left, read each period as if it stood alone, giving the name of each period, except the last.

EXERCISES.

Copy, point off into periods, and read:

2.	235,	3145,	43020,	304002.
3.	342,	4382,	50647,	870460.
4.	186,	4276,	93786,	2347896.
5.	207,	5678,	76081,	5070405.
6.	450,	8030,	89307,	52674032.
7.	789,	9671,	70035,	72130401.
8.	999,	7000,	89008,	100100100.
9.	1000,	6839,	487624780,	78742650004.
10.	8750,	24635,	3042678912,	800072900408.

TO WRITE NUMBERS IN FIGURES.

20. 1. Write in figures, five million, three hundred forty-six thousand, seventy-eight.

EXPLANATION.—Writing the 5 million in the units' place of the third period, 346 thousand in orders of the second period, and 078 in the orders of the first period, the figure 0 being placed before the significant figures to make the periods complete, and while writing, separating the periods by commas, we have 5,346,078.

RULE FOR NOTATION.—*Begin with the highest period to be expressed and write the hundreds, tens, and units of each period in their proper order, putting ciphers in all vacant places and periods.*

While writing, separate each period by a comma from the one that follows it.

EXERCISES.

Write in figures:

2. One hundred twenty.
3. One hundred forty-five.
4. Two hundred fifty.
5. Four hundred twenty-eight.
6. Seven hundred seventy-seven.
7. Eight hundred eight.
8. One thousand, two hundred fifty.
9. Three thousand, one hundred twenty-five.
10. Six thousand, five hundred forty-two.
11. Ten thousand, seven hundred sixteen.
12. Thirteen thousand, two hundred one.
13. Seventeen thousand, three hundred sixty.
14. Thirty thousand, four hundred. Forty-two thousand, six hundred.
15. Sixty-four thousand, five hundred twenty. Seventy thousand, three hundred ninety-four.
16. Four hundred twenty thousand. Eight hundred thirty-five thousand, one hundred.
17. Three hundred one thousand, six hundred. Three hundred fifty thousand.
18. Seven hundred fifty-eight thousand, two hundred eight. Seventy-nine thousand, five.
19. Nine hundred thousand, nine. Seventy-five thousand, seventy-five.
20. 35 million, 23 thousand; 27 million, 142 thousand, 240; 16 million, 15 thousand.
21. 40 million; 38 million, 500 thousand, 204; 38 million, 701 thousand, 280.
22. 312 million, 40 thousand, 400; 516 million.
23. 28 million, 600 thousand; 325 million, 30 thousand, 440; 355 million, 275.
24. 213 billion, 317 million, 412 thousand, 625.
25. *Six hu*ndred billion, five million, ninety-five.

26. One hundred billion, one hundred million.

27. Fifty billion, fifty million, fifty thousand, fifty.

28. Seventy billion, three hundred forty million, seven hundred seven.

29. Three trillion, two hundred four million, six hundred five thousand, twenty-five.

30. Seven hundred trillion, seven billion, seven million, seven hundred thousand, seventy.

NOTATION AND NUMERATION OF UNITED STATES MONEY.

21. The currency of the United States has a decimal system of notation. The dollars are written as whole numbers, and cents as decimal parts of a dollar.

22. The **Dollar Sign** is **$**. It is written before the number.

Thus, $24 is read, twenty-four dollars.

23. In notation of United States currency a period, called the *decimal point*, is placed before the number of cents.

24. The dollars are written at the left of the decimal point. The *first two* places at the right of the decimal point express *cents*, and the *third* place, *mills*.

Thus, $12.365 is read twelve dollars, thirty-six cents, five mills.

Mills are not coined. In business, in considering final results, five mills or more are called one cent, and less than five are disregarded.

25. When the number of cents is *less* than *ten*, a cipher must be written in the first place at the right of the decimal point.

Thus, six cents is written $.06; two cents, $.02: five dollars, seven cents, 5 mills is written $5.075.

EXERCISES.

Read the following:

1.	$26.35,	$125.75,	$362.455,	$1614.375.
2.	$17.82,	$362.50,	$872.532,	$2004.067.
3.	$35.10,	$512.87,	$890.727,	$8614.325.
4.	$29.17,	$739.56,	$967.305,	$2764.045.
5.	$30.03,	$709.80,	$985.625,	$3412.560.

Write the following:

1. Eight dollars, fifteen cents. Thirteen dollars, seven cents. Twenty dollars, *five cents.*

2. Forty-five dollars, forty cents. Thirty-seven dollars, seventeen cents. Fifty dollars, ten cents.

3. Two hundred dollars, thirty cents. Nine hundred dollars. Eighty dollars, eighty cents, one mill.

4. Three hundred sixty-five dollars, forty-three cents, five mills. Seventy-seven dollars, three cents.

5. Six hundred fifty dollars, sixty-five cents. Thirty-nine dollars, twenty cents, five mills.

6. One thousand two hundred dollars, thirty-one cents, seven mills. Two thousand five dollars.

7. Five thousand three hundred ten dollars, eighty-seven cents. Forty dollars, eight cents, two mills.

8. Fifteen thousand four hundred dollars, six cents. Twenty-eight dollars, three cents, nine mills.

9. Forty-two thousand seven hundred five dollars, thirty-five cents. Sixty thousand dollars.

10. Ninety thousand ninety dollars, nine cents, five mills. Two thousand dollars, two cents, two mills.

ROMAN NOTATION.

26. The **Roman System** of notation uses seven capital letters to express numbers.

Letters,	I,	V,	X,	L,	C,	D,	M.
Values,	1,	5,	10,	50,	100,	500,	1000.

27. To express other numbers these letters are combined according to the following:

PRINCIPLES.—1. *Repeating a letter repeats its value.*

Thus, I represents one; II, two; III, three; XX, twenty.

2. *When a letter is placed before one of a greater value, the difference of their values is the number represented.*

Thus, IV represents four; IX, nine; XL, forty.

3. *When a letter is placed after one of a greater value, the sum of their values is the number represented.*

Thus, VI represents six; XV, fifteen; LX, sixty.

4. *A line (‾) placed over a letter increases its value a thousand fold.*

Thus, $\overline{V}$ *represents* five thousand; $\overline{L}$, fifty thousand.

The following table illustrates the method of combination:

I...	1	XV..	15	XXIX..	29	DC...	600
II...	2	XVI..	16	XXX..	30	DCC...	700
III...	3	XVII..	17	XL..	40	DCCC...	800
IV...	4	XVIII..	18	L..	50	DCCCC...	900
V...	5	XIX..	19	LX..	60	M...	1000
VI...	6	XX..	20	LXX..	70	MD...	1500
VII...	7	XXI..	21	LXXX..	80	MM...	2000
VIII...	8	XXII..	22	XC..	90	MMM...	3000
IX...	9	XXIII..	23	C..	100	MMMM...	4000
X...	10	XXIV..	24	CC..	200	$\overline{V}$...	5000
XI...	11	XXV..	25	CCC..	300	$\overline{VI}$...	6000
XII...	12	XXVI..	26	CCCC..	400	$\overline{VII}$...	7000
XIII...	13	XXVII..	27	CD..	400	$\overline{VIII}$...	8000
XIV...	14	XXVIII..	28	D..	500	$\overline{IX}$...	9000

EXERCISES.

28. Read the following numbers:

XII; XXIV· XXVII; XIII; XLIV; LXI; LXX; CX; CXIX; XCVIII; CCLIV; CDXCVI; $\overline{V}$D; MDCC; MDCCCXCVIII.

Express the following numbers by the Roman Notation:

17,	44,	59,	67,	85,	127,	215,	342,
503,	618,	754,	861,	1060,	1859,	2000,	4675,
314,	410,	621,	735,	1171,	1463,	3101,	5344,
839,	515,	756,	978,	1365,	1776,	6792,	9999.

ADDITION.

29. Addition is the process of finding a number which is equal to the sum of two or more given numbers.

30. The **Sum** or **Amount** is the result obtained by adding two or more numbers.

31. The **Addends** are the numbers added.

32. A **Sign** is a mark used to denote what is to be done, or to express briefly an idea or direction.

33. The **Sign of Addition** is an upright cross +, and is read *plus*. It denotes that the numbers between which it is placed are to be added.

Thus, 2 + 1 is read 2 plus 1, and it shows that 2 and 1 are to be added.

34. The **Sign of Equality** is two short equal horizontal lines =, and is read *equals*, or *is equal to*.

Thus, 2 + 1 = 3, is read 2 plus 1 equals 3.

35. An **Equation** is an expression of equality.

Thus, 5 + 3 = 8 and 7 + 4 = 11 are equations.

36. Like or **Similar** numbers are those in which the units are alike.

Thus, $4 and $3 are like numbers; also 5 apples and 2 apples.

37. PRINCIPLES.—1. *Only like numbers can be added.*

2. *The sum must be like the numbers added.*

ADDITION TABLE.

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	13
2	3	4	5	6	7	8	9	10	11	12	13	14
3	4	5	6	7	8	9	10	11	12	13	14	15
4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	7	8	9	10	11	12	13	14	15	16	17
6	7	8	9	10	11	12	13	14	15	16	17	18
7	8	9	10	11	12	13	14	15	16	17	18	19
8	9	10	11	12	13	14	15	16	17	18	19	20
9	10	11	12	13	14	15	16	17	18	19	20	21
10	11	12	13	14	15	16	17	18	19	20	21	22
11	12	13	14	15	16	17	18	19	20	21	22	23
12	13	14	15	16	17	18	19	20	21	22	23	24

EXPLANATION.—The numbers in the left-hand column and across the top are the addends. The sums will be found in the horizontal columns opposite the numbers.

Thus, 1 and 0 are 1; 1 and 1 are two; 1 and 2 are 3; 1 and 3 are 4, etc.

The importance of thoroughly committing this table should not be overlooked. The neglect of it accounts, in large measure, for the waste of time and inaccuracy in adding columns of figures. The pupil should work out the sums for himself, thus investigating the results, and then thoroughly master the table.

DRILL EXERCISES.

38. The following list contains all the combinations of two numbers from 1 to 9.

If the student has thoroughly committed the addition table, he can tell the sums at a glance. If he can not, he should practice adding the numbers daily until able to do so.

2	4	6	2	7	3	6	2	6
5	3	4	2	4	9	7	8	5
4	8	6	3	4	3	3	2	1
5	9	8	7	2	5	3	7	1
2	7	1	6	3	4	8	4	9
9	8	2	6	1	9	5	4	5

6	5	4	9	1	3	7	9	7
3	5	8	1	4	8	7	9	9
2	3	8	8	7	5	6	6	5
6	2	1	8	1	1	1	9	7

39. 1. Count or add by 2's from 0 to 20, thus: 0, 2, 4, 6, etc.

2. Add by 2's from 1 to 33, thus: 1, 3, 5, 7, 9, etc.
3. Add by 3's from 0 to 30. From 1 to 40.
4. Add by 3's from 2 to 41. From 3 to 45.
5. Add by 4's from 0 to 44. From 1 to 52.
6. Add by 4's from 3 to 67. From 5 to 73.
7. Add by 5's from 0 to 60. From 1 to 76.
8. Add by 5's from 2 to 82. From 3 to 93.
9. Add by 5's from 4 to 89. From 6 to 91.
10. Add by 6's from 0 to 66. From 1 to 73.
11. Add by 6's from 2 to 80. From 3 to 99.
12. Add by 6's from 4 to 100. From 5 to 101.
13. Add by 7's from 0 to 70. From 1 to 85.
14. Add by 7's from 2 to 93. From 3 to 108.
15. Add by 7's from 4 to 95. From 5 to 103.
16. Add by 8's from 0 to 88. From 1 to 97.
17. Add by 8's from 2 to 106. From 3 to 115.
18. Add by 8's from 5 to 101. From 7 to 103.
19. Add by 9's from 0 to 90. From 1 to 100.
20. Add by 9's from 2 to 92. From 3 to 102.
21. Add by 9's from 4 to 112. From 5 to 122.
22. Add by 10's from 0 to 100. From 1 to 111.
23. Add by 10's from 2 to 122. From 3 to 113.

WRITTEN EXERCISES.

40. Copy and add from the bottom upwards, and then from the top downwards, each of the following:

In adding, name results only. Thus, in example 1 add as follows: 3, 7, 14, 19, 21 instead of 3 and 4 are 7, 7 and 7 are 14, 14 and 5 are 19, etc.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
2	4	6	3	7	5	4	3	4	3	2	6
5	6	4	2	3	4	2	5	5	4	2	1
7	7	2	1	3	6	7	4	7	6	5	3
4	8	5	7	1	7	1	6	1	8	6	7
3	1	7	9	2	3	5	7	2	3	4	5

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
5	2	7	3	5	6	7	8	5	1	8	9
6	7	2	6	7	3	2	1	3	6	5	1
8	3	4	8	2	7	6	3	4	2	4	4
7	4	5	7	1	4	4	5	2	5	2	7
3	6	8	3	8	5	3	2	6	7	6	2
4	2	1	6	4	9	7	6	7	3	1	5

25.	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.
5	4	3	6	5	8	9	1	2	3	7	4
2	1	7	2	4	6	1	8	1	2	2	8
1	3	6	7	3	4	3	7	5	4	3	5
3	7	4	3	5	1	6	2	3	5	6	7
4	6	5	4	2	3	1	1	7	1	4	3
6	5	8	2	1	5	2	3	2	8	1	2
2	1	3	5	7	2	4	2	4	1	8	5

37.	38.	39.	40.	41.	42.	43.	44.	45.	46.	47.	48.
3	5	1	7	4	7	6	8	1	5	3	7
4	1	5	4	1	3	4	3	4	6	5	4
5	3	3	3	5	1	2	4	3	3	7	1
6	2	6	5	2	4	5	7	2	4	2	3
7	6	4	2	6	3	7	2	7	1	4	5
8	7	1	6	3	8	1	4	1	3	2	6
9	1	2	1	8	1	6	1	6	7	1	5
2	3	3	7	1	7	7	6	4	3	4	8
1	4	5	4	4	3	4	5	7	8	6	9

49.	50.	51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
3	6	4	8	2	1	7	1	5	7	1	5
7	2	1	3	4	6	2	5	9	2	3	7
6	1	3	5	6	7	4	2	6	5	4	6
5	4	6	7	9	8	5	7	7	9	2	3
4	2	7	4	3	1	7	1	2	4	6	9
2	6	4	1	5	6	6	3	6	7	3	7
1	7	2	3	1	7	4	4	7	6	1	4
5	1	3	4	7	3	3	6	3	5	5	6
3	4	1	5	1	4	9	8	9	9	4	7
6	3	6	3	6	7	2	7	4	6	7	9
7	5	4	2	5	4	1	6	7	3	1	3
2	1	5	3	7	2	5	3	9	7	6	4

TO ADD LARGER NUMBERS.

41. 1. What is the sum of 132, 324, and 203 ?

132
324
203
659

EXPLANATION.—For convenience, we write the numbers so that all the figures of the same order stand in the same column. We draw a line beneath, and begin with the units to add. 3 units and 4 units are 7 units, and 2 units are 9 units, which we write under the column of units; 0 tens and 2 tens are 2 tens, and 3 tens are 5 tens, which we write under the column of tens; 2 hundreds and 3 hundreds are 5 hundreds, and 1 hundred are 6 hundreds, which we write under the column of hundreds. Hence, the sum is 6 hundreds, 5 tens, and 9 units, or six hundred fifty-nine.

2. What is the sum of 636, 873, and 945 ?

636
873
945
2454

EXPLANATION.—We write the figures of the same order in the same column as before and begin with the units to add. 5 units and 3 units are 8 units, and 6 units are 14 units, which equal 1 ten and 4 units; we write the 4 units under the column of units, and add the 1 ten to the column of tens. 1 ten and 4 tens are 5 tens, and 7 tens are 12 tens, and 3 tens are 15 tens, which equal 1 hundred and 5 tens; we write the 5 tens under the column of tens, and add the 1 hundred to the column of hundreds. 1 hundred and 9 hundreds are 10 hundreds and 8 hundreds are 18 hundreds, and 6 hundreds are 24 hundreds, which equal 2 thousands and 4 hundreds. We write the 4 hundreds under the column of hundreds, and place the 2 thousands on the left in the order of thousands. Hence, the sum is 2 thousands, 4 hundreds, 5 tens, and 4 units, or two thousand four hundred fifty-four.

1. In practice we name only the results. Thus, we say: 5, 8, 14; write the 4 and carry the 1; 1, 5, 12, 15; write the 5 and carry the 1; 1, 10, 18, 24; write 24. Hence the sum is 2454.

2. When the sum of any column is an exact number of tens, as 10, 20, 30, a cipher is written under the column added, and the 1, 2 or 3, etc., is added to the next column.

3. The addition of each column should be proved correct by adding it in the opposite direction from that in which it was first added.

RULE.— *Write the numbers to be added so that the figures of the same order stand in the same column, and draw a line beneath.*

Begin at the right, add the numbers expressed by the figures of each column separately, and write the sum underneath, if less than ten.

When the sum is ten or more, write the right-hand figure underneath, and add (carry) the number expressed by the other figure or figures with the numbers of the next column.

Write the whole sum of the last column.

PROOF.—*Add each column in the opposite direction from that in which it was added before, and if the work is correct the result of both additions will be the same.*

EXAMPLES.

Copy, add, and prove:

3.	4.	5.	6.	7.	8.
132	244	356	524	$2.46	4.35
325	421	542	361	7.31	6.40

9.	10.	11.	12.	13.	14.
126	347	716	$8.43	$7.24	8.95
321	526	813	7.65	5.68	7.27
428	413	594	4.72	2.47	6.16

15.	16.	17.	18.	19.	20.
4713	9275	4560	$73.82	$60.48	$95.65
3120	1413	1834	15.75	32.14	38.90
2235	2315	2643	12.40	75.37	57.26
1323	1241	3850	30.18	58.15	23.42

21.	22.	23.	24.	25.	26.
4216	2673	6107	8471	$96.42	$87.65
3724	6512	5462	6752	74.32	93.78
4637	7254	7126	7421	68.41	79.43
7152	6978	3418	5367	59.63	68.84
6208	5423	5726	8169	72.48	72.05
7531	4176	7345	6925	56.34	90.48
6143	3819	2964	8143	87.25	73.32
4265	8176	5873	5467	39.46	84.48

27.	28.	29.	30.	31.	32.
4632	2736	7216	7165	$65.47	$19.37
7184	1245	4734	4273	87.26	2.75
573	7802	2437	2568	92.35	.64
2764	3469	8	1673	46.52	37.29
87	726	19	6147	77.23	8.42
2418	83	4315	9432	86.29	16.37
503	7	782	708	71.54	29.15
5672	695	9641	67	89.91	.75

33. Add 2645, 7621, 4327, 6423, 5467.
34. Add 4856, 7358, 234, 579, 724, 3712, 48.
35. Add 3916, 4237, 8326, 47, 326, 516, 792.

36. Add 413, 6471, 3142, 718, 6437, 27, 5792.
37. Add 27, 15, 416, 592, 7126, 3748, 597, 694.
38. Add 723, 45, 9267, 486, 4172, 341, 7169, 764.
39. Add 7142, 37, 6415, 724, 9685, 723, 8476, 14.
40. Add 76, 437, 4816, 725, 789, 6438, 762, 897.

Copy, add, and prove:

41.	42.	43.	44.
274125	7146712	41267842	437654876
753671	3427164	62473149	721367480
246137	6435271	71341726	920672513
653946	8176325	46176317	568172486
271482	4264537	35724164	724650893
496317	3547362	72615736	546718047
720463	6128413	65423845	328571836
876517	7341867	76576437	872146827
914358	2368154	84370698	607234863

45.	46.	47.	48.
36782	432561	5764385	67245673
45613	641253	8120456	21346325
27164	302847	7346527	62513417
63843	576426	6842136	37142536
85267	752483	5634275	52834172
24651	634175	6724693	13521643
32789	863864	9653082	78143125
52438	732684	8724576	36254381
46275	680527	6387439	53672015
71523	923476	4871234	62132431
32764	391248	1726452	45326698
84657	728164	3874627	78093642
61246	815639	6732435	86245137
57438	234678	5873641	78901234
74219	756487	8462578	27486789
62105	602436	1723694	93827364
83246	575275	8237818	45361728
27513	634256	5426375	67273467
41673	456789	7367512	87928789
75918	836542	2435138	98346987

49. Add 46 thousand, 2 hundred; 72 thousand, 3 hundred 27; 65 thousand, 7 hundred 50; 87 thousand, 4 hundred 84; 25 thousand, 4 hundred 60.

50. Add 324 thousand, 6 hundred 85; 237 thousand, 4 hundred 18; 2 million, 46 thousand, 7 hundred 36; 14 million, 305 thousand, 4 hundred 90; 5 million, 6 thousand, 7 hundred 39.

PRACTICAL PROBLEMS.

42. 1. A farmer produced 75 bushels of rye, 250 bushels of wheat, 185 bushels of oats, and 360 bushels of corn. How many bushels of grain did he produce?

75
250
185
360
870

SOLUTION.—If he raised 75 bushels of rye, 250 bushels of wheat, 185 bushels of oats, and 360 bushels of corn, he raised in all the sum of 75 bushels, 250 bushels, 185 bushels, and 360 bushels, or 870 bushels.

2. A merchant bought beef for $560, pork for $275, and fish for $138. What was the entire cost?

3. In a certain book there are 330 pages, in another 360, and in another as many pages as in the other two. How many pages in the three books?

4. A man bought a horse for $250, a buggy for $175, and 8 cows for as much as he paid for the horse and buggy. What was the cost of all?

5. A farm that cost $4350, was sold for $850 more than it cost. How much was received for it?

6. A merchant paid his bookkeeper $1200 a year, two salesmen $650 each, and a boy $250. What amount of salaries did he pay?

7. A town lot was bought for $785. A house was built upon it which cost $2,350. For how much must the property be sold to gain $500?

8. In 1890, the population in Brooklyn was 806343. The population of New York City was 708958 more than that of Brooklyn. What was the population of both cities?

9. If Europe contains 3807195 square miles, Asia 17805146, Africa 11647428, America 14850400, and Oceanica 334780, how many square miles of land surface do these grand divisions comprise?

10. A grain dealer bought wheat for $7500, corn for $12080, and oats for $2000. He sold the wheat at a profit of $1500, the corn at a gain of $2020, and the oats at cost. What did he receive for the whole?

11. The area of New York is 49170 square miles, New Jersey 7815, Pennsylvania 45215, Delaware 2050, Maryland 12210, Vir-

ginia 42450, and West Virginia 24780 square miles. What is the total area of these states?

12. A man sold a house for $2780 and a lot for $875. He lost $510 on the house and $125 on the lot. What did he pay for both?

13. A, B, and C formed a partnership. A invested $2750, B invested $2875, and C invested as much as A and B together. How much did they all invest in the business?

14. A firm's resources are cash $1285, dry-goods $3769, personal accounts due the firm $1621, office furniture $205, and real estate $4760. What are the total resources of the firm?

15. A lady bought a stove for $23.50, a side-board for $18, a table for $9.50, dishes for $25, a bed-room suit for $40, a carpet for $13.75, a set of chairs for $9, a lamp for $2.65, and an organ for $85. What was the entire bill?

16. A dealer in live stock paid $3478 for horses, $2831.85 for cattle, and $1217.90 for sheep. He gained $850 on the sale of the horses, $760.90 on the cattle, and $215 on the sheep. How much did he receive for all?

17. Mr. M. deposited in the First National Bank of Philadelphia, on June 1, 1902, $375.50; on June 4, $680.75; on June 11, $700; on June 15, $225.65. He also deposited in the Globe National Bank, New York City, on June 18, $3167; on June 24, $1859. How much did he deposit in both banks?

SHORT METHODS IN ADDITION.

43. The process of addition may be much shortened by observing the following suggestions:

1. *In adding name merely the results.*

2. *When two or more numbers of a column can be easily grouped together, add their sum instead of adding each figure separately.*

3. *When a number is repeated consecutively in a column, multiply it by the number of times it is repeated, and add the result.*

4536	1ST METHOD.—Say, 3, 10, 16, 20, 22, 28; write the 8 and carry
3822	the 2; 2, 5, 10, 15, 20, 22, 25; write the 5 and carry the 2; etc.
7854	2D METHOD.—We may say 10, 20, 28, in which we group 3 and
1256	7, 6 and 4, and 2 and 6. In the second column, we may say 5, 15,
5657	22, 25, grouping 2 and 3, 5 and 5, 5 and 2; etc.
3633	3D METHOD.—In the second column with the 2 carried and
26758	grouping, five 5's are readily seen; in the third column, four 8's
322	and 5; in the last column, two 6's, 10 and 4; thus, 12, 22, 26.

ADDING BY TENS.

44. The method of *adding by tens* is helpful in the adding of long columns.

1. What is the sum of 897, 645, 257, 461, 798, 529, 397, and 489?

$8^{-}\ 9^{-}\ 7^{-}$	EXPLANATION.—In adding by 10's, the beginner should
$6\ 4\ 5$	place a dash at the upper right corner of each figure where
$2^{-}\ 5^{-}\ 7^{-}$	the sum equals or exceeds 10, thus keeping account of the
$4\ 6\ 1$	number of 10's and adding only the units which exceed each
$7^{-}\ 9^{-}\ 8^{-}$	10 of the column. Thus, in the first column, 9 and 7 are 16,
$5\ 2\ 9^{-}$	or 10 and 6. We place a dash at the 7 to mark the 10, and
$3^{-}\ 9^{-}\ 7^{-}$	consider only the 6; 6 and 9 are 10 and 5; mark for the 10
$4\ 8^{-}\ 9$	and consider only the 5, etc. Thus, in adding the first col-
$4\ 4\ 7\ 3$	umn, we say, 9, 6, 5, 3, 4, 1, 6, 3. We write the 3 and carry

to the next column 5, the number of dashes. Adding the second column, we say, 5 (carried), 3, 2, 4, 3, 9, 4, 8, 7. We write the 7, count the dashes and carry to the next column. Adding, we say, 5, 9, 2, 7, 4, 8, 0, 6, 4. We write the 4, and counting the dashes, write the number to the left, since it is the last column.

1. In adding, never fail to mark each 10 as you drop it, either by a dash or some other easily made mark.

2. The advantage in this method is that you can not only acquire the ability by a little practice, to add rapidly and accurately, but you may be interrupted at any point without inconvenience, if the unit figure is just placed to the right or above the last added figure; for the dashes marked while adding the column show the 10's of the column already added, so that there is really nothing to remember.

3. This method is recommended chiefly for beginners, who may find it difficult to carry in mind both the unit figure and the precise number of 10's accumulated. In all practice the student should attempt to keep clearly in mind the exact amount computed to any point, and a little effort of attention will, with experience, ensure this result. It is unnecessary to remark that the use of dashes is, like counting on the fingers, unworthy of the expert.

EXAMPLES.

Copy, add, and prove by all the methods:

2.	3.	4.	5.	6.	7.
34651	41328	31416	61428	32568	41368
42764	76241	14253	26374	41367	54267
51628	34156	63154	51413	52738	61287
34173	63415	41517	62197	31645	47513
71482	12413	13618	38412	74962	61438
31765	76235	73154	40987	35467	54321

8.	9.	10.	11.	12.	13.
71614	84678	98674	47650	12345	75438
26178	38496	8432	184	69827	42176
41571	64289	658	7256	35741	13715
25413	26437	31	34	69826	54123
68415	89762	8467	9	72413	21365
39186	62534	24003	34798	18576	64127
41542	40364	19867	1675	43214	37141
48698	50807	7046	18	35867	89376
73281	63987	29	637	72406	18421
17634	48071	7	6458	56829	39617
52683	32961	8360	17330	31724	89898

ADDING BY HUNDREDS.

45. The method of adding by hundreds is adapted to add long columns of figures. By this method a dash is placed at upper right hand of the figure at which each hundred is obtair

1. Find the sum of the following:

7 4 5	
8 6 7	
9 7 6	
7 8 9	
9 6 7	
8 3 8	
9 7 6	
6 9 4	
7 4 9	
9 5 8	
8 4 7	
7 7 6	
9 5 4	
6 4 7	
8 7 9	
7 8 6	
8 7 8	
9 9 6	
8 5 8	
7 9 6	
8 7 6	
9 8 7	
1 8 8 3 9	

EXPLANATION.—In adding the first column, we b at the bottom and say, 7, 13, 19, 27, 33, 41, 47, 56, 63 73, 80, 88, 97, 1 (dash), 7, 15, 22, 31, 37, 44, 49. Glan at the column we see 1 dash. Hence, the sum of the umn is 149. We write the 9 and carry 14 to the column.

Adding the second column we say, 14 (carried), 22 38, 43, 52, 59, 67, 74, 78, 83, 90, 94, 99, 3 (dash), 12, 19 28, 36, 43, 49, 53. Glancing at the column we see 1 d Hence the sum of the column is 153. We write th and carry 15 to the next column.

Adding the third column we say, 15 (carried) 24, 32 47, 56, 64, 71, 79, 85, 94, 1 (dash), 9, 18, 25, 31, 40, 48 64, 73, 81, 88. Glancing at the column we see 1 d Hence the sum of the column is 188, all of which is w ten since it is the last column.

Hence the sum is 18839.

1. The advantage of this method over the metho adding by tens is that scarcely any time is consume making dashes, and hence the additions may be n quickly performed.

2. For practice, the student should add by hund examples 45, 46, 47, and 48, under article 40, and m other exercises of long columns for himself.

RAPID ADDITION.

46. There are different methods of rapid addition, but all of them are based on *grouping* or *sum reading*. No one can attain the highest results in addition so long as he continues to add one figure at a time. This is *sum spelling* instead of *sum reading*.

Thus, instead of looking at 3 and 4 as separate and distinct figures, their sum, 7, should be seen with the readiness that we pronounce the word "seven" from a mere glance at the word. This is true of the sum of any two figures; and it is not a very difficult task to learn to recognize at a glance the sum of any three figures.

TABLE NO. 1.

47. This table shows all the different combinations of two numbers to produce sums from 2 to 10 inclusive.

1					
1					2
1	2				
2	1				3
1	2	3			
3	2	1			4
1	2	4	3		
4	3	1	2		5
1	4	3	2	5	
5	2	3	4	1	6
1	5	3	2	6	
6	2	4	5	1	7
1	6	3	4	5	
7	2	5	4	3	8
1	7	3	5	6	
8	2	6	4	3	9
1	7	2	5	4	
9	3	8	5	6	10

The student should not attempt to learn Table No. 2 until he has thoroughly mastered Table No. 1.

TABLE NO. 2.

48. This table shows all the different combinations of two numbers to produce sums from 10 to 18 inclusive.

1	8	3	6	5	
9	2	7	4	5	10
2	8	4	6	7	
9	3	7	5	4	11
3	8	5	6	9	
9	4	7	6	3	12
4	8	6	5	7	
9	5	7	8	6	13
5	8	7	6	9	
9	6	7	8	5	14
6	8	9	7		
9	7	6	8		15
7	8	9			
9	8	7			16
8	9				
9	8				17
9					
9					18

The student should practice upon Table No. 2 until he can call the sum of any combination instantly.

49. With the two tables mastered, the student will know the sum of any two numbers, below ten, at sight; and with a little practice he may know the sum of any four. If, for example, he knows 3 and 4 as 7, and 5 and 4 as 9, and 7 and 9 as 16, he will know the sum of 3 and 4 and 5 and 4 by simply looking at them.

TABLE NO. 3.

50. This table contains all groups of three figures. The student should thoroughly master them so that he can instantly call their sum without any effort at addition. Let the student practice upon them in the same manner as directed for the mastery of Tables Nos. 1 and 2.

1												
1												
1												3
1	1	2										
1	2	1										
2	1	1										4
1	1	1	2	3	2	1						
1	2	3	1	1	2	1						
3	2	1	2	1	1	3						5
1	1	2	3	1	2	1	4	2	3			
1	2	2	1	4	3	2	1	1	2			
4	3	2	2	1	1	3	1	3	1			6
1	5	3	2	1	2	3	1	2	3	3	4	
2	1	1	3	5	4	2	3	2	3	2	2	
4	1	3	2	1	1	2	3	3	1	2	1	7
1	2	1	3	2	1	1	4	3	4	2	2	
3	1	6	2	4	4	5	1	3	2	5	3	
4	5	1	3	2	3	2	3	2	2	1	3	8
1	4	2	1	5	2	3	2	3	1	5	4	
2	1	4	7	1	5	3	6	2	5	2	2	
6	4	3	1	3	2	3	1	4	3	2	3	9
1	3	3	2	3	4	3	1	7	1	4	4	
4	1	4	1	5	2	4	8	2	5	4	3	
5	6	3	7	2	4	3	1	1	4	2	3	10

1	2	4	5	2	3	3	4	5	1	2	4	
3	8	1	5	7	2	5	3	4	1	6	5	
7	1	6	1	2	6	3	4	2	9	3	2	11
1	2	9	4	3	5	5	2	3	4	1	4	
5	8	1	4	6	2	3	4	7	1	3	5	
6	2	2	4	3	5	4	6	2	7	8	3	12
1	4	3	8	4	6	3	2	5	2	5	2	
5	3	1	4	5	6	7	2	3	7	2	8	
7	6	9	1	4	1	3	9	5	4	6	3	13
1	5	4	6	1	3	2	3	8	6	3	2	5
5	4	1	4	6	5	3	4	2	6	8	5	6
8	5	9	4	7	6	9	7	4	2	3	7	3 14
1	4	2	4	5	6	5	7	2	7	3	8	5
6	5	4	7	5	3	1	1	8	2	3	4	3
8	6	9	4	5	6	9	7	5	6	9	3	7 15
1	5	7	4	6	3	4	5	6	6	2	2	4
7	3	7	3	1	7	8	4	5	4	5	6	7
8	8	2	9	9	6	4	7	5	6	9	8	5 16
1	6	2	5	6	1	2	4	8	7	3	5	8
7	5	6	5	4	8	7	9	3	3	5	8	2
9	6	9	7	7	8	8	4	6	7	9	4	7 17
1	4	9	6	8	5	3	4	5	3	6	8	2
8	5	2	6	2	6	6	7	8	7	4	3	7
9	9	7	6	8	7	9	7	5	8	8	7	9 18
1	4	8	5	6	7	6	3	6	3	8	6	9
9	7	2	9	5	5	7	8	4	7	6	9	7
9	8	9	5	8	7	6	8	9	9	5	4	3 19
2	6	8	6	5	8	9	5	7	9	9	8	7
9	8	4	7	7	3	4	9	8	2	6	8	5
9	6	8	7	8	9	7	6	5	9	5	4	8 20
3	8	7	8	9	6	8	5	9	6	5	9	7
9	5	7	4	5	9	7	7	4	6	8	7	6
9	8	7	9	7	6	6	9	8	9	8	5	8 21

4	8	5	7	8	9	8	8	9	7	9	6	7	
9	6	8	6	7	4	8	5	7	8	5	9	7	
9	8	9	9	7	9	6	9	6	7	8	7	8	22
5	8	9	7	9	8	8	9	8	6	7	9	7	
9	7	8	9	5	8	6	7	8	8	7	9	8	
9	8	6	7	9	7	9	7	7	9	9	5	8	23
6	8	9	9	7	9	7	9	8					
9	8	7	6	8	9	9	8	7					
9	8	8	9	9	6	8	7	9					24
7	8	9	8	9	9								
9	9	7	8	9	8								
9	8	9	9	7	8								25
8	9	9											
9	9	8											
9	8	9											26
9													
9													
9													27

DRILL EXERCISES.

51. The following list contains all the groups of three figures.

The student should so master these exercises that he may by merely glancing at the figures call their sum. In practicing learn to see and read the sum of each combination at a glance. Thus it may be seen at once that $\frac{1}{1}$ is 3; that $\frac{2}{1}$ is 9, and so on. Run over them rapidly. Practice with close attention is the key to success.

1	1	1	1	1	1	2	1	5	3	2
1	1	1	2	1	2	2	2	1	1	3
1	2	3	2	4	3	2	4	1	3	2
1	2	1	3	2	1	4	2	1	5	2
3	1	6	2	4	2	1	4	7	1	5
4	5	1	3	2	6	4	3	1	3	2
3	1	3	3	2	3	4	3	1	1	2
3	4	1	4	1	5	2	4	8	3	8
3	5	6	3	7	2	4	3	1	7	1

4	5	2	3	3	4	5	1	1	2	9
1	5	7	2	5	3	4	1	5	8	1
6	1	2	6	3	4	2	9	6	2	2
4	3	5	5	2	3	4	1	1	4	3
4	6	2	3	4	7	1	3	5	3	1
4	3	5	4	6	2	7	8	7	6	9
8	4	6	3	2	5	2	5	2	1	5
4	5	6	7	2	3	7	2	8	5	4
1	4	1	3	9	5	4	6	3	8	5
4	6	1	3	2	3	8	6	3	2	1
1	4	6	5	3	4	2	6	8	5	6
9	4	7	6	9	7	4	2	3	7	8
4	2	4	5	6	5	7	2	7	3	8
5	4	7	5	3	1	1	8	2	3	4
6	9	4	5	6	9	7	5	6	9	3
5	1	5	7	4	6	3	4	5	6	6
3	7	3	7	3	1	7	8	4	5	4
7	8	8	2	9	9	6	4	7	5	6
2	2	1	6	2	5	6	1	2	4	8
5	6	7	5	6	5	4	8	7	9	3
9	8	9	6	9	7	7	8	8	4	6
7	3	5	1	4	9	6	8	5	3	4
3	5	8	8	5	2	6	2	6	6	7
7	9	4	9	9	7	6	8	7	9	7
5	3	6	1	4	8	5	6	7	6	3
8	7	4	9	7	2	9	5	5	7	8
5	8	8	9	8	9	5	8	7	6	8
6	3	2	6	8	6	5	8	9	5	3
4	7	9	8	4	7	7	3	4	9	9
9	9	9	6	8	7	8	9	7	6	9
8	7	8	9	6	8	4	8	5	7	8
5	7	4	5	9	7	9	6	8	6	7
8	7	9	7	6	6	9	8	9	9	7

5	8	9	7	6	8	9	7	8	8	9
9	7	8	9	9	8	7	9	9	9	9
9	8	6	7	9	8	8	9	8	9	9

ADDING BY HUNDREDS.

52. If the student has now learned to see readily and accurately the sum of any combination of several figures found in the tables and exercises on pages 27 to 32 he will be able to apply the grouping method in adding by hundreds, which will now be explained.

To more clearly show the comparative advantage of grouping we will take the same example explained on p. 26.

	15	14	
	7	4	5
	8	6	7
	9	7	6
	7	8	9
	9	6	7
	8	3	8
	9	7	6
	6	9	4
	7	4	9
	9	5	8
	8	4	7
	7	7	6
	9	5	4
	6	4	7
	8	7	9
	7	8	6
	8	7	8
	9	9	6
	8	5	8
	7	9	6
	8	7	6
	9	8	7
1	8	8	3	9

EXPLANATION.—We will here begin at the top of the units column, instead of at the bottom as heretofore, and add downwards by groups of two figures. Thus, we say, 12, 27, 42, 52, 69, 82, 93, 8 (dash), 22, 36, 49. Glancing up the column, we see one dash. Hence, the sum is 149. Set down the 9 and carry 14 to the next column by writing it above the first figure of that column in light pencil figures. Before proceeding, it is well to prove the addition of the units column. This may be done by reversing the process just used. Commence at the bottom and add upwards by groups of two figures, saying, 13, 27, 41, 56, 67, 80, 97, 7, 22, 37, 49—149. The result is the same; therefore the correctness of both additions may be assumed.

Next add the tens column, saying 14 (carried) 24, 39, 48, 64, 73, 84, 93, 8 (dash), 24, 38, 53. Glancing over this column we see 1 dash—hence the sum is 153. We set down 3 and carry 15, writing it at the top of the next column, as before, and thus keeping the whole sum of the column in view as a convenient aid to the memory. Now prove the addition of this column in the same manner as the units column was proved by commencing at the bottom and adding upwards—thus, 14 (carried), 29, 43, etc.

Adding the hundreds column, we begin again at the top and say, 15 (carried), 30, 46, 63, 78, 94, 9 (dash), 24, 39, 56, 71, 88. We see only 1 dash on looking up the column. Hence the sum is 188. As this is the last column to be added, write these three figures under the column. Now prove as before by adding the column in the opposite direction.

To add the above example by groups of three figures we say, in footing up the units column—commencing at the top—18, 42, 61, 82, 2 (dash) 22, 49—149. Prove. Adding the tens column we say, 14 (carried), 31, 48, 68, 100 (dash), 24, 45, 53—153. Prove. Adding the hundreds column we say, 15 (carried), 39, 63, 85, 9 (dash), 32, 56, 79, 88—188. Prove.

2. In footing long columns always prove the correctness of your work

adding each column in the opposite direction before proceeding to the next column.

2. Instead of making a dash to mark the hundreds the sum in small figures may be written lightly in pencil at the upper right-hand corner of the topmost figure of the group at which the hundred is reached. This method is suggested as a time saver where the work of adding long columns is subject to frequent interruptions, as it indicates in writing the first number of each succeeding hundred count.

RECORDING PARTIAL ADDITIONS.

53. Accountants who are liable to interruptions when adding long columns may save themselves extra trouble by writing the sum of each column in light lead pencil figures to the right of the numbers to be added.

1. Find the sum of 576, 897, 679, 786, 878, and 697.

576	
897	
679	
786	48
878	51
697	45
4513	

EXPLANATION.—The sum of the units column is 43, which we place to the right in light lead pencil figures. Carrying 4, the sum of the tens column is 51, which is placed under the 43. Carrying the 5, the sum of the hundreds column is 45, which is placed under the 51. The total sum is found by taking the last number and the unit figure of the other partial sums, reading upward. Hence, the sum of the columns is 4513.

THE CASHIER'S METHOD.

54. The following method, which we prefer to call the Cashier's method (though it is known by different names) of recording the sums of the columns separately is worthy of consideration.

1. Find the sum of the following:

$35.67		
42.28		
26.45		
33.72	32	
54.16	23	
29.34	9	
$221.62	1 9	$ 2 2 1.6 2

EXPLANATION.—The sum of the first column is 32; of the second 23, which is written under the sum of the first column, and one place to the left of it. The sum of the third column is 29, which is written under the sum of the second column, and one place to the left of it; etc. Then, adding the partial sums, we have the entire sum.

By this method, in adding the columns, there is nothing to carry.

OTHER METHODS OF ADDING.

55. We give below perhaps the best methods that can be devised of adding several columns at once. But it should be remembered that in adding *long* columns in which there are few ciphers

no other method can equal the single column method. In adding two or more columns at the same time we never read the sum of more than two numbers at once, while in adding a single column at a time, as has already been shown, the sum of three or four numbers may be read at once.

HOW TO ADD TWO FIGURE COLUMNS.

56. The following method of finding the sum of two figure columns will be found of practical value to the student.

16	45	52
42	34	36
35	62	71
21	27	18

EXPLANATION.—Commence at the lower left corner and read to the upper right corner of the numbers to be added. Do not add in the ordinary way, but simply glance at the numbers and read the results 58, 79, 88, 56, etc.

The student should make many similar exercises and practice on them until he can read the sums as easily as printed words.

34	26
28	47
58	45
35	35

EXPLANATION.—When the sum of the digits to the right make as much as ten, which the student can see by a mere glance, he should call the sum of the digits to the left one more. Hence, the sums of the exercises here given are read 62, 73, 93, 80.

When the sum of the digits one place to the right is 9, it is necessary to look ahead two places to see if the sum of those digits is 10 or more. If so, we must call the sum of the digits to the left one more.

57. After the student has had abundant exercises similar to those just given, he should prepare a large number of other exercises like the following, and practice upon them until he can write the sums quickly and accurately from *left* to *right*.

1.	2.	3.	4.
24635	172156	54367	723589
47586	564678	87348	291423

5.	6.	7.	8.
374628	4236854	4653721	37145627
983245	2987431	6524786	23482195

HOW TO ADD TWO COLUMNS AT ONCE.

58. It will be found a great help in adding two columns at once to divide the numbers into syllables. Thus, instead of saying thirty-two (32) in adding, we say three–two (3–2); 46 call four–six (4–6); 63 call six–three (6–3), etc.

32 19-6
25 16-4
46 13-9
34 9-3
59
196

EXPLANATION.—Commencing at the lower left corner of the numbers to be added, we read nine-three (9-3). Now, conceiving the 9-3 to be under the 4-6, we read 13-9; the 13-9 under the 2-5, we read 16-4; and the 16-4 under the 3-2, we read 19-6, or 196, the sum of the two columns.

To add by this method the student should always fix his mind on the number conceived to be under the number to be added, and then read their sum. The numbers are to be *read*, not added. It will require considerable practice to acquire celerity in adding by this method.

Find the sum of the following by adding two columns at once:

1.	2.	3.	4.	5.	6.	7.	8.
21	13	15	23	34	22	31	42
34	41	32	15	22	31	23	14
23	32	41	23	11	40	12	36
42	57	16	40	26	34	47	73

9.	10.	11.	12.	13.	14.	15.	16.
24	31	42	23	34	17	21	33
12	13	17	46	53	32	30	45
13	22	30	55	15	54	32	16
22	33	53	34	25	20	14	22
10	20	45	43	31	53	42	34
25	24	21	20	42	12	27	43
37	45	14	61	36	65	30	25

59. HOW TO ADD THREE COLUMNS AT ONCE.

$1.23
2.31
3.40
5.32

EXPLANATION.—We place the pencil between the 3 and 4 of the second number from the bottom, and read $8.72; then up between the 2 and 3, read $11.03; and then between the 1 and 2, read $12.26.

60. HOW TO ADD FOUR COLUMNS AT ONCE.

$12.25
30.10
51.12
63.30
$156.77

EXPLANATION.—We place the pencil between the 1 and 1, and read $114.42; then up between the 0 and 1, read $144.52; and then up between the 2 and 2, read $156.77, which is the sum of the four columns.

The student who practices upon the examples given below and others similar to them, and who in so doing learns to concentrate his attention upon the work, will soon astonish himself and others at his skill in reading the sums of accounts of at least four items of figures.

l the sum of the following by adding all the columns o
xample at once:

1.	2.	3.	4.
$2.13	$3.20	$5.12	$8.25
1.10	2.13	6.03	1.13
5.21	1.15	7.10	4.22
3.14	4.31	8.41	3.60

5.	6.	7.	8.
$11.20	$14.13	$31.11	$13.15
13.15	20.12	10.40	21.20
22.02	25.30	16.12	30.14
32.13	17.25	21.13	46.30

9.	10.	11.	12.
$3.25	$2.12	$4.00	$13.15
5.30	4.20	5.10	14.10
6.00	5.10	6.30	25.10
7.15	6.13	7.25	36.43
2.10	3.40	4.50	42.27

13.	14.	15	16.
$22.30	$1.30	$13.20	$32.10
12.10	14.01	16.40	13.20
13.20	12.12	5.10	40.05
14.30	.30	.45	.30
15.15	.70	1.20	6.15
32.13	2.10	3.30	21.40

HOW TO ADD HORIZONTALLY.

61. It is sometimes necessary to add items horizontally. is especially true of books containing certain ruled colum the various kinds of items. To avoid the necessity of re-arra the items so as to add them perpendicularly, and to avoid as as possible the liability to err the following method is mended:

Add horizontally the following:

1. 2364, 156, 54, 1723, 631. Ans. 4928.

EXPLANATION.—Since it is customary to read from left to right, we begin at the left with the units figure, 4, of the first number, and add to it successively the units 6, 4, 3, and 1 of the other numbers, making 18. We write down the 8 and carry the 1 to the ten's figure, 6, of the first number, and add to it successively the tens 5, 5, 2, and 3 of the other numbers, making 22. We write down the 2 and carry the other 2 to the hundred's figure, 3, of the first number, and add to it successively the hundreds 1, 7, and 6 of the other numbers containing hundreds, making 19. We write down the 9 and carry the 1 to the thousand's figure 2, of the first number, and add it to the thousand's figure, 1, of the fourth number (the other numbers not having a thousand's place), making 4, which we write. Hence, the sum of the numbers is 4928.

Copy and add horizontally the following:

2. 4678, 537, 642, 40354, 6342, 96.
3. 734, 865, 732, 672, 8904, 6742.
4. 1043, 27, 462, 8409, 76431, 27, 4896.
5. 720, 42, 76, 871, 5630, 4725, 87.
6. 1726, 307, 4268, 9725, 32, 403, 26.
7. 34268, 28, 135, 6724. 5962, 306.
8. 59260, 172, 427, 3964, 7685, 703.
9. 14032, 675, 1089, 7638, 976472.
10. 89642, 4235, 72168, 73, 5, 987.
11. 64356, 742345, 67345, 9643, 756, 7856, 723.
12. 72, 814, 1734, 6896, 4672, 596, 754, 9147.
13. 7627, 42306, 76248, 964, 505, 273, 45, 1005.
14. 53678, 904, 672, 96, 7, 49, 365, 27, 652.
15. 345, 768, 9764, 642, 567, 8952, 354, 1024.

16. Add vertically and horizontally the following statement of receipts:

	Mon.	Tues.	Wed.	Thurs.	Friday.	Sat.	Total.
	$732.14	$462.48	$800.45	$100.36	$1000.50	$465.30	
	10.45	103.24	74.34	9.46	741.16	786.52	
	16.76	16.35	85.16	972.18	806.30	394.15	
	503.41	17.52	98.45	76.87	14.72	617.25	
	608.56	167.18	127.42	14.72	315.45	872.48	
	796.45	19.76	87.69	176.14	96.72	13.65	
	1264.72	134.67	537.42	985.76	568.15	1132.80	
	4367.20	573.62	18.17	314.83	87.43	614.20	
	7415.36	1432.35	38.42	12.64	15.16	872.82	
Total...							

SUBTRACTION.

62. Subtraction is the process of finding the difference between two numbers.

63. The **Minuend** is the number from which we subtract.

64. The **Subtrahend** is the number to be subtracted, or taken from the minuend.

65. The **Difference**, or **Remainder**, is the result obtained after subtracting.

66. The **Sign of Subtraction** is a short horizontal line —, and is read *minus*. It denotes that the number after it is to be taken from the number before it.

Thus, $7 - 4 = 3$ is read 7 minus 4 equals 3, and means that 4 subtracted from 7 leaves 3.

67. PRINCIPLES.—1. *Only like numbers can be subtracted.*

2. *The difference or remainder must be like the minuend and subtrahend.*

3. *The sum of the remainder and the subtrahend is equal to the minuend.*

SUBTRACTION TABLE.

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	13
2	3	4	5	6	7	8	9	10	11	12	13	14
3	4	5	6	7	8	9	10	11	12	13	14	15
4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	7	8	9	10	11	12	13	14	15	16	17
6	7	8	9	10	11	12	13	14	15	16	17	18
7	8	9	10	11	12	13	14	15	16	17	18	19
8	9	10	11	12	13	14	15	16	17	18	19	20
9	10	11	12	13	14	15	16	17	18	19	20	21
10	11	12	13	14	15	16	17	18	19	20	21	22
11	12	13	14	15	16	17	18	19	20	21	22	23
12	13	14	15	16	17	18	19	20	21	22	23	24

EXPLANATION.—This table is made the same as the addition table. The numbers in the left-hand column may be regarded as the subtrahend, and the numbers in the horizontal columns opposite the subtrahends as the minuends. The remainders will be found across the top.

Thus, 1 from 1 leaves 0; 1 from 2 leaves 1; 1 from 3 leaves 2; 1 from 4 leaves 3; 1 from 5 leaves 4, etc.

The order may be changed so that the numbers across the top may be regarded as the subtrahends, and the numbers on the left as the remainders. The pupil should work out the remainders for himself, and then thoroughly master the table.

DRILL EXERCISES.

68. The student should practice these exercises daily until he can tell the results instantly.

1.	$8 + ? = 15$	$15 - 8 = ?$	$8 + ? = 18$	$18 - 8 = ?$	
2.	$7 + ? = 16$	$16 - 7 = ?$	$9 + ? = 15$	$15 - 9 = ?$	
3.	$4 + ? = 12$	$12 - 4 = ?$	$5 + ? = 16$	$16 - 5 = ?$	
4.	$6 + ? = 15$	$15 - 6 = ?$	$7 + ? = 15$	$15 - 7 = ?$	
5.	$9 + ? = 13$	$13 - 9 = ?$	$8 + ? = 17$	$17 - 8 = ?$	
6.	$8 + ? = 17$	$17 - 8 = ?$	$9 + ? = 19$	$19 - 9 = ?$	
7.	$7 + ? = 13$	$13 - 6 = ?$	$5 + ? = 14$	$14 - 9 = ?$	
8.	$9 + ? = 16$	$16 - 9 = ?$	$8 + ? = 14$	$14 - 6 = ?$	
9.	$11 + ? = 18$	$18 - 7 = ?$	$10 + ? = 19$	$19 - 7 = ?$	
10.	$12 + ? = 21$	$21 - 8 = ?$	$12 + ? = 23$	$20 - 9 = ?$	
11.	$8 - 5$	$10 - 7$	$5 - 3$	$9 - 3$	$7 - 5$
12.	$9 - 8$	$8 - 2$	$5 - 2$	$10 - 9$	$9 - 7$
13.	$10 - 4$	$11 - 3$	$12 - 4$	$6 - 5$	$11 - 9$
14.	$16 - 6$	$12 - 5$	$10 - 5$	$17 - 6$	$15 - 8$
15.	$17 - 4$	$18 - 6$	$13 - 9$	$14 - 4$	$12 - 7$
16.	$18 - 8$	$14 - 2$	$11 - 7$	$13 - 7$	$14 - 3$
17.	$16 - 7$	$13 - 2$	$15 - 4$	$19 - 7$	$11 - 8$
18.	$10 - 6$	$14 - 9$	$17 - 5$	$14 - 6$	$13 - 4$
19.	$19 - 7$	$18 - 7$	$17 - 4$	$16 - 8$	$9 - 9$
20.	$15 - 6$	$16 - 7$	$18 - 9$	$19 - 8$	$17 - 8$

21. Subtract by 2's from 24 to 2; thus, 24, 22, 20, 18, etc.
22. Subtract by 3's from 30 to 0. From 37 to 1.
23. Subtract by 4's from 48 to 8. From 46 to 2.
24. Subtract by 5's from 60 to 0. From 57 to 2.
25. Subtract by 6's from 72 to 6. From 69 to 3.
26. Subtract by 7's from 84 to 0. From 75 to 5.
27. Subtract by 8's from 96 to 8. From 86 to 6.

WRITTEN EXERCISES.

69. 1. Subtract 245 from 698.

Minuend, 698
Subtrahend, 245
Remainder, 453

EXPLANATION.—For convenience, we write the subtrahend under the minuend, placing figures of the same order in the same column. We draw a line beneath and begin with units to subtract. 5 units from 8 units leave 3 units, which we write under the units; 4 tens from 9 tens leave 5 tens, which we write under the tens; 2 hundreds from 6 hundreds leave 4 hundreds, which we write under the hundreds. Hence, the difference, or remainder is 453.

PROOF.—453, the remainder, plus 245, the subtrahend, equals 698, the minuend. Hence, the result in correct (Prin. 3).

2. From 852 subtract 437.

852
437
415

METHOD BY BORROWING.—We write the subtrahend under the minuend as before, and begin at the right to subtract. We cannot take 7 units from 2 units; but we can take 1 ten, which is equal to 10 units, from the 5 tens and unite it with the 2 units, making 12 units. 7 units from 12 units leave 5 units, which we write under the units.

Since 1 ten was taken from the 5 tens, there are but 4 tens left. 3 tens from 4 tens leave 1 ten, which is written under the tens. 4 hundreds from 8 hundreds leave 4 hundreds, which are written under the hundreds. Hence, the remainder is 415.

PROOF.—415, the remainder, plus 437, the subtrahend, equals 852, the minuend. Hence, the result is correct.

METHOD BY ADDING TEN.—Since 7 units cannot be subtracted from 2 units, we will therefore add 10 units to the 2 units, making 12 units. 7 units from 12 units leave 5 units. Now, since we have added 10 units, or 1 ten, to the minuend, our remainder will be 1 ten too large. Hence, to secure the correct remainder, we must add 1 ten to the subtrahend. 1 ten and 3 tens are 4 tens. 4 tens from 5 tens leave 1 ten; and 4 hundreds from 8 hundreds leave 4 hundreds. Hence, the remainder is 415.

In practice, we say, 7 from 2, impossible, but 7 from 12 leaves 5, 3 from 4 (or 4 from 5) leaves 1, and 4 from 8 leaves 4. Answer, 415.

RULE.— *Write the subtrahend under the minuend, so that figures of the same order stand in the same column, and draw a line beneath.*

Begin at the right, subtract the number expressed by each figure of the subtrahend from the number expressed by the figure above it, and write the difference beneath.

If any figure of the subtrahend express a greater number than the figure above it, add ten to the latter, and then subtract. Take one from the number expressed by the next figure of the minuend or

add one to the number expressed by the next figure of the subtrahend, and proceed as before.

PROOF.—*Add the remainder to the subtrahend and, if the work is correct, the sum will equal the minuend.*

EXAMPLES.

Copy, subtract, and prove:

3.	4.	5.	6.	7.
45	76	87	68	96
21	34	45	34	52
8.	**9.**	**10.**	**11.**	**12.**
235	427	381	536	687
123	214	150	324	444
13.	**14.**	**15.**	**16.**	**17.**
453	368	576	637	789
132	124	323	315	427
18	**19.**	**20.**	**21.**	**22.**
693	894	987	423	725
231	741	567	218	492
23.	**24.**	**25.**	**26.**	**27.**
$8.35	$7.75	$6.05	$8.68	$9.38
6.50	5.85	3.72	4.59	4.45
28.	**29.**	**30.**	**31.**	**32.**
3057	4876	5064	8470	9632
1735	2493	2178	5492	5827
33.	**34.**	**35.**	**36.**	**37.**
$75.26	$67.45	$59.72	$84.57	$48.37
64.19	43.80	38.57	65.25	29.28
38.	**39.**	**40.**	**41.**	**42.**
$88.65	$57.32	$63.42	$72.16	$70.85
75.49	39.27	35.37	48.50	48.68
43.	**44.**	**45.**	**46.**	**47.**
37456	58317	67593	83486	98375
21567	27582	52768	28915	84692

48. From 7000 subtract 4565.

$$\begin{array}{r} {}^{6}\,{}^{9}\,{}^{9}\,{}^{10} \\ 7\,0\,0\,0 \\ 4\,5\,6\,5 \\ \hline 2\,4\,3\,5 \end{array}$$

EXPLANATION.—We cannot subtract 5 units from 0 units, and since there are no tens nor hundreds, 1 thou-and must be changed into hundreds, leaving 6 thousand; 1 of the hundreds must be changed into tens, leaving 9 hundreds; and one of the tens into units, leaving 9 tens. The expression 6 thousands, 9 hundreds, 9 tens, and 10 units is thus equal to the minuend. 5 units from 10 units leave 5 units, which we write under the units; 6 tens from 9 tens leave 3 tens, which we write under the tens; 5 hundreds from 9 hundreds leave 4 hundreds, which we write under the hundreds; 4 thousands from 6 thousands leave 2 thousands, which we write under the thousands. Hence, the result is 2435.

Subtract and prove:

49. 6000 — 3847.
50. 7000 — 4672.
51. 3500 — 1743.
52. 4600 — 2478.
53. 8000 — 6345.
54. 5000 — 3408.
55. 9000 — 7356.
56. 46031 — 11234.
57. 40637 — 21463.
58. 74006 — 53764.
59. $4500.75 — $1735.39.
60. $6000.50 — $2400.70.
61. $7080.48 — $4236.95.
62. $8006.00 — $5237.45.
63. $9050.60 — $7234.56.
64. 67843500 — 1278934.
65. 82000400 — 5234638.
66. 59003040 — 7030425.
67. 30040506 — 8214306.
68. 90007003 — 6400294.

PRACTICAL PROBLEMS.

70. 1. A man had 380 bushels of corn, and sold 165 bushels. How many bushels had he left?

$$\begin{array}{r} 380 \\ 165 \\ \hline 215 \end{array}$$

SOLUTION.—If a man had 380 bushels of corn and sold 165 bushels, there remained the difference between 380 bushels and 165 bushels, which is 215 bushels.

2. A farmer who owned 750 acres of land, sold 275 acres. How many acres remained?

3. A merchant sold for $4563 goods which cost him $3645. How much did he gain?

4. A ship that cost $35742 was sold for $40000. What was the gain?

5. George Washington was born in 1732, and died in 1799. How old was he at his death? How long since he died?

6. Galileo was born in 1564, and invented the telescope in 1610. How old was he when he invented it? How long since the tel*escope was invented?*

7. A man owned property valued at $65475, and owed $18869. How much was he worth more than he owed?

8. In 1900 the population of New York was 3437202, and that of Chicago 1698575. How much greater was the population of New York than that of Chicago?

9. A man having a fortune of $18000, willed $7500 to his wife, $4500 to his daughter, and the remainder to his son. How much did he leave to his son?

10. A merchant having $12360 in a bank, drew out $3475, and afterward put in $4720. How much had he then in the bank?

11. Mr. A bought a farm for $8450. He spent $2340 for improvements, paid $87 for taxes, and then sold it for $12000. What was the gain?

12. A grocer's profits last year were on fruits $350; on sugars $227; on potatoes $420; on syrups $165; on fish $137; on general merchandise $3785. He paid for rent $300, insurance $125, salaries $775, and incidental expenses $195. What was his net profit?

13. I went to a store and bought a hat for $1.75, tie for $0.45, 6 collars for $1.25, 6 linen handkerchiefs for $1.87, and a cane for $2.25. I handed the clerk a twenty-dollar bill. What change was given back?

14. A firm's total resources are $27350, and the total liabilities $9875. What is the net capital of the firm?

15. I had in bank on Monday morning $750. On Tuesday I deposited $230; on Wednesday I deposited $475; Thursday I checked out $322.60; Friday I checked out $172.25. How much remained in bank?

16. A farmer received $750 for his wheat, $145.85 for his oats, $190 for his rye, and $213.75 for his corn. He paid $115 for taxes, $80 for labor, and $165 for other expenses. How much did he save?

17. A merchant bought goods of a manufacturer amounting to $3465, and paid him cash $1637.85. How much did the merchant still owe?

18. Mr. H. began business with $4000. The first year he lost $1450, the second year he gained $675, the third year he gained $2285.68, and the fourth year he gained $1750. What was his capital at the end of the fourth year?

MULTIPLICATION.

71. Multiplication is a short process of finding the result where equal numbers are added together a given number of times.

Multiplication is a short way of adding equal numbers.

72. The **Multiplicand** is the number to be multiplied.

73. The **Multiplier** is the number by which to multiply.

74. The **Product** is the result of the multiplication.

75. The **Factors of the Product** are the multiplicand and the multiplier.

76. The **Sign of Multiplication** is an oblique cross $\times$. It is read *multiplied by* when the multiplicand precedes it, and *times* when the multiplier precedes the sign.

Thus, 5×2 is read 5 multiplied by 2 when 5 is the multiplicand, but it is read 5 times 2 when 5 is the multiplier.

77. PRINCIPLES.—1. *The multiplicand may be an abstract or a concrete number.*

2. *The multiplier must always be regarded as an abstract number.*

3. *Either factor may be used as the multiplier when both are abstract.*

4. *The product is always similar to the multiplicand.*

In practice, the smaller number is generally used as the multiplier.

MULTIPLICATION TABLE.

2	3	4	5	6	7	8	9	10	11	12
4	6	8	10	12	14	16	18	20	22	24
6	9	12	15	18	21	24	27	30	33	36
8	12	16	20	24	28	32	36	40	44	48
10	15	20	25	30	35	40	45	50	55	60
12	18	24	30	36	42	48	54	60	66	72
14	21	28	35	42	49	56	63	70	77	84
16	24	32	40	48	56	64	72	80	88	96
18	27	36	45	54	63	72	81	90	99	108
20	30	40	50	60	70	80	90	100	110	120
22	33	44	55	66	77	88	99	110	121	132
24	36	48	60	72	84	96	108	120	132	144

ANATION.—The numbers across the top may be regarded as the ands, and those in the left-hand column as the multipliers. The will be found in the column under each multiplicand.

3 ones are 3; 3 twos are 6; 3 threes are 9, etc.

upil should make the multiplication table for himself, work out the , and commit it to memory. Study, recitation, and frequent repe- re necessary to fix it in the mind. The pupil must repeat each line ble over and over, and be drilled upon it until he knows it. Writ- able frequently will assist the pupil in committing it.

DRILL EXERCISES.

What are the products of the following?

5	7 × 4	10 × 8	5 × 9	8 × 8
7	5 × 5	7 × 2	11 × 10	3 × 9
4	6 × 4	9 × 8	7 × 5	11 × 3
9	2 × 5	2 × 7	9 × 9	7 × 6
4	8 × 6	3 × 10	6 × 3	5 × 8
7	10 × 3	11 × 2	2 × 8	9 × 12
4	7 × 8	8 × 4	4 × 10	8 × 3
2	5 × 10	7 × 9	5 × 2	6 × 2
6	8 × 5	10 × 2	7 × 10	2 × 9
5	9 × 3	3 × 11	2 × 6	7 × 11
9	6 × 7	9 × 4	8 × 2	6 × 6
3	11 × 5	6 × 8	9 × 5	4 × 2
8	3 × 6	12 × 3	6 × 9	9 × 8

11 × 6	7 × 12	11 × 9	10 × 5	10 × 10
4 × 7	9 × 3	3 × 7	3 × 12	6 × 10
5 × 11	8 × 9	6 × 11	2 × 10	12 × 7
6 × 12	10 × 12	9 × 7	12 × 8	10 × 6
9 × 10	3 × 4	12 × 4	8 × 10	4 × 3
8 × 9	4 × 5	4 × 8	12 × 2	12 × 10
3 × 3	10 × 9	5 × 12	4 × 4	11 × 12
2 × 11	7 × 4	3 × 2	7 × 8	4 × 6
4 × 12	10 × 10	8 × 7	10 × 11	2 × 12
11 × 7	12 × 6	4 × 11	9 × 11	11 × 8
12 × 9	11 × 11	12 × 11	12 × 5	12 × 12
7 × 3	8 × 9	5 × 6	3 × 8	10 × 7

WRITTEN EXERCISES.

79. 1. Multiply 385 by 7.

Multiplicand, 385
Multiplier, 7
Product, 2695

EXPLANATION.—For convenience, we write the multiplier under the multiplicand, draw a line beneath, and begin at the right to multiply.

7 times 5 units are 35 units, or 3 tens and 5 units. We write the 5 units in units' place in the product, and reserve the 3 tens to add to the tens.

7 times 8 tens are 56 tens, plus the 3 tens are 59 tens, or 5 hundreds and 9 tens. We write the 9 tens in tens' place in the product, and reserve the 5 hundreds to add to the hundreds.

7 times 3 hundreds are 21 hundreds, plus the 5 hundreds, are 26 hundreds, or 2 thousands and 6 hundreds, which we write in the thousands' and hundreds' places in the product.

Hence, the product is 2695.

In practice, we say: 7 fives are 35; write 5 and carry 3. 7 eights are 56 and 3 are 59; write 9 and carry 5. 7 threes are 21 and 5 are 26; write 26. Answer, 2695.

RULE.—*Write the multiplier under the multiplicand, so that units may stand under units, tens under tens, etc. Draw a line beneath.*

Begin at the right, and multiply each figure of the multiplicand by the multiplier, writing the result and carrying as in addition.

EXAMPLES.

2.	3.	4.	5.	6.	7.
346	572	683	785	438	463
3	4	2	6	5	7

8.	9.	10.	11.	12.	13.
$4.35	$3.36	$7.23	$5.94	6.72	8.91
5	4	6	7	8	9

Find the products of the following:

14. 4×235	23. 4321×5	32. 6×25632
15. 3×423	24. 5243×7	33. 8×37143
16. 5×701	25. 6452×6	34. 5×41371
17. 6×642	26. 8246×4	35. 7×56135
18. 8×387	27. 5734×8	36. 9×47206
19. 7×420	28. 7365×7	37. 8×83041
20. 9×627	29. 6837×11	38. 7×73654
21. 11×734	30. 6243×12	39. 8×93742
22. 12×583	31. 8674×11	40. 9×84370

PRACTICAL PROBLEMS.

80. 1. What will 6 barrels of flour cost at $4.75 a barrel?

$4.75
6
$28.50

SOLUTION.—If one barrel of flour costs $4.75, 6 barrels will cost 6 times $4.75, which are $28.50.

2. How much must be paid for 5 tons of hay at $9.50 a ton?

3. A man sold 8 acres of land at $165 an acre. How much did he get for the land?

4. There are 231 cubic inches in a gallon. How many cubic inches in 7 gallons?

5. A farmer sold 6 cattle at an average of $42.65 a head. What was the amount received?

6. A merchant bought 8 stoves for $14.75 each. What did he pay for them?

7. There are 5280 feet in a mile. How many feet in 5 miles?

8. A clerk received $14.80 per week. How much did he receive in 7 weeks?

9. An agent sold 9 mowers for $35.50 each. How much did he receive for them?

10. Sound travels 1120 feet in a second. How far will it travel in 11 seconds?

11. A invested $1790 in grain, and B invested 9 times as much. What sum did B invest?

12. There are 12 units in a dozen, and 12 dozen in a gross. What is the number of pens in 12 gross, or a great gross?

DRILL EXERCISES.

81. 1. How many are 10 times 4? 10 times 5? 10 times 6? 10 times 7?

2. What is annexed to 4 to make it 40? To 5 to make it 50? To 6 to make it 60?

3. What then is annexed to a number when it is multiplied by 10?

4. How many are 10 times 11? 10 times 12? 10 times 13? 10 times 15?

5. How many are 10 times 17? 10 times 19? 10 times 20?

6. How many are 100 times 1? 100 times 2? 100 times 3? 100 times 4?

7. What is annexed to 1 to make it 100? To 2 to make it 200? To 4 to make it 400?

8. What then is annexed to a number when it is multiplied by 100?

9. How many are 1000 times 1? 1000 times 2? 1000 times 4? 1000 times 6?

10. What is annexed to 1 to make it 1000? To 4 to make it 4000? To 6 to make it 6000?

11. How then is a number multiplied by 1000?

12. How then is a number multiplied by 1 with ciphers annexed?

82. PRINCIPLE.—*A number is multiplied by 10, 100, 1000, etc., by annexing to the multiplicand as many ciphers as there are ciphers in the multiplier.*

WRITTEN EXERCISES.

83. 1. Multiply 236 by 10.

$236 \times 10 = 2360$ EXPLANATION.—Since annexing a cipher to the multiplicand multiplies it by ten, we merely indicate the operation and write out the result.

EXAMPLES.

What are the products of the following?

2. 247×10	312×100	4216×1000
3. 315×10	431×100	5317×1000
4. 562×10	729×100	5801×1000
5. 489×10	673×100	7124×10000
6. 576×10	485×100	5163×10000
7. 604×10	726×100	4075×10000
8. 817×10	801×100	8436×100000

10. Multiply 4235 by 3000.

4235
3000
12705000

EXPLANATION.—Since 3000 is equal to 1000 times 3, we first multiply 4235 by 3, giving a product of 12705, and this result is multiplied by 1000 by annexing three ciphers.

Find the products of the following:

11. 234×20	517×200	7235×2000
12. 321×40	428×500	4103×4000
13. 416×30	635×300	6251×5000
14. 635×50	582×700	7162×3000
15. 548×40	746×400	4819×7000
16. 736×60	653×800	7635×6000
17. 675×70	495×600	6914×8000
18. 487×80	714×700	3805×9000
19. 743×60	821×800	6072×7000
20. 681×90	572×900	8475×5000

21. Multiply 342 by 235.

342
235
1710
10260
68400
80370

EXPLANATION.—For convenience, we write the multiplier under the multiplicand, units under units, tens under tens, etc.

We cannot multiply 235 at one operation, hence we multiply by the parts, and then add the products. The parts by which we multiply are 5, 3 tens, or 30, and 2 hundreds, or 200.

5 times 342 are 1710, the *first* partial product; 30 times 342 are 10260, the *second* partial product; 200 times 342 are 68400, the *third* partial product. The sum of these partial products is 80370, the *entire* product.

342
235
1710
1026
684
80370

EXPLANATION.—In practice, we omit the ciphers at the right of the partial products, the significant figures occupying their proper places without the ciphers.

Thus, in multiplying by 3 *tens*, the lowest order in the product is tens; hence we write the first figure of the product under tens, and the rest in their proper order.

In multiplying by *hundreds*, the lowest order in the product is hundreds; hence we write the first figure of the product under hundreds, and the rest in their proper order.

RULE.—*Write the multiplier under the multiplicand, so that units may stand under units, tens under tens, etc., and draw a line beneath.*

Begin at the right and multiply the multiplicand by each significant figure of the multiplier, writing the first figure of each partial product under the figure of the multiplier which produces it.

Add the partial products, and their sum will be the entire product.

PROOF.—*Multiply the multiplier by the multiplicand; and the work is correct, the products will be the same.*

1. When there are ciphers between the significant figures of the multiplier, pass over them and multiply the significant figures only, taking care to place the first figure of each partial product under the figure of the multiplier used to obtain it.

2. When there are ciphers at the right of the numbers to be multiplied, place the right-hand significant figures directly under each other, find the product of the numbers denoted by the significant figures, and annex as many ciphers to the result as are found at the right of both numbers.

22.	23.	24.
2436	3754	4030
503	2006	5700
7308	22524	2821
12180	7508	2015
1225308	7530524	22971000

Multiply:

25. 1326 by 45.
26. 4517 by 38.
27. 6314 by 61.
28. 8143 by 73.
29. 4658 by 59.
30. 8792 by 84.
31. 3425 by 36.
32. 4316 by 42.
33. 2803 by 53.
34. 7142 by 67.
35. 5361 by 70.
36. 7432 by 85.
37. 6343 by 93.
38. 5718 by 69.
39. 8342 by 87.
40. 9674 by 99.
41. 2315 by 123.
42. 4152 by 214.
43. 7032 by 325.
44. 6324 by 316.
45. 5735 by 428.
46. 4203 by 380.
47. 6420 by 423.
48. 7045 by 546.
49. 8906 by 657.
50. 9827 by 809.
51. 34201 by 2134.
52. 43712 by 3200.
53. 57241 by 6354.
54. 63053 by 2345.
55. 68007 by 3012.
56. 70140 by 5400.
57. 435624 by 3005.
58. 384237 by 4250.
59. 516742 by 3900.
60. 670405 by 7304.
61. $415.24 by 2435.
62. $364.18 by 5042.
63. $530.35 by 6324.
64. $675.08 by 7143.
65. $7 6.42 by 6752.
66. $831.70 by 8300.
67. $689.87 by 7542.
68. $715.72 by 6854.
69. $819.01 by 4013.
70. $900.14 by 6095.

PRACTICAL PROBLEMS.

84. 1. How many pounds in 250 bushels of corn, if there are 56 pounds in a bushel?

2. How many oranges in 68 boxes, if each box contains 225 oranges?

3. If a bushel of wheat is worth $1.22, how much are 120 bushels worth?

4. How many bushels of oats will a field of 23 acres produce which averages 75 bushels to the acre?

5. If a man can earn $35 in a week, how much can he earn in 52 weeks, or a year?

6. A farm containing 148 acres was bought for $47 per acre. How much was paid for the farm?

7. If a saw-mill produces 2560 feet of boards in a day, how many feet can it produce in 104 days?

8. At an average cost of $4675 per mile, what would it cost to build 150 miles of railroad?

9. There are 5280 feet in a mile. How many feet in 315 miles?

10. A certain news office prints 7800 copies of its paper each day. How many copies at this rate could it print in 312 days?

11. If a certain factory can make 15263 buttons in a day, how many buttons at this rate could it make in 26 working days, or a month?

12. A planter put up his cotton into 375 bales, averaging 436 pounds each. How many pounds of cotton did he have?

13. A dealer bought 500 bushels of wheat at $1.15 a bushel, 300 bushels of corn at $.45 a bushel, and 275 bushels of oats at $.28 a bushel. How much hid he pay for all?

14. In a block of houses there are 38 buildings, each building containing 27 windows, and each window 12 panes of glass. How many panes of glass in the whole block of buildings.

15. How many minutes in the month of January?

16. A man bought 260 acres of cleared land at $65 an acre, and 40 acres of woodland at $74.75 an acre. How much did he pay for all of the land?

17. A shoe-dealer bought 27 cases of shoes, each case containing 20 pairs, at $1.35 a pair. What was the amount of the bill?

18. If a certain town consumes 35760 gallons of water in a day, how much at this rate will be required for 365 days, or a year?

DIVISION.

85. Division is the process of finding how many times one number is contained in another.

86. The **Dividend** is the number to be divided.

87. The **Divisor** is the number by which we divide.

88. The **Quotient** is the result obtained by the division. It shows how many times the dividend contains the divisor.

89. The **Remainder** is the part of the dividend which is sometimes left after dividing.

90. The **Terms** in Division are the *Dividend*, the *Divisor*, and the *Quotient*.

91. The **Sign of Division** is a short horizontal line (in the line of writing), with a dot above and another below the middle of it ÷. It denotes that the number before it is to be divided by the number after it.

Thus, 8 ÷ 2 is read 8 divided by 2. The forms 2) 8, and $\frac{8}{2}$ also indicate that 8 is to be divided by 2.

92. PRINCIPLES.—1. *The dividend and divisor are like numbers.*

2. *The quotient is an abstract number.*

3. *The divisor is abstract in dividing a number into equal parts.*

4. *The product of the divisor and quotient, plus the remainder, if any, is equal to the dividend.*

DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

EXPLANATION.—This table is made the same as the multiplication table. The numbers in the left hand column may be regarded as the divisors, the numbers to the right as the dividends, and those across the top as the quotients.

Thus, 2 in 2, 1 time; 2 in 4, 2 times; 2 in 6, 3 times, etc.

There is little need of committing a table of quotients, if the pupil readily recalls the products in multiplication. Should we wish to know how many *fives* there are in 35, if we recall that *five* sevens are 35, we have the answer at once. Five is contained in 35 *seven* times; because five 7's are 35. Nine is contained in 54 six times; because nine 6's are 54.

Hence, division is the reverse of multiplication, or *reverse multiplication*, the product being regarded as the dividend, and the two factors as divisor and quotient.

DRILL EXERCISES.

93. Tell at once the quotients of the following:

30 ÷ 6	25 ÷ 5	80 ÷ 10	45 ÷ 5	64 ÷ 8
56 ÷ 8	24 ÷ 6	14 ÷ 2	110 ÷ 11	27 ÷ 3
44 ÷ 11	10 ÷ 2	72 ÷ 9	35 ÷ 7	33 ÷ 11
36 ÷ 4	48 ÷ 8	14 ÷ 7	81 ÷ 9	42 ÷ 7
40 ÷ 10	30 ÷ 10	30 ÷ 3	18 ÷ 6	40 ÷ 5
49 ÷ 7	56 ÷ 7	22 ÷ 11	16 ÷ 2	72 ÷ 9
20 ÷ 5	50 ÷ 5	32 ÷ 8	40 ÷ 4	24 ÷ 8
18 ÷ 9	40 ÷ 8	63 ÷ 7	10 ÷ 5	12 ÷ 6
36 ÷ 6	27 ÷ 9	20 ÷ 10	70 ÷ 7	18 ÷ 2
15 ÷ 3	42 ÷ 6	33 ÷ 3	12 ÷ 2	77 ÷ 7

64 ÷ 8	55 ÷ 11	36 ÷ 9	16 ÷ 8	36 ÷ 6
66 ÷ 11	18 ÷ 3	48 ÷ 6	45 ÷ 9	8 ÷ 4
28 ÷ 7	84 ÷ 7	36 ÷ 12	54 ÷ 6	54 ÷ 9
55 ÷ 5	27 ÷ 9	99 ÷ 11	50 ÷ 10	100 ÷ 10
72 ÷ 6	72 ÷ 8	21 ÷ 3	36 ÷ 3	60 ÷ 6
90 ÷ 9	120 ÷ 10	66 ÷ 6	20 ÷ 2	84 ÷ 12
72 ÷ 8	12 ÷ 3	63 ÷ 9	96 ÷ 12	60 ÷ 10
9 ÷ 3	20 ÷ 4	48 ÷ 12	80 ÷ 8	12 ÷ 4
22 ÷ 2	90 ÷ 10	32 ÷ 4	24 ÷ 12	120 ÷ 12
48 ÷ 4	28 ÷ 7	60 ÷ 5	16 ÷ 4	132 ÷ 11
77 ÷ 11	100 ÷ 10	6 ÷ 3	56 ÷ 7	24 ÷ 4
108 ÷ 12	72 ÷ 12	56 ÷ 8	110 ÷ 10	88 ÷ 11
21 ÷ 7	121 ÷ 11	44 ÷ 4	99 ÷ 9	144 ÷ 12
54 ÷ 6	72 ÷ 9	132 ÷ 12	60 ÷ 12	24 ÷ 2
28 ÷ 4	64 ÷ 8	30 ÷ 5	24 ÷ 3	70 ÷ 7

94. When anything is divided into *two* equal parts, *each part* is called *one half;* when into *three* equal parts, *one third;* when into *four* equal parts, *one fourth*, etc.

95. A **Fraction** is one or more of the equal parts of anything.

96. Fractions are expressed as follows:

One half by $\frac{1}{2}$.	One third by $\frac{1}{3}$.	Two thirds by $\frac{2}{3}$.
One fourth by $\frac{1}{4}$.	Three fourths by $\frac{3}{4}$.	One fifth by $\frac{1}{5}$.
Two fifths by $\frac{2}{5}$.	Five sixths by $\frac{5}{6}$.	Ten twelfths by $\frac{10}{12}$.

97. 1. Read the following fractional expressions:

$\frac{4}{5}$	$\frac{7}{8}$	$\frac{5}{9}$	$\frac{8}{10}$	$\frac{11}{12}$	$\frac{9}{18}$	$\frac{7}{15}$	$\frac{12}{19}$
$\frac{17}{20}$	$\frac{18}{21}$	$\frac{19}{24}$	$\frac{15}{37}$	$\frac{14}{41}$	$\frac{23}{50}$	$\frac{29}{60}$	$\frac{35}{64}$
$\frac{27}{70}$	$\frac{43}{75}$	$\frac{51}{80}$	$\frac{35}{88}$	$\frac{52}{97}$	$\frac{77}{100}$	$\frac{85}{109}$	$\frac{31}{176}$

2. What is $\frac{1}{2}$ of 8?	10?	14?	16?	20?	24?
3. What is $\frac{1}{3}$ of 9?	15?	21?	27?	30?	36?
4. What is $\frac{1}{4}$ of 8?	16?	24?	32?	40?	44?
5. What is $\frac{1}{5}$ of 15?	25?	30?	40?	50?	55?
6. What is $\frac{1}{6}$ of 12?	24?	36?	48?	54?	72?
7. What is $\frac{1}{7}$ of 21?	35?	56?	70?	77?	84?
8. What is $\frac{1}{8}$ of 16?	32?	40?	56?	72?	88?
9. What is $\frac{1}{9}$ of 27?	36?	54?	63?	81?	108?
10. What is $\frac{1}{10}$ of 20?	40?	50?	80?	100?	110?
11. What is $\frac{1}{11}$ of 33?	55?	66?	88?	99?	132?

12. What is $\frac{1}{12}$ of 24? 48? 60? 96? 120? 144?
13. What is $\frac{1}{2}$ of 7?

EXPLANATION.—7 equals 6 + 1; $\frac{1}{2}$ of 6 is 3, $\frac{1}{2}$ of 1 is $\frac{1}{2}$. Hence, $\frac{1}{2}$ of 7 is $3\frac{1}{2}$.

14. What is $\frac{1}{2}$ of 3? 9? 11? 15? 19? 23?
15. What is $\frac{1}{3}$ of 4? 5? 7? 11? 17? 25?
16. What is $\frac{1}{4}$ of 9? 13? 15? 21? 23? 29?
17. What is $\frac{1}{5}$ of 8? 11? 13? 16? 19? 27?
18. What is $\frac{1}{6}$ of 11? 13? 17? 19? 31? 37?
19. What is $\frac{1}{7}$ of 24? $\frac{1}{8}$ of 35? $\frac{1}{9}$ of 28? $\frac{1}{9}$ of 58?
20. What is $\frac{1}{10}$ of 37? $\frac{1}{11}$ of 80? $\frac{1}{11}$ of 100? $\frac{1}{12}$ of 143?

WRITTEN EXERCISES.

98. 1. Divide 1373 by 3.

Divisor. Dividend.
3)1374
Quotient. 458

EXPLANATION.—For convenience, we write the divisor at the left of the dividend with a curve line between them, draw a line under the dividend, and begin at the left to divide.

3 is not contained in 1 thousand any thousand times; 1 thousand equals 10 hundreds, which with 3 hundreds are 13 hundreds. 3 is contained in 13 hundreds 4 hundreds times and 1 hundred remaining. We write the 4 hundreds under the line in hundreds' place.

1 hundred, the remainder, equals 10 tens, which added to 7 tens are 17 tens. 3 is contained in 17 tens 5 tens times and 2 tens remaining. We write the 5 tens under the line in tens' place.

2 tens, the remainder, equal 20 units, which added to 4 units are 24 units. 3 units is contained in 24 units 8 units times. We write the 8 under the line in units' place.

Hence, the quotient is 458.

In practice, we say: 3 in 13, 4 times; in 17, 5 times; in 24, 8 times. Answer, 458

Proof.—The quotient, 458, multiplied by 3, the divisor, is 1374. Hence, the work is correct (Prin. 4).

When the work is performed by writing only the dividend, divisor, and quotient, as above, the process is called *Short Division.*

99. Short Division is the method in which only the dividend, divisor, and quotient are written, the operation being performed mentally.

Short division should be employed when the divisor does not exceed 12.

RULE.— *Write the divisor at the left of the dividend with a curve line between them, and draw a line under the dividend.*

Beginning at the left, divide each figure of the dividend by the divisor, and place the quotient under the figure divided.

If there is a remainder after any division, regard it as prefixed to the next figure of the dividend, and divide as before.

If any partial dividend is less than the divisor, write a cipher in the quotient, and prefix such dividend to the next figure.

When there is a final remainder, write it after the quotient, or with the divisor beneath it as part of the quotient.

PROOF.—*Multiply the quotient by the divisor, and add the remainder, if any, to the product. If the work is correct the result will equal the dividend.*

We begin at the left to divide, so that the remainder can be united to the number of units of the next lower order, giving a new partial dividend.

EXAMPLES.

Divide by *short division:*

2.	3.	4.	5.
3)25092	5)21025	7)$304.50	9)207365
8364	4205	$43.50	$23040\frac{5}{9}$

6.	1929 ÷ 3	25.	$\frac{3048}{4}$	44.	674584 ÷ 7
7.	3820 ÷ 5	26.	$\frac{2735}{5}$	45.	341877 ÷ 8
8.	2304 ÷ 4	27.	$\frac{4032}{6}$	46.	918468 ÷ 9
9.	4470 ÷ 6	28.	$\frac{4179}{8}$	47.	891473 ÷ 11
10.	5375 ÷ 5	29.	$\frac{5360}{8}$	48.	876436 ÷ 12
11.	7251 ÷ 3	30.	$\frac{4136}{4}$	49.	$432.48 ÷ 6
12.	9142 ÷ 7	31.	$\frac{6276}{6}$	50.	$462.24 ÷ 8
13.	6048 ÷ 4	32.	$\frac{8344}{7}$	51.	$714.98 ÷ 7
14.	6105 ÷ 5	33.	$\frac{6525}{9}$	52.	$937.65 ÷ 5
15.	9234 ÷ 6	34.	$\frac{7380}{5}$	53.	$674.04 ÷ 6
16.	7042 ÷ 7	35.	$\frac{6384}{4}$	54.	$870.80 ÷ 8
17.	8364 ÷ 3	36.	$\frac{7375}{8}$	55.	$932.12 ÷ 7
18.	3428 ÷ 4	37.	$\frac{6478}{6}$	56.	$652.40 ÷ 8
19.	5024 ÷ 8	38.	$\frac{8107}{9}$	57.	$712.35 ÷ 5
20.	7110 ÷ 6	39.	$\frac{6482}{7}$	58.	$821.70 ÷ 9
21.	7385 ÷ 5	40.	$\frac{8072}{8}$	59.	$934.32 ÷ 6
22.	6223 ÷ 7	41.	$\frac{7348}{6}$	60.	$837.50 ÷ 10
23.	7400 ÷ 8	42.	$\frac{7689}{9}$	61.	$793.10 ÷ 11
24.	8325 ÷ 9	43.	$\frac{8176}{8}$	62.	$984.72 ÷ 12

PRACTICAL PROBLEMS.

100. 1. There are 3 feet in one yard. How many yards in 873 feet?

2. In one peck there are 8 quarts. How many pecks in 1952 quarts?

3. In one square yard there are 9 square feet. How many square yards in 2115 square feet?

4. In a certain box there were 1728 eggs. How many dozen did it contain?

5. At $6 a barrel, how many barrels of flour can be purchased for $1200?

6. The salaries of 7 men amounted to $5838. What was the average salary?

7. If a vessel sails 9 miles an hour, how long at this rate will it take her to sail 1089 miles?

8. I bought 6 lots for $5484. What was the average price of each lot?

9. At 8 cents a yard, how many yards of muslin can be bought for $3.04?

10. A father divided $19480 equally among his 8 children. How much did each child receive?

11. A man bought 7 cows for $171.50. What was the average price paid for each cow?

DRILL EXERCISES.

101. 1. How many 10's are there in 40? In 50? In 60? In 70? In 100? In 150? In 200? In 300?

2. What must be cut off from 40 to make the number 4? From 50 to make it 5? From 100 to make it 10? From 300 to make it 30?

3. How many ciphers in the divisor 10? How many figures cut off from the right of the dividend will give the quotient when divided by 10?

4. How many 100's in 400? In 500? In 1500? In 2500?

5. What must be cut off from 400 to make the number 4? From 500 to make it 5? From 1500 to make it 15? From 2500 to make it 25?

6. How many ciphers in the divisor 100? And how many figures are cut off from the right of the dividend in dividing by 100?

7. How many 1000's in 2000? In 6000? In 10000? In 18000?

8. How many ciphers in 1000? And how many figures are cut off from the right of the dividend to secure the quotient in dividing by 1000?

9. How many figures did you cut off in dividing numbers by 10? In dividing by 100? In dividing by 1000?

10. How then may a number by divided by 10, 100, 1000, etc.

102.—*A number may be divided by 10, 100, 1000, etc., by cutting off from the right of the dividend as many figures as there are ciphers at the right of the divisor.*

WRITTEN EXERCISES.

103. Divide the following:

1.	2.	3.
1|00)246|00	1|00)421|37	1|000)38|769
246	$421\frac{37}{100}$	$38\frac{769}{1000}$

4. 36700 ÷ 100	9. 73145 ÷ 1000	14. 65371 ÷ 10000
5. 41500 ÷ 100	10. 64732 ÷ 1000	15. 51234 ÷ 10000
6. 26734 ÷ 100	11. 72158 ÷ 1000	16. 73418 ÷ 10000
7. 51451 ÷ 100	12. 94730 ÷ 1000	17. 81437 ÷ 10000
8. 73640 ÷ 100	13. 42564 ÷ 1000	18. 67483 ÷ 10000

19. Divide 54743 by 300.

3|00)547|43
182—143,
or $182\frac{143}{300}$

EXPLANATION.—We first cut off the ciphers at the right of the divisor and as many figures at the right of the dividend. The remaining part of the dividend is then divided by 3, the remaining part of the divisor, giving a quotient of 182 and a remainder of 1 hundred. 1 hundred plus 43, the partial remainder cut off, gives the entire remainder 143. Hence, the quotient is $182\frac{143}{300}$.

Divide:

20. 4356 by 20	27. 35867 by 200	34. 426781 by 3000
21. 2481 by 40	28. 41635 by 400	35. 546735 by 4000
22. 7635 by 60	29. 53643 by 500	36. 764324 by 6000
23. 8147 by 70	30. 72430 by 600	37. 843162 by 8000
24. 5746 by 50	31. 48763 by 700	38. 437527 by 7000
25. 6174 by 80	32. 87354 by 800	39. 728379 by 8000
26. 7493 by 90	33. 93673 by 900	40. 567145 by 9000

41. Divide 4819 by 23.

Divisor. Dividend. Quotient
23) 4819 ($209\frac{12}{23}$
46
219
207
12

EXPLANATION.—23 is not contained in 4 thousands any thousands times; hence, there are no thousands in the quotient. 4 thousands and 8 hundreds are 48 hundreds. 23 is contained in 48 hundreds 2 hundreds times. We write the 2 hundreds in the quotient at the right of the dividend and multiply the divisor by them. 2 hundreds times 23 are 46 hundreds, which subtracted from 48 hundreds leave 2 hundreds.

2 hundreds and 1 ten are 21 tens. 23 is not contained in 21 tens any tens times; th rite 0 in the tens's place in the quotient.

21 tens and 9 units are 219 units. 23 is contained in 219 units 9 units times. We write the 9 units in the quotient and multiply the divisor by them. 9 units times 23 units are 207 units, which subtracted from 219 units leave a remainder of 12 units. To indicate the division of the remainder by 23, we write the 12 over the 23 with a line between them.

Hence, the quotient is $209\frac{12}{23}$.

In practice we say: 23 in 48, 2 times; write 2 in the quotient. $2 \times 23 = 46$, which subtracted from 48 leaves 2. Bring down 1. 23 in 21, 0 times; write 0 in the quotient. Bring down 9. 23 in 219, 9 times; write 9 in the quotient. $9 \times 23 = 207$, which subtracted from 219 leaves 12. Answer, $209\frac{12}{23}$.

Proof. $(209 \times 23) + 12 = 4819$. Hence, the work is correct (Prin. 4).

When in dividing the work is written out in full, as above, the process is called *Long Division*.

104. Long Division is the method of dividing when the work is written out in full.

Long division is generally employed when the divisor exceeds 12.

RULE.—*Draw a curve line at both sides of the dividend, and write the divisor at the left.*

Divide the number expressed by the fewest figures at the left of the dividend that will contain the divisor, and write the quotient at the right of the dividend.

Multiply the divisor by the quotient; subtract the product from the part of the dividend used, and to the remainder bring down the next figure of the dividend.

If any partial dividend will not contain the divisor, place a cipher in the quotient, bring down the next figure of the dividend, and proceed as before.

When there is a final remainder, write it after the quotient, or with the divisor beneath it as part of the quotient.

PROOF.—The proof is the same as in Short Division.

1. The student should carefully observe the *four* steps in the process of long division: 1st, *Divide;* 2d, *Multiply;* 3d, *Subtract;* 4th, *Bring down.*

2. *To find the quotient figure*, see how often the first *figure* at the left in the divisor is contained in the *first figure*, or *first two figures*, at the left of the partial dividend, making allowance for the addition of the tens from the product of the second figure of the divisor.

3. If after multiplying the divisor by the quotient figure, we find the product greater than the partial dividend from which it is to be subtracted, the quotient figure is *too large*, and must be diminished.

4. Each remainder must be less than the divisor. When the remainder is as great as the divisor, the quotient figure is *too small*, and must be increased.

5. When there is no remainder, the divisor is called an *exact divisor* of the dividend.

Find the quotients:

42. 156 ÷ 13	60. 5103 ÷ 42	78. 46713 ÷ 54
43. 210 ÷ 15	61. 7382 ÷ 51	79. 72148 ÷ 84
44. 748 ÷ 22	62. 8135 ÷ 61	80. 98732 ÷ 94
45. 768 ÷ 32	63. 6871 ÷ 71	81. 64372 ÷ 46
46. 837 ÷ 27	64. 5496 ÷ 81	82. 81735 ÷ 38
47. 864 ÷ 24	65. 7352 ÷ 91	83. 124356 ÷ 45
48. 899 ÷ 29	66. 6873 ÷ 32	84. 143761 ÷ 55
49. 952 ÷ 34	67. 8342 ÷ 42	85. 201342 ÷ 65
50. 943 ÷ 41	68. 7880 ÷ 53	86. 314216 ÷ 75
51. 999 ÷ 37	69. 9876 ÷ 64	87. 401324 ÷ 85
52. 2925 ÷ 13	70. 38723 ÷ 72	88. 287692 ÷ 95
53. 2760 ÷ 15	71. 51635 ÷ 62	89. 537148 ÷ 47
54. 4473 ÷ 21	72. 71406 ÷ 82	90. 672041 ÷ 58
55. 5450 ÷ 25	73. 38674 ÷ 92	91. 713257 ÷ 49
56. 8424 ÷ 27	74. 61483 ÷ 43	92. 637415 ÷ 87
57. 4236 ÷ 31	75. 73810 ÷ 53	93. 714326 ÷ 89
58. 2847 ÷ 41	76. 84152 ÷ 73	94. 842352 ÷ 97
59. 3185 ÷ 51	77. 50460 ÷ 63	95. 736485 ÷ 99

96. 13245072 ÷ 101	107. 234605372 ÷ 876
97. 24132413 ÷ 201	108. 420037204 ÷ 987
98. 34605328 ÷ 301	109. 567820346 ÷ 798
99. 36145671 ÷ 401	110. 876543210 ÷ 999
100. 50361426 ÷ 212	111. 346241356 ÷ 1010
101. 43254214 ÷ 311	112. 472630425 ÷ 1122
102. 62141637 ÷ 510	113. 351648172 ÷ 2134
103. 71034142 ÷ 608	114. 643715243 ÷ 3240
104. 48762035 ÷ 821	115. 731641278 ÷ 5163
105. 32460574 ÷ 735	116. 846732412 ÷ 4276
106. 56387263 ÷ 852	117. 570007003 ÷ 6087

PRACTICAL PROBLEMS.

105. 1. At $35 each, how many cows can be bought for $840?

```
35) 840 (24
    70
   ---
   140
   140
   ---
```

SOLUTION.—If $35 will buy one cow, $840 will buy as many cows as $35 are contained times in $840, which are 24.

2. If a steamboat sails 18 miles an hour, how long will it take to go 576 miles?

3. There are 32 quarts in a bushel. How many bushels are there in 3936 quarts?

4. *How many* days are there in 4128 hours?

5. I wish to place 1440 pens in boxes of 36 pens each. How many boxes will be required?

6. How many farms of 50 acres each can be made from a tract of land containing 1200 acres?

7. A certain cistern contains 4725 gallons of water. How many hogsheads of 63 gallons each would be required to hold it?

8. A field of 80 acres produced 2160 bushels of wheat. What was the average yield per acre?

9. There are 56 pounds in a bushel of corn. How many bushels in 19320 pounds of corn?

10. At the rate of 136 miles a day, how many days would it require to travel 5712 miles?

11. There are 128 cubic feet in one cord. How many cords are there in 10112 cubic feet of wood?

12. A gallon contains 231 cubic inches. How many gallons in a vessel that contains 7392 cubic inches?

13. A cotton grower raised 154224 pounds of cotton. How many bales averaging 324 pounds each could be made from it?

14. The stock of a railroad company is $3750000. If it is divided into shares of $500 each, how many shares are there?

14. If the combined wealth of 2135 men is $6419945, what is the average wealth of each man?

16. The product of two numbers is 515522, and the smaller is 322. What is the larger number.

17. The product of two numbers is 433825428, and the larger is 71873. What is the smaller number.

18. The state of Pennsylvania has an area of 45215 square miles. Rhode Island has an area of only 1250 square miles. How many states of the size of Rhode Island could be formed out of Pennsylvania, and how many square miles would be left over?

19. In 1900, the inhabitants of Pennsylvania numbered 6302115. About how many persons, on an average, were living on a square mile?

THE SIGNS OF AGGREGATION.

106. The Signs of Aggregation are the Parenthesis, ( ); the Vinculum, ——; the Brackets, [ ]; and the Braces, { }. They show that the numbers included by them are to be subjected to the same operation.

Thus, each of the expressions $(3+4)\times 5$, $\overline{3+4}\times 5$, $[3+4]\times 5$, and $\{3+4\}\times 5$, shows that the sum of 3 and 4, or 7, is to be multiplied by 5; but $3+4\times 5$ means that 3 is to be added to 4 times 5.

When numbers are under the vinculum, or are included within any the other signs of aggregation, they are commonly said to be *in parenthesi*

107. The **Terms** of an expression are the parts connected by th signs + or —.

Thus, the expression 4 + 5 contains two terms. (4 + 5) × 2 + (6 — 3) 4 also contains but two terms; for the expression included in each pare thesis is treated as a single number.

1. 4 + 8 ÷ 2 = 4 + 4 or 8; but (4 + 8) ÷ 2 = 12 ÷ 2, or
2. 6 + 4 × 7 — 5 = 6 + 28 — 5, or 29; (6 + 4) × 7 — 5 = 70 — 5, or 65; and (6 + 4) × (7 — 5) = 10 × 2, or 20.

Hence, to find the value of expressions containing parenthetic forms:

1st. Simplify the expressions within the parenthesis by perform ing the operations indicated by the signs. 2d. Perform the mu tiplication or division indicated. 3d. Combine the terms i indicated by the signs + and —.

When expressions contain two or more parentheses, brackets, etc., pe form the operations indicated by the inner ones first.

EXAMPLES.

Find the values of the following:

3. (4 + 5) × 6
4. (4 + 5) × 7 — 3
5. (6 + 2) × 4 + 3
6. (3 + 7) × (3 + 2)
7. (8 + 6) ÷ 2 + 5
8. (4 + 2) × 6 — (3 + 2) × 4
9. (5 + 7) × 4 + (6 + 8) ÷ 7
10. 8 + 12 ÷ 4 — (4 + 6 ÷ 2)
11. (6 + 3) × 5 + (10 — 6) ÷ 2
12. $\overline{12 + 8} \div 4 - \overline{4 + 6} \div 2$
13. 10 + 3 × [7 + 3 × (8 — 2) + 5]
14. [(65 — 15) × 2 — 16 + 4] ÷ 5
15. 7 × {(184 — 36) ÷ 4 — $(\overline{9 \times 3} + 4)$ + 24 ×

RELATION OF TERMS IN DIVISION.

108. The value of the quotient depends upon the dividend a the divisor. If either is changed, the quotient will be change If both are changed, the quotient may or may not be changed.

Take the equation, 32 ÷ 8 = 4. If we multiply the dividend by 2, have 64 ÷ 8 = 8, which multiplies the quotient by 2. If we divide the divis by 2, we have 32 ÷ 4 = 8, which also multiplies the quotient by 2.

If we divide the dividend by 2, we have 16 ÷ 8 = 2, which divides t quotient by 2. If we multiply the divisor by 2, we have 32 ÷ 16 = 2, whi *also divides the quotient* by 2.

If we multiply both dividend and divisor by 2, we have 64 ÷ 16 =

which does not change the quotient. If we divide both dividend and divisor by 2, we have $16 \div 4 = 4$, which also does not change the quotient.

These facts may be shown to be true of any equation. Hence, the following:

109. PRINCIPLES.—1. *Multiplying the dividend or dividing the divisor by any number, multiplies the quotient by that number.*

2. *Dividing the dividend or multiplying the divisor by any number, divides the quotient by that number.*

3. *Multiplying or dividing both dividend and divisor by the same number, does not change the quotient.*

ARITHMETICAL ANALYSIS.

110. Analysis is the process of solving problems by a comparison of their parts to each other directly; or indirectly, through their relation to the unit.

1. If 5 apples cost 7 cents, what will 10 apples cost?.

ANALYSIS.—If 5 apples cost 7 cents, 10 apples, which are 2 times 5 apples, will cost 2 times 7 cents, or 14 cents.

2. If 6 combs cost 9 cents, what will 12 combs cost?
3. If 4 lemons cost 12 cents, what will 16 lemons cost?
4. How much will 15 oranges cost, if 3 oranges cost 8 cents?
5. What must be paid for 32 barrels of flour, if 8 barrels cost $40.
6. If 7 tons of coal cost $22, what will 21 tons cost?
7. If 12 yards of muslin cost 66 cents, how much would 4 yards cost?

ANALYSIS.—If 12 yards of muslin cost 66 cents, 4 yards, which is $\frac{1}{3}$ of 12 yards, would cost $\frac{1}{3}$ of 66 cents, or 22 cents.

8. If 18 pigs cost $30, how much must be paid for 9 pigs?
9. If 14 pencils cost 24 cents, what will 7 pencils cost?
10. How much must be paid for 9 sheep, if 36 sheep are worth $140?
11. In 15 quires of paper there are 360 sheets. How many sheets are there in 5 quires?
12. If 48 bushels of corn cost $24, what must be paid for 12 bushels?
13. If 5 pounds of rice cost 35 cents, what will 9 pounds cost?

ANALYSIS.—If 5 pounds of rice cost 35 cents, 1 pound will cost $\frac{1}{5}$ of 35 cents, or 7 cents; and since 1 pound costs 7 cents, 9 pounds will cost 9 times 7 cents, or 63 cents.

14. James walked 36 miles in 9 hours. How far, at this rate, could he walk in 11 hours?

15. How many tons of hay will a drover feed in 12 weeks, at the rate of 10 tons in 5 weeks?

16. If 8 men can dig 32 rods of ditch in a day, how much can 15 men dig in the same time?

17. If 7 bushels of wheat are worth $5.60, how much are 20 bushels worth?

18. A farmer raised 360 bushels of wheat on 12 acres of land. How much wheat, at this rate, could he raise on 15 acres?

REVIEW.

111. 1. A man gave $2150 for a house, $750 for a lot, and had $1700 left. How much money had he before purchasing?

2. Mr. M. had $6000. He gave to each of his four children $1250. How much money had he left?

3. My farm contains 175 acres. At what price per acre must I sell it to receive $14175?

4. John went to a store and bought 28 pounds of sugar at 5 cents a pound; 3 pounds of butter at 16 cents a pound; 5 dozen eggs at 17 cents a dozen; and 7 pounds of oatmeal at 5 cents a pound. He gave the clerk a 10-dollar bill. What change was given back?

5. A dealer bought 17 mowers at $25 each, and gave in exchange 12 tons of old metal at $5.25 a ton, and the balance in cash. How much cash did he pay?

6. A company invested a million dollars in bonds of $1000 each. How many bonds did the sum secure?

7. Mr. R's barn cost $2350; his house cost 3 times as much as his barn; and his farm cost as much as both. What was the cost of the farm?

8. A dealer bought 176 barrels of oil at $3.20 per barrel. He lost 21 barrels by a fire, and sold the remainder at $4.45 a barrel. Did he gain, or lose, and how much?

9. A clerk receives a salary of $60 a month, and his expenses are $45 a month. In how many years, at this rate, can he pay for a house which cost him $1800?

10. Mr. Mancor has $10000 which he wishes to invest in real estate. If he should buy a house for $4500, how many acres of land could he buy with the balance at $110 an acre?

11. A farmer wished to pay a mortgage of $300. He sold 180 bushels of wheat at $.90 per bushel, and enough clover seed at $3 per bushel to lift the mortgage. How many bushels of clover seed did he sell?

12. A drover had 360 sheep. He sold to one man 120 sheep, and to another 75, and then bought enough to make his number 500. How many did he buy?

13. Mr. B. sold 3450 bushels of wheat, then bought 1200 bushels, and then had 1750 bushels. How many bushels had he at first?

14. A ship whose rate of sailing is 112 miles a day is already 540 miles on its course, when another vessel whose rate is 148 miles a day leaves the same port for the point as the other. In how many days will the former vessel be overtaken by the latter?

15. Two men start from the same place at the same time and travel in opposite directions, the one at the rate of 23 miles a day, and the other 26 miles. How far will they be apart at the end of 6 days?

16. The distance from New York City to London by postal route is 3740 miles, and the mail time is about 6 days. What is the rate of sailing per hour, in whole numbers?

17. Mr. H. sold from his farm 265 bushels of wheat at $.80 per bushel; 400 bushels of corn at $.35 per bushel, 130 bushels of oats at $.25 per bushel, and 7 tons of hay at $8.50 per ton. How much did he realize from the farm?

18. I sold 75 acres of land at $90 per acre, and invested the money in land at $50 per acre. How many acres did I purchase?

19. If Mr. Row's field of 16 acres should yield 27 bushels of wheat to the acre, what would the grain be worth at $.98 per bushel?

20. A teacher's salary is $100 per month. His expenses per week average $19.35. How much can he save in a term of 9 months?

PROPERTIES OF NUMBERS.

112. Numbers are either *odd* or *even*, *prime* or *composite*.

113. **An Odd Number** is a number that cannot be exactly divided by 2.

The unit *figure of an odd* number is always, 1, 3, 5, 7, or 9; as 5, 21, 33.

114. An Even Number is a number that can be exactly divided by 2.

The unit figure of an even number is always 0, 2, 4, 6, or 8; as 4, 20, 36.

115. A Prime Number is a number that can be exactly divided only by itself and 1.

Thus, 1, 3, 5, 7, 11, 13, 17, etc., are prime numbers.

116. A Composite Number is a number that can be exactly divided by others besides itself and 1.

Thus, 4, 6, 8, 10, 12, 14, 15, etc., are composite numbers.

DRILL EXERCISES.

117. 1. Tell which of the following numbers are odd or even: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 27, 31, 36, 39, 40, 42, 45, 43, 46, 48, 50.

2. Tell which of the following numbers are prime or composite: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30.

3. Name the prime numbers from 31 to 67. From 67 to 101.

4. Name the composite numbers from 20 to 50. From 50 to 75. From 75 to 100.

5. Select all of the prime numbers from the following: 8, 10, 11, 17, 15, 18, 19, 23, 41, 45, 20, 21, 27, 29, 38, 42, 36, 35, 31, 43, 45, 47, 63, 61, 67.

6. Select all of the composite numbers from the following: 13, 19, 20, 21, 24, 25, 29, 26, 27, 32, 34, 37, 39, 58, 71, 73, 78, 80, 47, 45, 84, 87, 93, 99.

DIVISIBILITY OF NUMBERS.

118. A number is *divisible* by another when the latter will divide the former without a remainder.

119. Any number is divisible—

1. By 2, if it is an even number.
2. By 3, if the sum of the digits is divisible by 3.

Thus, 231 is divisible by 3, for 2 + 3 + 1, or 6, is divisible by 3.

3. By 4, if the two right hand figures are ciphers or express a number divisible by 4.

Thus, 300, 516, 928, etc., are divisible by 4.

sum of the digits is

number and 4 + 3 + 2, or

re ciphers, or express a

y 8.

divisible by 9.

).

digits are ciphers or are divis-

hand digits are ciphers, or ex-

RCISES.

the exact divisors of the following

11. 31416	16. 53712
12. 43700	17. 48224
13. 27125	18. 70227
14. 32536	19. 67401
15. 74375	20. 74000

FACTORING.

f a number are the numbers which multi-
oduce it.

ors of 18.

ors are prime numbers used as factors or exact

is the process of separating a number into its

me Factors of a composite number are the prime
multiplied together will produce it.

d 3, are the prime factors of 18.

124. An **Exponent** is a small figure written a little above a to the right of a number to indicate how many times the numb is used as a factor.

Thus, 3 used as a factor 2 times is indicated by 3^2.

125. PRINCIPLES.—1. *A number is divisible by its prime facto or by any product of them.*

2. *A number is divisible only by its prime factors or by so product of them.*

The number itself and unity are not considered factors of a number.

WRITTEN EXERCISES.

126. 1. What are the prime factors of 42?

$$\begin{array}{r|l} 2 & 42 \\ 3 & 21 \\ & 7 \end{array}$$

EXPLANATION.—We find (Art. 119) that 2 is a prime factor of 4 Dividing the quotient or other factor by 3, we find that 3 and are factors of 21. Since 2, 3, and 7 are prime numbers they a the prime factors of 42.

Proof.—$2 \times 3 \times 7 = 42$ (Art. 123).

RULE.—*Divide the given number by any prime number great than 1, that is an exact divisor. Divide the quotient in the sam manner, and so continue until the quotient is prime.*

The divisors and the last quotient will be the prime factors.

EXAMPLES.

Find the prime factors of the following:

2. 84	8. 428	14. 4336	20. 6486
3. 108	9. 512	15. 5048	21. 4832
4. 252	10. 630	16. 6300	22. 7357
5. 125	11. 1204	17. 7435	23. 5463
6. 240	12. 2340	18. 6124	24. 8145
7. 312	13. 3274	19. 5733	25. 9500

CANCELLATION.

127. Cancellation is a process of shortening computations b rejecting common factors from both the dividend and the divisor

128. PRINCIPLE.—*Rejecting common factors from both dividend and divisor does not change the quotient.*

WRITTEN EXERCISES.

129. 1. Divide $24 \times 30 \times 7$ by $16 \times 27 \times 11$.

$$\frac{\overset{3}{\cancel{24}} \times \overset{5}{\cancel{30}} \times 7}{\underset{2}{\cancel{16}} \times \underset{3}{\cancel{27}} \times 11} = \frac{35}{33} = 1\tfrac{2}{33}$$

EXPLANATION.—We write the dividend above the divisor with a line between them as in division.

We reject the common factor 8 from both 24 and 16, writing 3, the other factor of 24, above 24, and 2, the other factor of 16, below 16. We then reject 3 from 3 and 27, writing 9 below the 27. In like manner, we reject 3 from 30 and 9, writing 10 and 3. Since 2 is a factor of 10, we reject it, leaving 5.

The product of the factors not canceled in the dividend is 35, and those in the divisor 33. Hence, the quotient is $\frac{35}{33}$, or 35 ÷ 33, or $1\frac{2}{33}$.

RULE.—*Cancel the common factors from the dividend and divisor. Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

When any factor is canceled, 1 is understood to remain. It need only be written when the last of the factors in the dividend is canceled.

EXAMPLES.

Divide, using cancellation:

2. $8 \times 9 \times 10 \times 12$ by $4 \times 3 \times 2 \times 6$.
3. $7 \times 8 \times 15 \times 28$ by $2 \times 5 \times 7 \times 4$.
4. $14 \times 12 \times 21 \times 24$ by $7 \times 3 \times 4 \times 6$.
5. $18 \times 24 \times 30 \times 16$ by $3 \times 8 \times 5 \times 4$.
6. $35 \times 22 \times 16 \times 18 \times 14$ by $5 \times 11 \times 32 \times 9$.
7. $15 \times 9 \times 8 \times 6 \times 25$ by $3 \times 27 \times 4 \times 5$.
8. $40 \times 35 \times 27 \times 24$ by $8 \times 7 \times 9 \times 6 \times 5$.
9. $28 \times 15 \times 32 \times 26$ by $4 \times 5 \times 8 \times 13$.
10. $21 \times 18 \times 24 \times 38$ by $19 \times 6 \times 9 \times 7$.
11. $36 \times 14 \times 45 \times 29$ by $48 \times 21 \times 3 \times 5$.
12. $48 \times 26 \times 17 \times 50$ by $32 \times 13 \times 34 \times 40$.
13. $39 \times 18 \times 42 \times 37$ by $9 \times 13 \times 14 \times 3$.
14. $85 \times 30 \times 24 \times 16$ by $17 \times 6 \times 3 \times 8$.
15. $65 \times 40 \times 36 \times 27$ by $35 \times 13 \times 9 \times 18$.
16. $130 \times 64 \times 33 \times 49$ by $24 \times 10 \times 7 \times 8$.
17. $95 \times 145 \times 78 \times 54$ by $29 \times 26 \times 19 \times 9$.
18. $625 \times 9 \times 46 \times 38$ by $27 \times 92 \times 19 \times 125$.
19. $1500 \times 144 \times 5 \times 36$ by $365 \times 100 \times 45 \times 72$.
20. $1225 \times 40 \times 8 \times 63$ by $75 \times 60 \times 24 \times 21 \times 15$.

PRACTICAL PROBLEMS.

130. 1. How many yards of gingham at 15 cents a yard ca bought for 20 pounds of butter at 18 cents a pound?

$$\frac{\overset{6}{\cancel{18}} \times \overset{4}{\cancel{20}}}{\underset{5}{\cancel{15}}} = 24$$

EXPLANATION.—If 1 pound of butter is worth 18 c 20 pounds are worth 20 × 18 cents. At 15 cents a for 20 × 18 cents, as many yards of gingham ca bought as 15 is contained times in 18 × 20, which w by cancellation to be 24.

2. How many pounds of coffee at 28 cents a pound shoul given for 8 pounds of butter at 21 cents a pound?

3. How many bushels of oats at 35 cents a bushel must be g in exchange for 5 cords of wood at $3.15 a cord?

4. If 36 bushels of potatoes are worth $27, what are 72 bu worth?

5. How many bushels of wheat at 90 cents a bushel, mu given for 6 barrels of sugar, 200 pounds each, at 5 cents pound?

6. How many boxes of tea, each containing 24 pounds a cents a pound, must be given in exchange for 30 firkins of b of 56 pounds each, at 18 cents a pound?

7. A soap manufacturer sold 10 boxes of soap containin pounds each at 6 cents a pound, receiving for pay 40 can caustic soda, each containing 4 pounds. How much did he per pound for the soda?

8. If 18 men can do a piece of work in 15 days working 8 h a day, in how many days can 12 men do the same work if work 10 hours a day?

9. A farmer exchanged 20 bags of wheat containing 2 bu each at 90 cents a bushel, for 60 sacks of bran containing 3 b els each. What was the price of the bran per bushel?

10. Mr. L. sold 6 piles of brick, each containing 10 thou at $7 per thousand, and was paid in wood, 5 ranges at $3 per c How many cords were in each range?

GREATEST COMMON DIVISOR.

131. A **Common Divisor** of two or more numbers is any n ber that will exactly divide each of them.

Thus, 3 is a common divisor of 6, 9, and 12; 4 is a common divisor 12, and 16; and 5 is a common divisor of 10, 15, and 20.

132. The **Greatest Common Divisor** of two or more num *is the largest* number that will exactly divide each of them.

Thus, 8 is the greatest common divisor of 16, 24, and 36; and 12 is the greatest common divisor of 24, 48 and 72.

The initials G. C. D. may be used for Greatest Common Divisor.

133. PRINCIPLE.—*The greatest common divisor of two or more numbers is the product of all their common prime factors.*

WRITTEN EXERCISES.

134. 1. What is the greatest common divisor of 56, 84, and 112?

$56 = 2 \times 2 \times 2 \times 7$
$84 = 2 \times 2 \times 3 \times 7$
$112 = 2 \times 2 \times 2 \times 2 \times 7$
G. C. D. $= 2 \times 2 \times 7 = 28$

2	56	84	112
2	28	42	56
7	14	21	28
	2	3	4

G. C. D. $= 2 \times 2 \times 7 = 28$

EXPLANATION.—Since the greatest common divisor of two or more numbers is equal to the product of all the prime factors common to the given numbers, the numbers are separated into their prime factors. The only common prime factors are 2, 2, and 7. Hence their product, 28, is the greatest common divisor of the given numbers.

The common prime factors may also be found by placing the numbers in a horizontal line and dividing them successively by the prime factors until the quotients contain no common factor.

Find the greatest common divisor of:

2. 36, 90.	4. 72, 96.	6. 216, 312, 460.
3. 42, 63.	5. 144, 196.	7. 350, 410, 540.

135. In business it is seldom necessary to find the greatest common divisor of numbers greater than 100, so that almost any one with some knowledge of arithmetic can tell the greatest common divisor by inspecting the numbers. The method given below, though seldom used, applies to numbers which cannot be readily factored.

1. What is the greatest common divisor of 329 and 1551?

329) 1551 (4
1316
235) 329 (1
235
94) 235 (2
188
47) 94 (2
94

EXPLANATION.—We know that the G. C. D., whatever it may be, must divide 329, and if it divides 329 it will divide 4 times 329, or 1316. Since the G. C. D., divides 1316 and 1551, it will divide their difference, or 235. Since the G. C. D. divides 235 and 329, it will divide their difference, or 94. If it divides 94, it will divide twice 94, or 188. Since it divides 188 and 235, it will divide their difference, or 47. Since the G. C. D. divides 47 and 94, and as 47 is an exact divisor of 94, it is the largest number that will divide itself *and 94. Hence,* 47 is the G. C. D.

Rule.—*Divide the greater number by the less, and if there is a remainder, divide the divisor by it, and continue dividing the last divisor by the last remainder, until nothing remains. The last divisor will be the greatest common divisor.*

If more than two numbers are given, find the greatest common divisor of two of them, then of this divisor and one of the other numbers, and so on, to the last. The last divisor will be the greatest common divisor.

Find the greatest common divisor of:

2. 192, 480.
3. 247, 323.
4. 945, 1050.
5. 1560, 2808.
6. 294, 357, 399.
7. 592, 918, 1029.

Reduce the following fractions to their lowest terms by finding the greatest common divisor of their terms and then dividing both numerator and denominator by the G. C. D.:

8. $\frac{145}{203}$
9. $\frac{224}{280}$
10. $\frac{282}{810}$
11. $\frac{344}{559}$
12. $\frac{495}{891}$
13. $\frac{546}{882}$
14. $\frac{1781}{4884}$
15. $\frac{49558}{78892}$

16. A farmer has three tracts of land containing 240, 320, 440 acres, respectively, which he wishes to divide into the largest possible fields of equal size. How many acres will there be in each field, and how many fields in each tract?

LEAST COMMON MULTIPLE.

136. A **Multiple** of a number is any number that will exactly contain it.

Thus, 15 is a multiple of 5; 63 is a multiple of 9; and 80 is a multiple of 16.

137. A **Common Multiple** of two or more numbers is any number that will exactly contain each of them.

Thus, 48 is a common multiple of 6, 8, and 12; and 60 is a common multiple of 5, 10, and 20.

138. The **Least Common Multiple** of two or more numbers is the least number that will exactly contain each of them.

Thus, 12 is the least common multiple of 3, 4, and 6; and 40 is the least common multiple of 5, 8, and 10.

The initials L. C. M. may be used for least common multiple.

139. Principle.—*The least common multiple of two or more numbers is equal to the product of all the different prime factors, each used the greatest number of times they occur in any one number.*

WRITTEN EXERCISES.

140. 1. Find the least common multiple of 12, 14, and 20.

$12 = 2 \times 2 \times 3$
$14 = 2 \times 7$
$20 = 2 \times 2 \times 5$
L. C. M. $= 2 \times 2 \times 3 \times 5 \times 7 = 420$

EXPLANATION.—The numbers are first separated into prime factors. The factor 2 occurs the greatest number of times but twice in any one number, so that it must appear twice as factors of the least common multiple. The factors 3, 5, and 7 are the other different factors, and they appear but once in any number, hence each of them must appear but once as factors of the least common multiple. Therefore the product of 2, 2, 3, 5, and 7, or 420, is the least common multiple of the numbers.

2	12	14	20
2	6	7	10
	3	7	5

L. C. M. $= 2 \times 2 \times 3 \times 7 \times 5 = 420$

EXPLANATION.—The least common multiple of the numbers may also be found by placing them in a horizontal line and dividing them by any prime number that will exactly divide at least two of them, and thus continue until quotients are found that are prime to one another. The product of the divisors and quotients will be the least common multiple.

In finding the least common multiples of numbers, all numbers that are factors of other given numbers may be disregarded. Thus, the least common multiple of 12, 15, 36, and 90 is the same as the least common multiple of 36 and 90; for 12 is a factor of 36, and 15 is a factor of 90.

Find the least common multiple of:

2. 18, 20, 24.
3. 48, 60, 56.
4. 33, 27, 88.
5. 25, 60, 45.
6. 16, 56, 32, 112.
7. 21, 45, 84, 15.
8. 54, 72, 86, 24.
9. 128, 114, 192, 285.

Find the least common multiple of the denominators of the following fractions:

10. $\frac{3}{7}, \frac{5}{21}, \frac{11}{12}$.
11. $\frac{4}{11}, \frac{3}{8}, \frac{27}{55}$.
12. $\frac{7}{15}, \frac{9}{20}, \frac{13}{45}$.
13. $\frac{5}{9}, \frac{4}{27}, \frac{20}{81}, \frac{42}{45}$.
14. $\frac{2}{3}, \frac{7}{15}, \frac{8}{75}, \frac{57}{120}$.
15. $\frac{6}{13}, \frac{7}{25}, \frac{14}{35}, \frac{15}{91}$.

16. What is the smallest piece of land that can be divided into fields of 16, 18, and 20 acres each?

141. In the operations of practical arithmetic it is seldom necessary to find the least common multiple of numbers greater than 100. In such cases the factors composing the least common multiple *may be obtained by inspection.*

When numbers cannot be readily factored by inspection, the factors may be found by the method of finding the greatest common divisor of such numbers (Art. 131).

1. Find the least common multiple of 345 and 391.

345) 391 (1
345
46) 345 (7
322
G. C. D. = 23) 46 (2
46
23) 345 391
15 17
L. C. M. = 23 × 15 × 17 = 5865

EXPLANATION.—Since the factors can not be obtained easily by inspection, the G. C. D. is found, which is 23.

Dividing the given numbers by 23 the quotients 15 and 17 are obtained, which factors are all prime to one another. Hence, the product of 23, 15, and 17, or 5865, is the L. C. M.

RULE.—*Find the greatest common divisor of the numbers and divide each of the given numbers by it. Then take the product of the greatest common divisor and the quotients. The result will be the least common multiple.*

Find the least common multiple of:

2. 147, 171.
3. 189, 204.
4. 321, 411.
5. 342, 399.
6. 357, 425.
7. 414, 460.
8. 837, 972.
9. 1023, 1271.
10. 2565, 3285.

11. A can walk around a certain island in 16 hrs., B in 18 hrs., and C in 20 hrs. If they start together and keep walking each at his own rate, how many days will elapse before they are all three together at the starting point, and how many times will each have gone around the island?

12. The periods of three planets which move uniformly in similar orbits around the sun, are respectively 275, 325, and 365 days. Supposing all three in conjunction at a given moment, in how many days will they be in conjunction again?

FRACTIONS.

142. A **Fraction** is one or more of the equal parts of anything.

A fraction is expressed by two numbers, written one above the other with a line between them. Thus, $\frac{3}{4}$ is a fraction.

143. The **Denominator** of a fraction shows into how many parts a thing has been divided.

It is written below the line.

Thus, in the fraction $\frac{3}{4}$, 4 is the denominator, and shows that something has been divided into 4 equal parts.

144. The **Numerator** of a fraction shows how many parts form the fraction.

It is written above the line.

Thus, in the fraction $\frac{3}{4}$, 3 is the numerator. It shows that the fraction contains 3 of the 4 equal parts.

145. The **Terms of a Fraction** are the numerator and denominator together.

146. A **Proper Fraction** is one whose numerator is less than its denominator.

Thus, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{13}{15}$ are proper fractions.
The value of a proper fraction is less than 1.

147. An **Improper Fraction** is one whose numerator equals or exceeds its denominator.

Thus, $\frac{5}{5}$, $\frac{8}{7}$, $\frac{21}{20}$, and $\frac{43}{9}$ are improper fractions.
The value of an improper fraction is 1 or more than 1.

148. A **Mixed Number** is a number expressed by an integer and a fraction.

Thus, $3\frac{1}{2}$, $7\frac{3}{4}$, *and* $18\frac{7}{11}$ are mixed numbers.

149. The **Unit of the Fraction** is the unit which is divided into equal parts.

1. A *Common Fraction* is one whose unit has been divided into *any number* of equal parts.
2. A *Decimal Fraction* is one whose unit has been divided into tenths, hundreds, thousands, etc.

150. A **Fractional Unit** is one of the equal parts into which a unit has been divided.

151. An **Integer** may be expressed fractionally by writing 1 under it as a denominator.

Thus, $7 = \frac{7}{1}$ and may be written instead of 7.

152. The **Reciprocal** of a number is 1 divided by that number.

Thus, the reciprocal of 4 is $\frac{1}{4}$.

1. A fraction may be regarded as *a number of equal parts of a unit, a number of equal parts* of *one* thing, or *one equal part* of a *number of things.* Thus, three fourths may be regarded as three fourths of one, three fourths of one thing, or one fourth of three things.
2. A fraction also expresses *unexecuted division.*

153. Fractions are read by naming first the number of fractional units, and then the kind.

Thus, $\frac{5}{9}$ is read five ninths; $\frac{6}{41}$ is read six forty-firsts.

Copy, read, and tell the kind of fractions:

$\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{7}$ $\frac{10}{18}$ $\frac{14}{17}$ $\frac{21}{31}$ $\frac{36}{51}$ $\frac{48}{67}$ $\frac{56}{87}$ $\frac{66}{91}$

$\frac{4}{15}$ $\frac{7}{8}$ $\frac{9}{5}$ $\frac{11}{32}$ $\frac{18}{19}$ $\frac{28}{25}$ $\frac{27}{37}$ $\frac{52}{78}$ $\frac{90}{41}$ $\frac{17}{16}$

$\frac{19}{124}$ $\frac{42}{212}$ $\frac{314}{45}$ $\frac{408}{407}$ $\frac{114}{467}$ $\frac{219}{516}$ $\frac{315}{628}$ $\frac{511}{718}$ $\frac{781}{869}$

Express in figures:

1. Five sixths. Six ninths. Eight elevenths.
2. Three tenths. Twelve elevenths. Four sevenths.
3. Eight twentieths. Seventeen nineteenths.
4. Nine and two thirds. Five and three fifths.
5. Sixty-two forty-eighths. Thirteen twenty-fifths.

154. To **Analyze** a fraction is to explain what is expressed by it.

1. Analyze the fraction $\frac{3}{4}$.

EXPLANATION.—$\frac{3}{4}$ represents 3 of 4 equal parts of a thing; or it means *three fourths of 3* or 3 divided by 4. It is read three fourths.

Analyze the following:

2. $\frac{2}{5}$ $\frac{3}{7}$ $\frac{5}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{15}$ $\frac{15}{14}$ $\frac{14}{15}$ $\frac{15}{16}$ $\frac{16}{17}$

3. $\frac{9}{21}$ $\frac{18}{35}$ $\frac{42}{56}$ $\frac{19}{22}$ $\frac{54}{63}$ $\frac{49}{61}$ $\frac{75}{88}$ $\frac{87}{91}$ $\frac{98}{98}$ $\frac{98}{99}$

155. A fraction may be regarded as indicated division (Art. 153), the numerator answering to the dividend, and the denominator to the divisor. Hence,

156. PRINCIPLES.—1. *Multiplying the numerator or dividing the denominator of a fraction by any number multiplies the value of the fraction by that number.*

2. *Dividing the numerator or multiplying the denominator of a fraction by any number divides the value of the fraction by that number.*

3. *Multiplying or dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction.*

REDUCTION.

157. **Reduction of Fractions** is the process of changing the forms of fractions without changing their values.

158. **To reduce whole or mixed numbers to improper fractions.**

1. Reduce $8\frac{2}{3}$ to thirds.

$8\frac{2}{3}$
3
$\overline{26}$
$\overline{3}$

EXPLANATION.—In *one* there are 3 thirds, and in 8 there are 8 times 3 thirds, or $\frac{24}{3}$, to which we add the $\frac{2}{3}$. $\frac{24}{3}$ plus $\frac{2}{3}$ are $\frac{26}{3}$.

In practice, we multiply the integer, 8, by the denominator, 3, and to the product add the numerator, 2; then write the sum over the denominator, 3.

RULE.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and under the result write the denominator.*

Reduce the following to improper fractions:

2. $7\frac{3}{5}$	9. $28\frac{3}{7}$	16. $95\frac{17}{21}$	23. $123\frac{3}{8}$
3. $9\frac{3}{4}$	10. $31\frac{4}{9}$	17. $78\frac{12}{25}$	24. $214\frac{7}{11}$
4. $12\frac{5}{6}$	11. $43\frac{2}{3}$	18. $86\frac{52}{61}$	25. $345\frac{14}{25}$
5. $13\frac{3}{4}$	12. $51\frac{5}{6}$	19. $67\frac{64}{75}$	26. $500\frac{27}{31}$
6. $15\frac{5}{8}$	13. $63\frac{4}{7}$	20. $59\frac{81}{82}$	27. $640\frac{65}{88}$
7. $17\frac{3}{8}$	14. $74\frac{11}{12}$	21. $87\frac{78}{91}$	28. $786\frac{87}{96}$
8. $20\frac{1}{9}$	15. $83\frac{14}{15}$	22. $99\frac{98}{99}$	29. $960\frac{91}{100}$

30. Change 15 to 4ths; 20 to 6ths; 36 to 10ths; 25 to 12ths; 34 to 15ths; 45 to 20ths; 50 to 25ths; 60 to 20ths.

159. To reduce improper fractions to whole or mixed numbers.

1. Reduce $\frac{75}{8}$ to a mixed number.

$\frac{75}{8} = 75 \div 8 = 9\frac{3}{8}$

or

$8\overline{)75}\ 9\frac{3}{8}$

EXPLANATION.—In *one* there are 8 eighths, and in $\frac{75}{8}$ there are as many ones as 8 is contained times in 75, or $9\frac{3}{8}$.

RULE.—*Divide the numerator by the denominator.*

Reduce to whole or mixed numbers:

2. $\frac{25}{6}$	8. $\frac{135}{15}$	14. $\frac{1428}{27}$	20. $\frac{213465}{124}$
3. $\frac{34}{5}$	9. $\frac{179}{20}$	15. $\frac{28315}{86}$	21. $\frac{3241416}{130}$
4. $\frac{48}{7}$	10. $\frac{212}{45}$	16. $\frac{17388}{56}$	22. $\frac{4381175}{214}$
5. $\frac{57}{8}$	11. $\frac{318}{81}$	17. $\frac{81142}{61}$	23. $\frac{5106368}{864}$
6. $\frac{65}{9}$	12. $\frac{427}{48}$	18. $\frac{43177}{75}$	24. $\frac{7283411}{4115}$
7. $\frac{838}{12}$	13. $\frac{5388}{51}$	19. $\frac{53455}{80}$	25. $\frac{8156682}{500}$

160. To reduce fractions to higher terms.

A fraction is expressed in higher terms when the numerator and denominator are expressed by larger numbers.

1. How many 24ths in $\frac{5}{6}$?

$24 \div 6 = 4$
$5 \times 4 = 20$
$6 \times 4 = 24$

EXPLANATION.—Since multiplying both terms of a fraction by the same number does not change its value (Prin. 3), we multiply both terms by the number which will cause the denominator 6 to become 24.

By dividing 24 by 6 we see that the multiplier is 4. Hence, $\frac{5}{6} = \frac{20}{24}$.

RULE.—*Multiply both terms of the fraction by the number that will give the required denominator.*

Change the following:

2. $\frac{5}{8}$ to 16ths.	8. $\frac{17}{21}$ to 84ths.	14. $\frac{51}{60}$ to 120ths.
3. $\frac{7}{9}$ to 27ths.	9. $\frac{23}{33}$ to 99ths.	15. $\frac{72}{88}$ to 249ths.
4. $\frac{11}{12}$ to 36ths.	10. $\frac{15}{24}$ to 120ths.	16. $\frac{67}{78}$ to 156ths.
5. $\frac{13}{14}$ to 28ths.	11. $\frac{27}{34}$ to 136ths.	17. $\frac{85}{91}$ to 273rds.
6. $\frac{13}{20}$ to 40ths.	12. $\frac{15}{48}$ to 144ths.	18. $\frac{29}{38}$ to 190ths.
7. $\frac{14}{15}$ to 75ths.	13. $\frac{31}{45}$ to 180ths.	19. $\frac{89}{57}$ to 228ths.

161. To reduce fractions to lower terms.

162. A fraction is expressed in lower terms when its numerator and denominator are expressed in smaller numbers.

163. A fraction is expressed in its lowest terms when its numerator and denominator have no common divisor.

1. Reduce $\frac{25}{30}$ to its lowest terms.

$5)\frac{25}{30} = \frac{5}{6}$

EXPLANATION.—Since dividing both terms of a fraction by the same number does not change its value (Prin. 3), we may reduce $\frac{25}{30}$ to lower terms by dividing both numerator and denominator by 5. Dividing we have $\frac{25}{30} = \frac{5}{6}$. Since 5 and 6 have no common divisor, the fraction is in its lowest terms.

RULE.—*Divide the terms of the fraction successively by their common factors until the numerator and denominator have no common divisor.*

Reduce to their lowest terms:

2. $\frac{12}{16}$, $\frac{14}{21}$.
3. $\frac{18}{27}$, $\frac{20}{25}$.
4. $\frac{24}{30}$, $\frac{27}{33}$.
5. $\frac{32}{40}$, $\frac{35}{49}$.
6. $\frac{48}{72}$, $\frac{50}{75}$.
7. $\frac{63}{84}$, $\frac{72}{96}$.
8. $\frac{75}{100}$, $\frac{100}{110}$.
9. $\frac{112}{124}$, $\frac{124}{186}$.
10. $\frac{186}{148}$, $\frac{148}{156}$.
11. $\frac{160}{180}$, $\frac{172}{196}$.
12. $\frac{184}{198}$, $\frac{160}{172}$.
13. $\frac{188}{196}$, $\frac{175}{200}$.
14. $\frac{192}{216}$, $\frac{176}{220}$.
15. $\frac{240}{360}$, $\frac{288}{480}$.
16. $\frac{195}{210}$. $\frac{272}{424}$.
17. $\frac{252}{396}$, $\frac{315}{378}$.
18. $\frac{364}{488}$, $\frac{375}{500}$.
19. $\frac{416}{582}$, $\frac{470}{510}$.
20. $\frac{306}{408}$, $\frac{288}{504}$.
21. $\frac{340}{408}$, $\frac{365}{511}$.
22. $\frac{420}{588}$, $\frac{399}{456}$.
23. $\frac{320}{448}$, $\frac{413}{581}$.
24. $\frac{576}{720}$, $\frac{425}{595}$.
25. $\frac{644}{736}$, $\frac{760}{855}$.

164. To reduce dissimilar fractions to similar fractions.

165. Similar Fractions are those which have like denominators.

166. Dissimilar Fractions are those which have different denominators.

167. Fractions have a **Common Denominator** when all their denominators are alike.

168. Fractions have their **Least Common Denominator** when expressed in their lowest common denominator.

1. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to similar fractions.

$\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$

$\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$

$\frac{7 \times 3}{8 \times 3} = \frac{21}{24}$

EXPLANATION.—Since the fractions are to be reduced to a common denominator, we must multiply the terms of each fraction by some number that will cause them to have the same denominator (Prin. 3).

By inspecting the denominators 4, 6, and 8, we find that 24 will contain each of them as exact divisors. Hence, changing each fraction to 24ths (Art. 160), we have the similar *fractions* $\frac{18}{24}$, $\frac{20}{24}$, and $\frac{21}{24}$.

Reduce to similar fractions:

2. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$.
3. $\frac{2}{3}, \frac{5}{6}, \frac{5}{12}$.
4. $\frac{1}{3}, \frac{3}{5}, \frac{1}{6}$.
5. $\frac{3}{4}, \frac{1}{8}, \frac{5}{16}$.
6. $\frac{3}{5}, \frac{7}{10}, \frac{2}{3}$.
7. $\frac{3}{4}, \frac{1}{2}, \frac{3}{24}$.
8. $\frac{4}{5}, \frac{2}{3}, \frac{4}{15}$.
9. $\frac{2}{9}, \frac{3}{4}, \frac{1}{18}$.
10. $\frac{3}{4}, \frac{2}{5}, \frac{7}{8}$.
11. $\frac{5}{6}, \frac{3}{7}, \frac{1}{14}$.
12. $\frac{3}{8}, \frac{5}{16}, \frac{1}{4}$.
13. $\frac{3}{10}, \frac{2}{5}, \frac{5}{6}$.
14. $\frac{3}{7}, \frac{5}{8}, \frac{5}{14}$.
15. $\frac{2}{9}, \frac{3}{4}, \frac{5}{36}$.
16. $\frac{1}{4}, \frac{3}{8}, \frac{1}{32}$.
17. $\frac{3}{10}, \frac{4}{5}, \frac{7}{15}$.
18. $\frac{3}{20}, \frac{4}{5}, \frac{7}{8}$.
19. $\frac{1}{4}, \frac{3}{11}, \frac{9}{22}$.
20. $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}$.
21. $\frac{2}{9}, \frac{7}{10}, \frac{1}{3}$.
22. $\frac{7}{15}, \frac{5}{6}, \frac{1}{5}$.
23. $\frac{3}{14}, \frac{4}{7}, \frac{5}{18}$.
24. $\frac{7}{8}, \frac{11}{12}, \frac{13}{14}$.
25. $\frac{3}{4}, \frac{14}{25}, \frac{13}{20}$.

26. Reduce $\frac{1}{3}, \frac{3}{8}, \frac{5}{12}$, and $\frac{7}{16}$ to their least common denominator.

3	3	8	12	16
2		8	4	16
2		4	2	8
2		2		4
				2

$3 \times 2 \times 2 \times 2 \times 2$, or $3 \times 2^4 = 48$.

EXPLANATION.—The least common denominator cannot always be easily found by inspection. It may then be found by taking the product of all the different prime factors of the denominators as in the margin.

3 is a prime of 3 and 12, and therefore a factor of the least common denominator. Dividing by 3, and writing below the quotients, when more than a unit, and numbers of which 3 is not a factor, we have 8, 4, 16. We continue in the same manner to divide by prime factors until no two numbers having a common prime factor remain. The factors of the least common denominator are found to be the divisors 3, 2, 2, 2, and the factor 2 in the last row. Taking the product of the factors, we have 48 as the least common denominator. Changing each fraction to 48ths (Art. 160), we have $\frac{16}{48}, \frac{18}{48}, \frac{20}{48}, \frac{21}{48}$.

1. Care should be taken to see that the fractions are in their lowest terms before finding their least common denominator.

2. In finding the factors of the least common denominator, a number that is a factor of another number may be disregarded. Thus, since 8 is a factor of 16, it might have been disregarded, and the factors of 3, 12 and 16 only found. By cancelling a number that is divided by the same number we also avoid unnecessary work.

Reduce to their least common denominator:

27. $\frac{2}{3}, \frac{5}{6}, \frac{2}{9}, \frac{7}{12}$.
28. $\frac{1}{5}, \frac{1}{3}, \frac{7}{10}, \frac{4}{15}$.
29. $\frac{3}{4}, \frac{9}{10}, \frac{5}{6}, \frac{7}{12}$.
30. $\frac{3}{7}, \frac{2}{3}, \frac{11}{14}, \frac{5}{28}$.
31. $\frac{5}{6}, \frac{3}{5}, \frac{13}{20}, \frac{7}{24}$.
32. $\frac{3}{14}, \frac{9}{20}, \frac{13}{35}, \frac{7}{10}$.
33. $\frac{4}{5}, \frac{5}{6}, \frac{11}{12}, \frac{7}{8}$,
34. $\frac{7}{9}, \frac{9}{16}, \frac{11}{48}, \frac{5}{18}$.
35. $\frac{3}{5}, \frac{7}{35}, \frac{22}{75}, \frac{2}{3}$.
36. $\frac{3}{4}, \frac{7}{8}, \frac{11}{12}, \frac{13}{15}$.
37. $\frac{5}{7}, \frac{4}{9}, \frac{7}{21}, \frac{5}{14}$.
38. $\frac{4}{11}, \frac{3}{33}, \frac{8}{44}, \frac{2}{5}$.
39. $\frac{7}{20}, \frac{3}{5}, \frac{16}{80}, \frac{2}{3}$.
40. $\frac{9}{18}, \frac{12}{27}, \frac{8}{36}, \frac{17}{45}$.

ADDITION.

169. Addition of Fractions is the process of finding the sum of two or more fractions.

170. PRINCIPLE.—*Only similar fractions can be added.*

1. What is the sum of $\frac{3}{9}$, $\frac{4}{9}$, and $\frac{7}{9}$?

$\frac{3}{9}+\frac{4}{9}+\frac{7}{9}=\frac{14}{9}=1\frac{5}{9}$

EXPLANATION.—Since the fractions are similar, we add their numerators and write the sum, 14, over the common denominator, making $\frac{14}{9}$, or $1\frac{5}{9}$.

2. What is the sum of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{5}{8}$?

$\frac{3}{4}+\frac{5}{6}+\frac{5}{8}=$
$\frac{18}{24}+\frac{20}{24}+\frac{15}{24}=$
$\frac{53}{24}=\frac{17}{8}=2\frac{1}{8}$

EXPLANATION.—Since the fractions are dissimilar, they must be changed to similar fractions before adding.

The least common denominator of the given fractions is 24. $\frac{3}{4}=\frac{18}{24}$; $\frac{5}{6}=\frac{20}{24}$; and $\frac{5}{8}=\frac{15}{24}$. Hence, the sum is $\frac{53}{24}$, or $\frac{17}{8}$, or $2\frac{1}{8}$.

3. What is the sum of $5\frac{1}{2}$, $6\frac{3}{8}$, and $7\frac{3}{5}$?

$5\frac{1}{2}=5\frac{5}{10}$
$6\frac{3}{8}=6\frac{20}{80}$
$7\frac{3}{5}=7\frac{48}{80}$
$19\frac{23}{80}$

EXPLANATION.—Since the numbers are composed of both integers and fractions, we may add them separately and take their sums. The sum of the fractions is $\frac{73}{80}$, or $1\frac{23}{80}$; the sum of the integers is 18. The sum of both is $19\frac{23}{80}$.

RULE.—*Reduce the given fractions to similar fractions, add the numerators, and write the sum over the common denominator.*

When there are mixed numbers, add the fractions and the integers separately, and then unite their sums.

1. Before adding see that the given fractions are in their lowest terms.

2. Sometimes only the numerator is written to express fourths, or quarters. Thus, 6^1 for $6\frac{1}{4}$; 9^3 for $9\frac{3}{4}$.

Find the sum of the following:

4. $\frac{3}{13}$, $\frac{4}{13}$, $\frac{5}{13}$, $\frac{7}{13}$.

5. $\frac{2}{5}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{2}$.

6. $\frac{3}{5}$, $\frac{7}{8}$, $\frac{5}{12}$, $\frac{1}{8}$.

7. $\frac{4}{7}$, $\frac{1}{4}$, $\frac{9}{14}$, $\frac{11}{28}$.

8. $\frac{2}{5}$, $\frac{6}{8}$, $\frac{9}{12}$, $\frac{4}{15}$.

9. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{18}$, $\frac{2}{6}$,

10. $\frac{7}{10}$, $\frac{3}{20}$, $\frac{2}{15}$, $\frac{9}{30}$.

11. $\frac{5}{9}$, $\frac{7}{8}$, $\frac{5}{36}$, $\frac{11}{18}$.

12. $\frac{7}{12}$, $\frac{5}{8}$, $\frac{7}{16}$, $\frac{10}{24}$.

13. $\frac{8}{9}$, $\frac{2}{8}$, $\frac{11}{15}$, $\frac{7}{10}$.

14. $\frac{4}{13}$, $\frac{6}{39}$, $\frac{4}{26}$, $\frac{8}{52}$.

15. $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$.

16. $2\frac{1}{2}$, $5\frac{3}{4}$, $7\frac{1}{8}$.

17. $9\frac{2}{5}$, $8\frac{3}{4}$, $12\frac{1}{2}$.

18. 6^1, 9^3, 18^2.

19. 8^3, 7^1, 25^3.

20. $6\frac{2}{3}$, $8\frac{3}{4}$, $7\frac{5}{8}$.

21. $5\frac{3}{5}$, $7\frac{2}{3}$, $6\frac{3}{7}$.

22. $46\frac{2}{3}$, $25\frac{3}{16}$, $41\frac{5}{12}$.

23. $16\frac{4}{9}$, $48\frac{1}{10}$, $65\frac{5}{6}$.

24. $112\frac{3}{4}$, $125\frac{3}{5}$, $180\frac{5}{6}$.

25. $205\frac{3}{16}$, $378\frac{4}{7}$, $437\frac{5}{8}$.

PRACTICAL PROBLEMS.

171. 1. The exact weights of three boxes of goods were $78\frac{1}{2}$ pounds, $134\frac{3}{4}$ pounds, and $206\frac{5}{8}$ pounds, respectively. What was the total weight?

2. The four quarters of a beef weighed respectively $146\frac{1}{2}$, $150\frac{7}{8}$, $172\frac{3}{4}$, and $174\frac{3}{16}$ pounds. How much was the entire weight of the four quarters?

3. Peter has $\$5\frac{1}{5}$; George, $\$8\frac{1}{10}$; Jacob, $\$2\frac{3}{4}$; and their father, $\$27\frac{1}{2}$. How much have they together?

4. Mr. G. walked $21\frac{3}{4}$ miles on Monday, $22\frac{1}{2}$ miles on Tuesday, $24\frac{5}{32}$ miles on Wednesday, and 75 miles during the next three days. How far did he walk in the whole week?

5, Cyrus bought $75\frac{3}{56}$ bushels of corn, $41\frac{1}{4}$ bushels of oats, $20\frac{1}{2}$ bushels of rye, and 10 bushels of barley. How many bushels of grain did he buy?

6. Harry picked $6\frac{5}{8}$ bushels of apples from one tree, $5\frac{7}{12}$ bushels from another, $10\frac{3}{10}$ bushels from another, and $15\frac{5}{6}$ bushels from another. How many bushels did he pick in all?

7. The distance from A to B is $3\frac{1}{4}$ miles, from B to C $5\frac{9}{16}$ miles, from C to D $8\frac{7}{64}$ miles, and from D to E $11\frac{19}{40}$ miles. How many miles from A to E?

8. A school-room is $40\frac{5}{8}$ feet long and $32\frac{2}{3}$ ft. wide. What is the distance around the room?

9. A merchant sold 5 pieces of cloth containing $28\frac{3}{4}$, $26\frac{4}{9}$, $27\frac{5}{12}$, $30\frac{7}{8}$, and $32\frac{5}{18}$ yards, respectively. How many yards did he sell?

10. Mr. H. has a farm of five fields containing, respectively, $18\frac{3}{4}$ acres, $20\frac{4}{5}$ acres, $21\frac{7}{40}$ acres, $22\frac{5}{16}$ acres, and $24\frac{9}{32}$ acres. How large is the farm?

SUBTRACTION.

172. Subtraction of Fractions is the process of finding the difference between two fractions.

173. PRINCIPLE.—*Only similar fractions can be subtracted.*

1. From $\frac{7}{12}$ subtract $\frac{5}{12}$.

$\frac{7}{12} - \frac{5}{12} = \frac{2}{12} = \frac{1}{6}$

EXPLANATION.—Since the fractions are similar, we take the difference between their numerators.

$\frac{7}{12} - \frac{5}{12} = \frac{2}{12}$, or $\frac{1}{6}$.

2. From $\frac{4}{5}$ subtract $\frac{3}{4}$.

$\frac{4}{5} - \frac{3}{4} =$
$\frac{16}{20} - \frac{15}{20} = \frac{1}{20}$

EXPLANATION.—Since the fractions are dissimilar, they must be changed to similar fractions before subtracting.

The least common denominator of the given fractions is 20. $\frac{4}{5} = \frac{16}{20}$ and $\frac{3}{4} = \frac{15}{20}$. $\frac{16}{20} - \frac{15}{20} = \frac{1}{20}$.

3. From $5\frac{2}{3}$ subtract $3\frac{4}{5}$.

$5\frac{2}{3} = 5\frac{10}{15}$
$3\frac{4}{5} = 3\frac{12}{15}$
$1\frac{13}{15}$

EXPLANATION.—Since the numbers are composed of integers and fractions, we subtract them separately.

We cannot subtract $\frac{12}{15}$ from $\frac{10}{15}$, hence we take 1 or $\frac{15}{15}$ from 5 and add it to the $\frac{10}{15}$, making $\frac{25}{15}$. $\frac{12}{15}$ from $\frac{25}{15}$ leaves $\frac{13}{15}$, and 3 from 4 leaves 1. Hence, the remainder is $1\frac{13}{15}$.

RULE.—*Reduce the given fractions to similar fractions, and write the difference of the numerators over the common denominator.*

When there are mixed numbers, subtract the fractions and the integers separately.

Solve:

4. $\frac{15}{17} - \frac{9}{17}$	13. $\frac{17}{20} - \frac{11}{15}$	22. $11\frac{1}{4} - 8\frac{3}{4}$
5. $\frac{5}{6} - \frac{2}{3}$	14. $\frac{23}{25} - \frac{11}{20}$	23. $20\frac{2}{8} - 16\frac{3}{4}$
6. $\frac{5}{9} - \frac{3}{7}$	15. $\frac{40}{63} - \frac{17}{56}$	24. $48\frac{2}{7} - 35\frac{3}{5}$
7. $\frac{4}{5} - \frac{5}{8}$	16. $\frac{69}{70} - \frac{33}{40}$	25. $87\frac{7}{12} - 53\frac{3}{9}$
8. $\frac{8}{9} - \frac{3}{8}$	17. $\frac{71}{78} - \frac{41}{50}$	26. $63\frac{5}{13} - 49\frac{5}{6}$
9. $\frac{11}{12} - \frac{9}{11}$	18. $\frac{65}{81} - \frac{13}{27}$	27. $75\frac{5}{8} - 54\frac{11}{14}$
10. $\frac{13}{15} - \frac{5}{7}$	19 $\frac{75}{88} - \frac{55}{77}$	28. $80\frac{2}{7} - 70\frac{7}{12}$
11. $\frac{13}{14} - \frac{7}{12}$	20. $\frac{87}{90} - \frac{49}{60}$	29. $91\frac{13}{21} - 78\frac{9}{14}$
12. $\frac{9}{16} - \frac{5}{18}$	21. $\frac{90}{100} - \frac{50}{70}$	30. $99\frac{999}{1000} - 90\frac{109}{110}$

PRACTICAL PROBLEMS.

174. 1. Mr. A. sold $17\frac{3}{4}$ yards of muslin from a piece containing $32\frac{1}{2}$ yards. How many yards were left?

2. John gave $\frac{3}{10}$ of his apples to David and $\frac{3}{8}$ to Jacob. What part of his apples had he left?

3. Mr. King's lot contains an acre of land. If he should sell $\frac{1}{2}$ of an acre to one man, and $\frac{1}{5}$ of an acre to another, how much would remain?

4. A farmer had 100 bushels of wheat. He sold at one time $35\frac{17}{60}$ bushels, and at another time $43\frac{21}{60}$ bushels. How many bushels had he left?

5. A tub of butter weighed $63\frac{1}{4}$ pounds. The weight of the tub alone was $4\frac{5}{8}$ pounds. What was the weight of the butter?

6. From a pole whose length was $70\frac{7}{10}$ feet, $13\frac{7}{8}$ feet were broken off. How long was the pole after the break?

7. A man having $95\frac{1}{4}$ gallons of oil, sold $13\frac{3}{8}$ gallons. How much had he left?

8. James and Henry together have $1\frac{1}{4}$ bushels of chestnuts. James claims $\frac{7}{8}$ of a bushel. How much is Henry's share?

MULTIPLICATION.

175. Multiplication of Fractions is the process of finding the product when one or both factors are fractions.

176. To multiply a fraction by an integer.

PRINCIPLE.—*Multiplying the numerator or dividing the denominator of a fraction by any number multiplies the value of the fraction by that number* (*Prin. 1, Art. 156*).

1. At $\$\frac{3}{8}$ a yard, what will 4 yards of cloth cost?

$\$\frac{3}{8} \times 4 = \$\frac{12}{8} = \$1\frac{4}{8} = \$1\frac{1}{2}$

Or,

$\$\frac{3}{8} \times 4 = \frac{\$3}{8 \div 4} = \$\frac{3}{2} = \$1\frac{1}{2}$

EXPLANATION.—If one yard of cloth costs $\$\frac{3}{8}$, 4 yards will cost 4 times $\$\frac{3}{8}$ or $\$\frac{12}{8}$, equal to $1\frac{4}{8}$, or $\$1\frac{1}{2}$.

Or, since dividing the denominator of a fraction by any number multiplies the value of the fraction by that number (Prin.), we divide the denominator of $\$\frac{3}{8}$ by 4, which gives us $\$\frac{3}{2}$ or $\$1\frac{1}{2}$.

The latter process may be shortened by cancellation.

Multiply:

2. $\frac{7}{12}$ by 4.	5. $\frac{4}{15}$ by 5.	8. $\frac{5}{7}$ by 12.
3. $\frac{5}{14}$ by 7.	6. $1\frac{3}{8}$ by 9.	9. $1\frac{3}{4}$ by 21.
4. $\frac{8}{11}$ by 3.	7. $\frac{6}{17}$ by 4.	10. $1\frac{2}{15}$ by 35.

11. If a bushel of corn costs $\$\frac{2}{3}$, what will be the cost of 75 bushels?

12. If a man earns $\$\frac{7}{20}$ in an hour, how much can he earn in 10 hours?

13. What will be the cost of 100 arithmetics at $\$\frac{3}{5}$ each?

14. A farmer sold 60 bushels of rye at $\$1\frac{3}{10}$ per bushel. How much did he receive for it?

177. To multiply an integer by a fraction.

178. To multiply an integer by a fraction is to take the same number of parts of the integer as the fraction is of a unit.

1. If a ton of coal costs \$6, what will $\frac{3}{4}$ of a ton cost?

$\$6 \times \frac{3}{4} = \$\frac{6 \times 3}{4} = \$\frac{18}{4} = \$4\frac{2}{4} = \$4\frac{1}{2}$

EXPLANATION.—If 1 ton of coal costs \$6, $\frac{1}{4}$ of a ton will cost $\frac{1}{4}$ of \$6. $\frac{1}{4}$ of \$6 is $\$\frac{6}{4}$, and $\frac{3}{4}$ of \$6 are 3 times $\$\frac{6}{4} = \$\frac{18}{4}$, $= \$4\frac{1}{2}$.

2. Multiply 9 by $\frac{3}{5}$, or find $\frac{3}{5}$ of 9.

$9 \times \frac{3}{5} = \frac{9 \times 3}{5} = \frac{27}{5}$, or $5\frac{2}{5}$

EXPLANATION.—To multiply 9 by $\frac{3}{5}$ is to find $\frac{3}{5}$ of 9, or 3 times $\frac{1}{5}$ of 9. $\frac{1}{5}$ of 9 is $\frac{9}{5}$, and 3 times $\frac{9}{5}$ are $\frac{27}{5}$, or $5\frac{2}{5}$.

Multiply:

3. 12 by $\frac{3}{4}$.
4. 18 by $\frac{5}{6}$.
5. 20 by $\frac{4}{5}$.
6. 35 by $\frac{6}{7}$.
7. 19 by $\frac{3}{5}$.
8. 42 by $\frac{7}{8}$.
9. 72 by $\frac{4}{9}$.
10. 120 by $\frac{7}{12}$.
11. 127 by $\frac{14}{20}$.

12. If a ton of hay is worth \$18, how much is $\frac{3}{4}$ of a ton worth?

13. A certain farm is valued at \$8000, and Mr. M. owns $\frac{4}{5}$ of it. What is the value of his share of the farm?

179. To multiply a fraction by a fraction.

1. Find $\frac{4}{9}$ of $\frac{3}{5}$, or multiply $\frac{3}{5}$ by $\frac{4}{9}$.

$\frac{4}{9}$ of $\frac{3}{5} = \frac{3}{5} \times \frac{4}{9} = \frac{4}{15}$

EXPLANATION.—To multiply $\frac{3}{5}$ by $\frac{4}{9}$ is to find $\frac{4}{9}$ of $\frac{1}{5}$, or 4 times $\frac{1}{9}$ of $\frac{1}{5}$. $\frac{1}{9}$ of $\frac{1}{5}$ is $\frac{1}{45}$ (Prin. 2, Art. 156), and 4 times $\frac{1}{9}$ of $\frac{1}{5} = 4$ times $\frac{1}{45} = \frac{4}{45}$, or $\frac{4}{15}$ (Prin. 3, Art. 156).

RULE.—*Reduce all integers and mixed numbers to improper fractions.*

Find the product of the numerators for the numerator of the product, and of the denominators for its denominator, cancelling common factors.

1. The word *of*, between fractions, is equivalent to the *sign of multiplication*. Thus, $\frac{4}{9}$ of $\frac{3}{5}$ is equal to $\frac{3}{5} \times \frac{4}{9}$.

2. A *fraction of a fraction*, as $\frac{3}{5}$ of $\frac{4}{9}$, is sometimes called a *compound fraction*.

3. Integers may be expressed as improper fractions, by writing 1 as a denominator. Thus, 7 may be written as $\frac{7}{1}$.

Find the products:

2. $\frac{2}{3}$ of $\frac{3}{4}$.
3. $\frac{4}{5}$ of $\frac{7}{8}$.
4. $\frac{8}{9}$ of $\frac{3}{16}$.
5. $\frac{11}{15}$ of $\frac{12}{14}$.
6. $\frac{14}{17}$ of $\frac{8}{7}$.
7. $\frac{20}{33}$ of $\frac{22}{30}$.
8. $\frac{32}{45}$ of $\frac{15}{16}$.
9. $\frac{27}{38}$ of $\frac{19}{36}$.
10. $\frac{25}{36} \times \frac{6}{10}$.
11. $\frac{32}{42} \times \frac{49}{56}$.
12. $2\frac{1}{2} \times \frac{4}{5}$.
13. $2\frac{3}{4} \times 2\frac{2}{3}$.
14. $3\frac{1}{4} \times \frac{8}{13}$.
15. $7\frac{1}{2} \times 5\frac{1}{3}$.
16. $5\frac{1}{4} \times 2\frac{1}{7}$.
17. $4\frac{2}{5} \times 2\frac{3}{11}$.
18. $\frac{4}{5} \times \frac{10}{12} \times \frac{3}{5}$.
19. $\frac{3}{7} \times \frac{14}{15} \times \frac{21}{24}$.
20. $\frac{12}{15} \times \frac{3}{4} \times \frac{7}{9}$.
21. $\frac{8}{19} \times \frac{18}{32} \times \frac{15}{24}$.
22. $\frac{7}{11} \times \frac{3}{4} \times \frac{20}{21}$.
23. $\frac{14}{15} \times \frac{2}{3} \times 6$.
24. $\frac{28}{44} \times \frac{11}{24} \times 2\frac{2}{3}$.
25. $\frac{45}{64} \times \frac{16}{35} \times 35$.

26. Multiply $3\frac{2}{5}$ by 12.

$3\frac{2}{5}$
12
$\frac{14}{5}$
36
$40\frac{4}{5}$

EXPLANATION.—Beginning at the right, as in whole numbers, to multiply, we have 12 times $\frac{2}{5}$ are $\frac{24}{5}$, or $4\frac{4}{5}$. 12 times 3 are 36. Adding the partial products, we have $40\frac{4}{5}$, the entire product. Or the product may be found by the general rule. Thus,

$3\frac{2}{5} = \frac{17}{5}$; and $\frac{17}{5} \times \frac{12}{1} = \frac{204}{5}$, or $40\frac{4}{5}$.

Multiply:

27. $8\frac{2}{3}$ by 9.
28. $12\frac{3}{4}$ by 6.
29. $14\frac{5}{8}$ by 7.
30. $18\frac{3}{5}$ by 8.
31. $21\frac{4}{7}$ by 14.
32. $35\frac{8}{9}$ by 27.
33. $26\frac{5}{18}$ by 39.
34. $44\frac{7}{16}$ by 36.
35. $27\frac{14}{25}$ by 35.
36. $19\frac{17}{33}$ by 44.
37. $38\frac{15}{27}$ by 36.
38. $54\frac{30}{56}$ by 48.

39. Multiply 12 by $3\frac{2}{5}$.

12
$3\frac{2}{5}$
$\frac{4}{5}$
36
$40\frac{4}{5}$

EXPLANATION.—Beginning at the right, $\frac{2}{5}$ times $12 = \frac{24}{5}$, or $4\frac{4}{5}$ times $12 = 36$. Adding, we have $40\frac{4}{5}$, the entire product. Or, by general rule: $\frac{12}{1} \times \frac{17}{5} = \frac{204}{5}$, or $40\frac{4}{5}$.

1. In practice, the products may often be added mentally. Thus, $\frac{2}{5}$ of $12 = \frac{24}{5}$, or $4\frac{4}{5}$; write $\frac{4}{5}$ and carry 4. 3 twelves are 36, and 4 40; write 40. Answer, $40\frac{4}{5}$.

2. In multiplying by the fraction, it is often shorter to divide the multiplicand by the denominator before multiplying it by the numerator.

Multiply:

40. 16 by $5\frac{3}{4}$.
41. 21 by $4\frac{5}{7}$.
42. 25 by $6\frac{4}{5}$.
43. 30 by $7\frac{5}{6}$.
44. 42 by $8\frac{3}{7}$.
45. 36 by $5\frac{5}{9}$.
46. 44 by $7\frac{6}{11}$.
47. 56 by $4\frac{3}{8}$.
48. 60 by $5\frac{7}{10}$.
49. 72 by $7\frac{5}{12}$.
50. 84 by $6\frac{11}{14}$.
51. 97 by $8\frac{7}{13}$.

52. Multiply $131\frac{3}{4}$ by $23\frac{1}{2}$.

$131\frac{3}{4}$
$23\frac{1}{2}$
$\frac{3}{8}$
$65\frac{1}{2}$
$17\frac{1}{4}$
393
262
$3096\frac{1}{8}$

EXPLANATION.—Beginning at the right, we multiply first by the fraction, and then by the whole number. Thus, $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$. $\frac{1}{2} \times 131 = 65\frac{1}{2}$. $23 \times \frac{3}{4} = \frac{69}{4}$, or $17\frac{1}{4}$. Then multiply 131 by 23 as in ordinary multiplication. Adding all of the partial products, we have $3096\frac{1}{8}$, the entire product.

The method just shown is adapted to larger mixed numbers. When both mixed numbers are small, reduce them to improper fractions, and solve by the general rule.

Multiply:

53. $36\frac{1}{2}$ by $7\frac{1}{2}$.
54. $49\frac{1}{2}$ by $8\frac{1}{4}$.
55. $74\frac{1}{4}$ by $6\frac{3}{4}$.
56. $75\frac{1}{3}$ by $5\frac{1}{5}$.
57. $82\frac{2}{3}$ by $8\frac{1}{2}$.
58. $90\frac{3}{4}$ by $7\frac{1}{8}$.
59. $63\frac{3}{4}$ by $24\frac{1}{8}$.
60. $85\frac{2}{5}$ by $20\frac{1}{4}$.
61. $56\frac{1}{2}$ by $44\frac{1}{2}$.
62. $72\frac{3}{5}$ by $35\frac{5}{8}$.
63. $120\frac{4}{7}$ by $49\frac{5}{6}$.
64. $216\frac{5}{9}$ by $78\frac{3}{8}$.

PRACTICAL PROBLEMS.

180. 1. If a man earns $\$1\frac{3}{4}$ per day, how much can he earn in 26 days?

2. At $\$1\frac{7}{8}$ dollars each, how much will 80 Bibles cost?

3. Mr. C. sold 15 loads of hay at $\$8\frac{3}{4}$ a load. How much did he receive for the hay?

4. When hay is worth $\$15\frac{1}{2}$ per ton, how much will $\frac{7}{8}$ of a ton cost?

5. Mr. Jones owns $\frac{3}{5}$ of a farm valued at $13465. What is the value of his share of the farm?

6. What must be paid for $20\frac{3}{4}$ bushels of apples, at $\$\frac{5}{8}$ a bushel?

7. How much will 3 pieces of muslin cost at $6\frac{1}{4}$ cents a yard, if each piece contains 28 yards?

8. A man bought $15\frac{1}{2}$ tons of coal at $\$4\frac{1}{4}$ a ton. How much did the coal cost him?

9. Mr. H. bought $\frac{1}{2}$ of a piece of cloth containing $19\frac{1}{4}$ yards, at $\$4\frac{1}{4}$ per yard. *How much did* it cost him?

DIVISION.

181. Division of Fractions is the process of dividing when one or both of the terms are fractional.

182. To divide a fraction by an integer.

PRINCIPLE.—*Dividing the numerator or multiplying the denominator of a fraction by any number divides the value of the fraction by that number* (Prin. 2, Art. 156).

1. If 3 pounds of tea cost $\$\frac{9}{10}$, what will 1 pound cost?

$\$\frac{9}{10} \div 3 = \$\frac{9 \div 3}{10} = \$\frac{3}{10}$

Or,

$\$\frac{9}{10} \div \frac{3}{1} = \$\frac{9}{10} \times \frac{1}{3} = \$\frac{3}{10}$

EXPLANATION.—If 3 pounds of tea cost $\$\frac{9}{10}$, 1 pound will cost $\frac{1}{3}$ as much as 3 pounds, or $\frac{1}{3}$ of $\$\frac{9}{10}$, or $\$\frac{3}{10}$.

Since dividing the numerator of a fraction divides the fraction (Prin.), we divide the numerator of $\$\frac{9}{10}$ by 3. Hence, the result is $\$\frac{3}{10}$.

2. A man divided $\$\frac{1}{2}$ equally between 2 boys. What part of a dollar did each receive?

$\$\frac{1}{2} \div 2 = \$\frac{1}{2 \times 2} = \$\frac{1}{4}$

Or,

$\$\frac{1}{2} \div \frac{2}{1} = \$\frac{1}{2} \times \frac{1}{2} = \$\frac{1}{4}$

EXPLANATION.—If $\$\frac{1}{2}$ was divided equally between 2 boys, each boy received $\frac{1}{2}$ of $\frac{1}{2}$ or $\frac{1}{4}$.

The numerator of the fraction $\$\frac{1}{2}$ can not be divided by 2 an exact number of times; but since multiplying the denominator of a fraction divides the fraction (Prin.), we multiply the denominator of $\$\frac{1}{2}$ by 2. Hence the result is $\$\frac{1}{4}$.

Divide:

3. $\frac{6}{10}$ by 3.
4. $\frac{15}{21}$ by 5.
5. $\frac{14}{32}$ by 7.
6. $\frac{36}{51}$ by 9.
7. $\frac{11}{12}$ by 4.
8. $\frac{17}{25}$ by 5.
9. $\frac{9}{16}$ by 6.
10. $\frac{18}{47}$ by 5.
11. $\frac{61}{85}$ by 3.

12. If 4 brooms cost $\$\frac{9}{10}$, what is the cost of each broom?

13. A girl paid $\$\frac{4}{5}$ for 5 dozen eggs. What was the price per dozen?

183. To divide an integer by a fraction.

1. How many pencils can by bought for $2 at $\$\frac{1}{4}$ each?

$\$2 \div \$\frac{1}{4} = \frac{2 \times 4}{1} = 8$

Or,

$2 = \frac{8}{4}$; $\frac{8}{4} \div \frac{1}{4} =$

$8 \div 1 = 8$

EXPLANATION.—At $\$\frac{1}{4}$ each, $2 will buy as many pencils as $\$\frac{1}{4}$ is contained in $2. As $\frac{1}{4}$ is contained 4 times in 1, in 2 it is contained 2 times 4 or 8 times. Hence, at $\$\frac{1}{4}$ each, for $2 you can buy 8 pencils.

Or, since in $2 there are $\$\frac{1}{4}$, as many pencils *can be bought as $\$\frac{1}{4}$, the price of one pencil, is contained times in $\$\frac{1}{4}$, which are 8 times, or 8 pencils.*

2. Divide 6 by $\frac{3}{5}$, or find how many times $\frac{3}{5}$ is contained in 6.

$$6 \div \frac{3}{5} = \frac{6 \times 5}{3} = \frac{30}{3} = 10$$

Or,

$$6 \div \frac{3}{5} = \frac{\overset{2}{\cancel{6}}}{1} \times \frac{5}{\cancel{3}} = 10$$

EXPLANATION.—As $\frac{1}{5}$ is contained 5 times in 1, in 6 it is contained 6 times 5 or 30 times. If $\frac{1}{5}$ is contained is 6 30 times, $\frac{3}{5}$, which is 3 times $\frac{1}{5}$, is contained $\frac{1}{3}$ as often, or $\frac{1}{3}$ of 30 times, which are 10 times.

In these examples, it may be seen that the result in each case is obtained by multiplying the integer by the denominator (which reduces integer and fraction to a common denominator), and dividing this product by the numerator of the fraction; that is, by multiplying the integer by the fraction *inverted*.

Divide:

3. 10 by $\frac{2}{3}$.
4. 18 by $\frac{3}{4}$.
5. 24 by $\frac{6}{7}$.
6. 15 by $\frac{3}{5}$.
7. 42 by $\frac{7}{11}$.
8. 54 by $\frac{3}{11}$.
9. 136 by $\frac{5}{8}$.
10. 148 by $\frac{18}{19}$.
11. 214 by $\frac{28}{40}$.

12. How many books at $\$\frac{5}{8}$ each can be bought for \$15?

13. If a boy should spend $\$\frac{3}{4}$ daily, in how many days would he spend \$21?

184. To divide a fraction by a fraction.

1. Divide $\frac{2}{3}$ by $\frac{3}{4}$, or find how many times $\frac{3}{4}$ is contained in $\frac{2}{3}$.

$\frac{1}{3} \div \frac{3}{4} =$
$\frac{1}{3} \times \frac{4}{3} = \frac{8}{9}$

EXPLANATION.—1 is contained in $\frac{3}{4}$, $\frac{4}{3}$ times; and $\frac{1}{4}$ is contained in $\frac{4}{3}$, 4 times $\frac{4}{3}$ times, or $\frac{8}{3}$ times. Since $\frac{1}{4}$ is contained in $\frac{4}{3}$, $\frac{1}{3}$ times, $\frac{1}{3}$ is contained in $\frac{8}{3}$, $\frac{1}{3}$ of $\frac{8}{3}$ times, or $\frac{8}{9}$ times.

Or, reduce the fractions to a common denominator and divide the numerator of the dividend by the numerator of the divisor. Thus, $\frac{2}{3} \div \frac{3}{4} = \frac{8}{12} \div \frac{9}{12} = \frac{8}{9}$; or following the rule given below, $\frac{8}{12} \div \frac{9}{12} = \frac{8}{12} \times \frac{12}{9} = \frac{8}{9}$ (cancelling the common factor 12). Inverting the divisor and multiplying is therefore *a short process* of reaching the same result.

RULE.—*Reduce all integers and mixed numbers to improper fractions.*

Invert the divisor and proceed as in multiplication of fractions.

Find the quotients:

2. $\frac{3}{4} \div \frac{3}{5}$
3. $\frac{5}{7} \div \frac{3}{5}$
4. $\frac{8}{15} \div \frac{4}{5}$
5. $\frac{7}{24} \div \frac{3}{8}$
6. $\frac{1}{2}\frac{2}{5} \div \frac{4}{5}$
7. $\frac{2}{3}\frac{9}{7} \div \frac{6}{7}$
8. $\frac{24}{25} \div \frac{8}{15}$
9. $\frac{32}{35} \div \frac{4}{19}$
10. $\frac{27}{42} \div \frac{9}{28}$
11. $\frac{46}{64} \div \frac{23}{40}$
12. $\frac{55}{72} \div \frac{18}{35}$
13. $\frac{65}{98} \div \frac{45}{49}$
14. $\frac{3}{9} \div 4$
15. $\frac{15}{22} \div 5$
16. $\frac{18}{25} \div 8$
17. $6 \div \frac{4}{5}$
18. $8 \div \frac{3}{7}$
19. $7 \div \frac{5}{6}$
20. $3\frac{1}{2} \div 4$
21. $6 \div 5\frac{2}{3}$
22. $4\frac{1}{2} \div 3\frac{1}{2}$
23. $5\frac{2}{3} \div 4\frac{2}{5}$
24. $8\frac{3}{4} \div 2\frac{1}{5}$
25. $9\frac{5}{7} \div 6\frac{4}{5}$

26. Divide $21\frac{2}{3}$ by 4.

$$4)\ 21\tfrac{2}{3}$$
$$5\tfrac{5}{12}$$

EXPLANATION.—4 is contained in $21\frac{2}{3}$, 5 times with a remainder of $1\frac{2}{3}$, or $\frac{5}{3}$; and $\frac{5}{3}$ divided by 4 equals $\frac{5}{12}$. Hence, the quotient is $5\frac{5}{12}$.

This method is often better than to solve by the general rule.

Divide:

27. $18\frac{1}{2}$ by 5.	31. $44\frac{2}{5}$ by 7.	35. $67\frac{2}{9}$ by 6.
28. $25\frac{1}{4}$ by 6.	32. $38\frac{3}{7}$ by 6.	36. $74\frac{3}{5}$ by 8.
29. $34\frac{2}{3}$ by 8.	33. $45\frac{2}{3}$ by 8.	37. $83\frac{4}{7}$ by 5.
30. $27\frac{3}{4}$ by 5.	34. $56\frac{3}{4}$ by 9.	38. $98\frac{5}{8}$ by 9.

39. Divide 32 by $4\frac{3}{4}$.

$$\begin{array}{r|l} 4\frac{3}{4} & 32 \\ 4 & 4 \\ \hline 19\ ) & 128\ (6\frac{14}{19} \\ & 114 \\ & \overline{14} \\ & \overline{19} \end{array}$$

EXPLANATION.—It is often convenient in dividing by a mixed number to reduce both the divisor and dividend to the denominator of the fraction, and then to divide by the numerators.

Thus, $4\frac{3}{4} = 19$ fourths, and $32 = 128$ fourths.

Then, $128 \div 19 = 6\frac{14}{19}$, the quotient.

Divide:

40. 26 by $3\frac{1}{2}$.	44. 28 by $4\frac{3}{4}$.	48. 58 by $6\frac{2}{3}$.
41. 34 by $4\frac{1}{3}$.	45. 36 by $3\frac{2}{5}$.	49. 61 by $8\frac{4}{5}$.
42. 42 by $6\frac{1}{2}$.	46. 53 by $7\frac{1}{2}$.	50. 74 by $5\frac{4}{5}$.
43. 37 by $5\frac{2}{3}$.	47. 65 by $3\frac{2}{7}$.	51. 89 by $7\frac{5}{6}$.

52. Divide $124\frac{3}{4}$ by $15\frac{1}{6}$.

$$\begin{array}{r|l} 15\frac{1}{6} & 124\frac{3}{4} \\ 12 & 12 \\ \hline 182) & 1497\ (8\frac{41}{182} \\ & 1456 \\ & \overline{41} \\ & \overline{182} \end{array}$$

EXPLANATION.—Since multiplying both dividend and divisor by the same number does not change the quotient (Art. 156, Prin. 3), we multiply both by 12, the least common denominator of the fractions, and divide as in whole numbers.

This method is especially convenient, when the numbers are large and the denominators of the fractions are expressed in low numbers.

Divide:

53. $132\frac{1}{2}$ by $13\frac{1}{2}$.	57. $242\frac{1}{3}$ by $23\frac{1}{4}$.	61. $347\frac{1}{4}$ by $12\frac{1}{5}$.
54. $144\frac{2}{3}$ by $21\frac{1}{2}$.	58. $235\frac{3}{4}$ by $18\frac{2}{3}$.	62. $538\frac{2}{5}$ by $20\frac{3}{4}$.
55. $160\frac{3}{4}$ by $31\frac{2}{3}$.	59. $317\frac{3}{5}$ by $29\frac{1}{2}$.	63. $781\frac{5}{6}$ by $43\frac{2}{3}$.
56. $216\frac{1}{2}$ by $20\frac{3}{4}$.	60. $418\frac{2}{3}$ by $34\frac{1}{2}$.	64. $1000\frac{3}{6}$ by $52\frac{3}{5}$.

PRACTICAL PROBLEMS.

185. 1. If a cord of wood cost $3\frac{1}{4}$, how many cords can be bought for $29\frac{1}{4}$?

2. A man raised 56 bushels of potatoes on $\frac{2}{3}$ of an acre of land. What was the yield per acre?

3. How many quarts of vinegar can be bought for 75 cents, at $18\frac{3}{4}$ cents a quart?

4. If $4\frac{3}{4}$ yards of ribbon cost 95 cents, how much will 3 yards cost?

5. How much must be paid for 21 books at the rate of $9 per dozen?

6. If 7 yards of cloth cost $13.44, what will be the cost of $\frac{5}{8}$ of a yard?

7. Mr. Knox bought $\frac{4}{5}$ of a ton of hay for $10. At this rate, what would be the cost of 6 tons?

8. If a bushel of potatoes cost $\$\frac{5}{8}$, how many bushels can be bought for $41\frac{1}{4}$?

9. A farmer sold $\frac{3}{4}$ of a bushel of seed corn for $\$\frac{3}{4}$. How much at this rate would he get for $27\frac{3}{4}$ bushels?

10. Mr. B. bought $5\frac{1}{2}$ tons of coal for $30\frac{1}{4}$. How much would $16\frac{1}{2}$ tons cost at the same rate?

11. A lady bought 9 yards of silk for $11.25. What would $6\frac{2}{3}$ yards of the same goods cost?

12. If $7\frac{1}{2}$ barrels of apples cost $11\frac{1}{4}$, what will be the cost of $37\frac{1}{2}$ barrels?

13. Mr. Earnest earns $2\frac{3}{4}$ a day, and his expenses are $7\frac{1}{4}$ a week. How many weeks must he labor to pay for a lot worth $138\frac{1}{4}$?

14. A house was bought for $5000 and sold for $\frac{1}{4}$ above cost. For how much was it sold?

15. Mr. M. had $1280. He bought goods with $\frac{5}{8}$ of it, and a horse with $\frac{1}{3}$ of the remainder. How much money had he left?

16. If $573\frac{3}{4}$ will buy $12\frac{3}{4}$ tons of iron, how many tons will $1237\frac{1}{2}$ buy?

COMPLEX FRACTIONS.

186. A Complex Fraction is one whose numerator is a fraction or a mixed number.

Thus $\frac{\frac{3}{4}}{5}$ and $\frac{3\frac{1}{2}}{7}$ are complex fractions.

Complex fractional expressions, such as $\frac{\frac{3}{4}}{\frac{5}{6}}$, $\frac{\frac{1}{2}+\frac{3}{5}}{\frac{3}{4}\text{ of }\frac{1}{4}}$, etc., are often, though incorrectly, called complex fractions. They are expressions which indicate division; the part above the line being the dividend, and the part below the divisor.

1. Reduce $\frac{\frac{3}{4}}{\frac{5}{6}}$ to a simple fraction.

$\frac{\frac{3}{4}}{\frac{5}{6}} = \frac{3}{4} \div \frac{5}{6} = \frac{3}{\cancel{4}_{2}} \times \frac{\cancel{6}^{3}}{5} = \frac{9}{10}$

EXPLANATION.—This expression means that $\frac{3}{4}$ is to be divided by $\frac{5}{6}$. Inverting the divisor and multiplying, we have $\frac{3}{4} \times \frac{6}{5}$, which equals $\frac{9}{10}$.

2. Reduce $\frac{\frac{2}{3}\text{ of }\frac{3}{5}}{\frac{1}{2}+\frac{2}{3}}$ to its simplest form.

$\frac{\frac{2}{3}\text{ of }\frac{3}{5}}{\frac{1}{2}+\frac{2}{3}} = \frac{\frac{2}{5}}{\frac{7}{6}} = \frac{2}{5} \times \frac{6}{7} = \frac{12}{35}$

EXPLANATION.—The expression indicates that $\frac{2}{3}$ of $\frac{3}{5}$ is to be divided by $\frac{1}{2}+\frac{2}{3}$. Simplify the dividend and the divisor by performing the multiplication indicated in the former and the addition in the latter, and we have $\frac{2}{5}$ divided by $\frac{7}{6}$. Inverting the divisor and multiplying, we have $\frac{12}{35}$.

Reduce the following expressions to their simplest form:

3. $\frac{21\frac{3}{4}}{7}$

4. $\frac{5\frac{3}{4}}{12}$

5. $\frac{18\frac{3}{4}}{25}$

6. $\frac{\frac{4}{5}}{\frac{6}{7}}$

7. $\frac{\frac{9}{10}}{\frac{3}{5}}$

8. $\frac{6\frac{1}{2}}{19\frac{1}{2}}$

9. $\frac{\frac{3}{4}\text{ of }\frac{8}{9}}{\frac{4}{7}\text{ of }\frac{7}{11}}$

10. $\frac{\frac{3}{4}-\frac{2}{5}}{\frac{2}{5}+\frac{5}{6}}$

11. $\frac{\frac{7}{8}\times\frac{5}{14}}{\frac{9}{10}\div\frac{3}{5}}$

12. $\frac{\frac{2}{5}\text{ of }3\frac{1}{2}}{\frac{3}{5}\text{ of }\frac{3}{5}}$

13. $\frac{\frac{4}{9}\text{ of }5\frac{3}{5}}{\frac{5}{6}\text{ of }7\frac{1}{2}}$

14. $\frac{\frac{8}{11}}{\frac{5}{16}} \times \frac{9\frac{1}{6}}{20\frac{4}{5}}$

DECIMAL FRACTIONS.

187. A **Decimal Fraction** is a fraction whose denominator is ten or some power of ten—as tenths, hundredths, thousandths, etc.

A decimal fraction is generally expressed by placing a period, called the *decimal point*, before the numerator and omitting the denominator.

Thus, .7 expresses $\frac{7}{10}$; .07 expresses $\frac{7}{100}$; .007, $\frac{7}{1000}$; etc.

188. The **Decimal Point** always indicates the decimal fraction and separates it from the whole number.

189. The **Decimal Places** are the places to the right of the decimal point. The first place to the right of the decimal point is *tenths*, the second place is *hundredths*, the third, *thousandths*, etc.

Thus, .2 represents $\frac{2}{10}$; .23 represents $\frac{23}{100}$; .234, $\frac{234}{1000}$, etc.

$.3 = \frac{3}{10}$	$.04 = \frac{4}{100}$	$.005 = \frac{5}{1000}$	$.0006 = \frac{6}{10000}$
$.5 = \frac{5}{10}$	$.06 = \frac{6}{100}$	$.003 = \frac{3}{1000}$	$.0056 = \frac{56}{10000}$
$.7 = \frac{7}{10}$	$.08 = \frac{8}{100}$	$.006 = \frac{6}{1000}$	$.0356 = \frac{356}{10000}$
$.9 = \frac{9}{10}$	$.07 = \frac{7}{100}$	$.009 = \frac{9}{1000}$	$.2356 = \frac{2356}{10000}$

190. The relation of decimal fractions to the unit and to one another may be seen in the following:

NUMERATION TABLE.

10th.	5	*Billions.*	5	*Hundred-millions.*	5	Ten-millions.	5	Millions.	5	Hundred-thousands.	5	Ten-thousands.	5	Thousands.	5	Hundreds.	5	Tens.	5	Units.
9th.	5		5		5		5		5		5		5		5		5		5	
8th.	5		5		5		5		5		5		5		5		5		5	
7th.	5		5		5		5		5		5		5		5		5		5	
6th.	5		5		5		5		5		5		5		5		5		5	
5th.	5		5		5		5		5		5		5		5		5		5	
4th.	5		5		5		5		5		5		5		5		5		5	
3d.	5		5		5		5		5		5		5		5		5		5	
2d.	5		5		5		5		5		5		5		5		5		5	
1st.	5		5		5		5		5		5		5		5		5		5	
Integers.																				

1st.	5	Tenths.	5	Hundredths.	5	Thousandths.	5	Ten-thousandths.	5	Hundred-thousandths.	5	Millionths.	5	Ten-millionths.	5	Hundred-millionths.	5	Billionths.		
2d.	5		5		5		5		5		5		5		5		5			
3d.	5		5		5		5		5		5		5		5		5			
4th.	5		5		5		5		5		5		5		5		5			
5th.	5		5		5		5		5		5		5		5		5			
6th.	5		5		5		5		5		5		5		5		5			
7th.	5		5		5		5		5		5		5		5		5			
8th.	5		5		5		5		5		5		5		5		5			
9th.	5		5		5		5		5		5		5		5		5			
Decimals.																				

1. The number is read 5 billion 555 million 555 thousand million 555 thousand 555 billionths.

2. The orders of decimals below billionths are ten-billionth billionths, trillionths, ten-trillionths, hundred-trillionths, qu etc.

191. A Decimal is a decimal fraction expressed by t of decimal notation.

Thus, .5, .75, and .075 are decimals.

192. A Pure Decimal consists of decimal figures onl

Thus, .425 is a pure decimal.

193. A Mixed Decimal consists of an integer and a d

Thus, 3.5 is a mixed decimal.

194. A Complex Decimal contains a fraction at the r decimal.

Thus, .33⅓ is a complex decimal.

EXERCISES IN NUMERATION.

195. 1. Read the expression 3.425.

The whole expression is read, 3 and 425 thousandths.

RULE.—*Read the decimal as a whole number and give of the last place at the right.*

1. First determine the numerator by numerating *towards* point from right to left; then determine the denominator by towards the right from the decimal point.

2. In reading, the word "and" should be used only to s whole number from the decimal.

Read the following:

2. .426	10. 3.127	18. 13452.1
3. .534	11. 5.306	19. 3234.12
4. .702	12. 72.40	20. 142.634
5. .986	13. 678.5	21. 53.4215
6. .2346	14. 1356.4	22. 4.21381
7. .3124	15. 213.65	23. .523417
8. .0035	16. 43.158	24. 71.0003
9. .0006	17. 6.1437	25. 1000.00

EXERCISES IN NOTATION.

196. 1. Express decimally thirty-five thousandths.

EXPLANATION.—Since thousandths occupy the third place, three figures are necessary to express the decimal. We write the number 35, and prefix a cipher to cause the figures to occupy their proper place. Hence, the decimal is written .035.

RULE.— *Write the numerator as a whole number, prefix ciphers if necessary, so that the right hand figure will occupy the order named, and place the decimal point before tenths.*

Avoid using the word "and" in reading whole numbers. It should be used only in reading a mixed decimal. Thus, 125.6 is read one hundred twenty-five *and* six tenths. Keeping this in mind will aid in writing decimals.

Express decimally:

2. Four tenths. Seven tenths. Two hundredths. Five hundredths. Nine hundredths.

3. Twelve hundredths. Fourteen hundredths. Six thousandths. Two hundred seven thousandths. Fifteen ten-thousandths.

4. Two hundred six thousandths. Three hundred seventy-two ten thousandths.

5. Five hundred twenty-five millionths. Thirty-six ten-thousandths. Twenty-one hundred-thousandths.

6. Seventy-five, and five tenths. Thirty-two, and thirty-two thousandths. Three hundred forty, and five thousandths.

7. One thousand, and one thousandth. Two thousand sixty-three, and twenty-six hundred thousandths. Four hundred fifty, and six hundred eighty-one millionths.

8. 32 ten-thousandths. 123 millionths. 406 thousandths. 2147 hundred-thousandths. 6347 ten-millionths. 4004 millionths.

9. 34572 hundred-thousandths. 23465 ten-millionths. 47 billionths. 1035 hundred-millionths. 5 ten-billionths.

Express decimally:

10. $\frac{7}{10}$	14. $7\frac{6}{10}$	18. $\frac{3}{10}$	22. $\frac{3}{10}$
11. $\frac{21}{100}$	15. $37\frac{65}{100}$	19. $\frac{30}{100}$	23. $\frac{3}{100}$
12. $\frac{136}{1000}$	16. $100\frac{4}{100}$	20. $\frac{300}{1000}$	24. $\frac{3}{1000}$
13. $\frac{1456}{10000}$	17. $49\frac{9}{1000}$	21. $\frac{3000}{10000}$	25. $\frac{3}{10000}$

26. How do the fractions in numbers 18, 19, 20, and 21 compare in value? How do the decimals compare in value? How does *annexing* ciphers to a decimal effect it? Give the reason.

27. How do the fractions in numbers 22, 23, 24, and 25 compare in value? How then do the decimals compare in value? How does *prefixing* a cipher to a decimal effect it? Prefixing *two* ciphers? *Three* ciphers?

28.	1.000	10.00	100.0	1000.	
29.	0001	000.1	00.01	0.001	.0001

30. Read the numbers expressed in number 28. What is the effect of moving the decimal point one place to the right? Two places to the right? Three places?

31. Read the numbers expressed in number 29. What is the effect of moving the decimal point one place to the left? Two places to the left? Three places?

197. PRINCIPLES.—1. *Annexing ciphers to a decimal does not change its value.*

2. *A decimal cipher prefixed to a decimal, divides the decimal by ten; two decimal ciphers prefixed to the decimal, divide it by a hundred; etc.*

3. *Moving the decimal point one place to the right, multiplies the decimal by 10; two places, multiplies by 100; three places, multiplies by 1000; etc.*

4. *Moving the decimal point one place to the left, divides the decimal by 10; two places, divides by 100; three places, divides by 1000; etc.*

198. The currency of the United States is based on the decimal system of notation; and the cents, mills, etc., may be read as decimals of a dollar.

Thus, $5.25 may be read five and twenty-five hundredths dollars.

Read as decimals:

1. $4.46	4. $6.465	7. $7.375
2. $7.17	5. $8.372	8. $9.047
3. $5.05	6. $5.703	9. $8.008

REDUCTION.

199. To reduce dissimilar decimals to similar decimals.

1. Reduce .3, .75, 1.36, and 4.0432 to similar fractions.

.3 = .3000
.75 = .7500
1.36 =1.3600
4.0432=4.0432

EXPLANATION—The lowest order of decimals in the given numbers is ten-thousandths. Hence, all of the decimals must be reduced to ten-thousandths. This may be done by annexing ciphers (Prin. 1, Art. 190).

RULE—*Give all the decimals the same number of places by annexing ciphers.*

Reduce to similar decimals :

2. .5, .26, .125
3. .2, .128, .5063
4. .043, .217, 1.2, .07
5. .1304, .22, 3.05, .36
6. 4.02, 34.5, 1.635
7. 1.8, .1456, .2, 1.35
8. .2643, 6.5, .34
9. .7, .08, 3.4, .0005
10. .5, .05, .50.5, .5
11. .1, .01, .100, .0001

200. To reduce a common fraction to a decimal.

Reduce $\frac{3}{8}$ to a decimal.

8)3.000
.375

EXPLANATION—$\frac{3}{8}$ equals $\frac{1}{8}$ of 3. 3 equals 30 tenths, and $\frac{1}{8}$ of 30 tenths is 3 tenths and 6 tenths remaining. 6 tenths equals 60 hundreds and $\frac{1}{8}$ of 60 hundredths is 7 hundredths and 4 hundredths remaining. 4 hundredths equals 40 thousandths, and $\frac{1}{8}$ of 40 thousandths is 5 thousandths. Hence, $\frac{3}{8}$ equals .375.

RULE—*Annex ciphers to the numerator and divide by the denominator.*

Point off as many decimal places in the quotient as there are ciphers used.

1. In many cases the division will not terminate. Such decimals are called *interminate, infinite,* or *circulating* decimals. The remainder may be expressed as a common fraction or the sign + may be used to show that the division is not exact. Thus, $\frac{1}{3}$ = .33$\frac{1}{3}$, or .333+.

2. Common fractions in their *lowest terms* cannot be reduced to pure decimals when their denominators contain any prime factors besides 2 or 5.

Change to decimals:

2. $\frac{1}{4}$	8. $\frac{1}{6}$	14. $\frac{4}{11}$	20. $6\frac{1}{2}$
3. $\frac{1}{5}$	9. $\frac{3}{5}$	15. $\frac{7}{12}$	21. $12\frac{1}{2}$
4. $\frac{2}{3}$	10. $\frac{5}{6}$	16. $\frac{8}{15}$	22. $14\frac{2}{3}$
5. $\frac{3}{4}$	11. $\frac{4}{9}$	17. $\frac{17}{22}$	23. $3.0\frac{5}{8}$
6. $\frac{2}{5}$	12. $\frac{5}{8}$	18. $\frac{45}{60}$	24. $.27\frac{3}{4}$
7. $\frac{3}{7}$	13. $\frac{7}{9}$	19. $\frac{24}{96}$	25. $391\frac{9}{50}$

201. To reduce decimals to common fractions.

1. Reduce .35 to a common fraction.

$.35 = \frac{35}{100} = \frac{7}{20}$ EXPLANATION.—.35 expressed as a common fraction is $\frac{35}{100}$, or $\frac{7}{20}$ when reduced to its lowest terms.

RULE.— *Write the denominator under the decimal, omitting the decimal point, and reduce the fraction to its lowest terms.*

Reduce to common fractions:

2. .15	7. .125	12. .045	17. .2134
3. .45	8. .236	13. .305	18. .0375
4. .54	9. .345	14. .005	19. .0065
5. .64	10. .475	15. .075	20. .0004
6. .75	11. .632	16. .001	21. .050

22. Reduce $.37\frac{1}{2}$ to a common fraction.

$.37\frac{1}{2} = \frac{37\frac{1}{2}}{100} = \frac{\frac{75}{2}}{100} = \frac{75}{200} = \frac{3}{8}$

Or $.37\frac{1}{2} = .375 = \frac{375}{1000} = \frac{3}{8}$

EXPLANATION.—$.37\frac{1}{2}$ is hundredths, which, written with its denominator becomes $\frac{\frac{75}{2}}{100}$, or $\frac{75}{200}$ or $\frac{3}{8}$ when reduced to its lowest terms.

Or, $.37\frac{1}{2} = .375 = \frac{375}{1000}$, which reduced to its lowest terms equals $\frac{3}{8}$.

Reduce to common fractions or mixed numbers:

23. $.6\frac{1}{4}$	27. $.41\frac{2}{3}$	31. $.121\frac{1}{4}$	35. $4.33\frac{1}{3}$
24. $.12\frac{1}{2}$	28. $.57\frac{1}{2}$	32. $.137\frac{1}{2}$	36. $10.62\frac{1}{2}$
25. $.18\frac{3}{4}$	29. $.62\frac{1}{2}$	33. $.007\frac{1}{2}$	37. $12.66\frac{2}{3}$
26. $.37\frac{1}{2}$	30. $.74\frac{1}{3}$	34. $.018\frac{3}{4}$	38. $15.087\frac{1}{2}$

ADDITION.

202. 1. What is the sum of 14.8, 120.35, .025, and 4.05?

14.8
120.35
.025
4.05
139.225

EXPLANATION—We write the numbers so that units of the same order stand in the same column. Then beginning at the right, we add as in whole numbers, and place the decimal point in the result directly under those of the numbers.

RULE—*Write the numbers so that units of the same order stand in the same column.*

Add as in whole numbers, and place the decimal point between the units and tenths of the sum.

When there are complex decimals, extend, if they do not terminate, to at least five decimal places before beginning to add.

Find the sum of the following:

2. 4.12, 3.04, .45
3. 1.05, 041, 37.4
4. 3.1, 4.167, 53.04
5. ,14, 0.31, .04, .5
6. .0041, 7.2, 3.65, .2
7. .063, 4.2, 7.3, .081.
8. 7.43, .065, 3.4, .075
9. .364, 7.135, 1.43, .07
10. 3.4, .0032, 2.04, .034
11. .75, 6.37, 5, 6.03, 2.005.
12. 8.15, 60.405, 710.035, 2.5, 120
13. 675.08, 602.415, 10.0045, 1340.0075
14. \$25.50, \$43.09, \$2.18, \$100, \$.10
15. \$164.37½, \$57.62½, \$5.18¾, \$27.125
16. \$1.50, \$2.04½, \$.62½, \$37½, \$75

17. Find the sum of thirty-seven thousandths; two hundred sixty-five, and fifty-four hundred-thousandths; seventy-five, and seventy-five thousandths.

18. Find the sum of seven hundred fifty, and one hundred one ten-thousandths; three thousand sixty-seven, and twenty-two millionths; eight, and five thousandths.

19. In five piles of wood there are respectively 4.418 cords, 6.214 cords, 7⅛ cords, 9¼ cords, and 10⅝ cords. How many cords in all?

20. A farmer sold at one time 2.35 tons of hay, at another time 11.8 tons, at another time 4.05 tons, and at another time 6½ tons. How many *tons of hay did* he sell?

SUBTRACTION.

203. 1. From 57.24 subtract 39.625.

57.24	EXPLANATION.—We write the subtrahend under the minuend
39.625	so that units of the same order stand in the same column. We
17.615	then subtract as in whole numbers, and place the decimal point
	in the result directly under the points of the numbers.

RULE.— *Write the subtrahend under the minuend so that units of the same order stand in the same column.*

Subtract as in whole numbers, and place the decimal point between the units and tens of the remainder.

If the minuend has not so many decimal places as the subtrahend, annex ciphers or subtract as if decimal ciphers were annexed.

Find the values:

2. .434 — .216
3. .5035 — .3408
4. 1.325 — .684
5. 3.42 — 1.256
6. 5.316 — 3.08
7. 37.503 — 20.0465
8. 75 — 24.2346
9. 567.3 — .045
10. 100.1 — .0305
11. $25.85 — $9.16
12. $41.67 — $23.45
13. $68.345 — $17.26
14. $84½ — $43.75
15. $135¾ — $68⅝

16. From thirty-seven, and six thousandths subtract thirteen, and fifteen millionths.

17. From five, and five thousandths subtract five hundred seven thousandths.

18. Mr. Brown's income last year was $3465. He spent $2347.76. How much money did he save?

19. From a cistern containing 500 barrels of water 189.075 barrels were pumped out. How many barrels of water remained in the cistern?

20. The receipts of a railroad company for a certain year were $1340769.23, and the expenditures for the same year were $1123002.89. What were the profits?

21. A dealer bought flour to the amount of $3647.37½, and sold it for $5000. How much did he gain?

MULTIPLICATION.

204. Multiply 4.27 by .32.

4.2 7
.3 2
8 5 4
1 2 8 1
1.3 6 6 4

EXPLANATION.—4.27 multiplied by 32 equals 136.64; and multiplied by 32 *hundredths* the product is 1 hundredth as great. Expressing this by removing the decimal point two places to the left (Prin. 4, Art. 190), the result becomes 1.3664. Hence, $4.27 \times .32 = 1.3664$.

RULE.—*Multiply as in whole numbers, and from the right of the product point off as many figures as there are decimal places in both factors.*

If there are not as many figures in the product as there are decimal places in both factors, *prefix* ciphers to make as many.

Find the products:

2. $.23 \times .15$
3. $.35 \times 2.4$
4. $2.304 \times 12\frac{1}{2}$
5. $.325 \times .004$
6. 31.42×1.05
7. $21.35 \times .035$
8. $.0416 \times .0015$
9. $121.48 \times .00025$

10. Multiply 3.4256 by 200.

3.4256
200
685.12

EXPLANATION.—Since moving the decimal point two places to the right multiplies the decimal by 100 (Prin. 3, Art. 190), we need only multiply 3.4256 by 2, and place the decimal point two places to the right.

Hence, $3.4256 \times 200 = 685.12$.

11. 413.21×100
12. 21.234×1000
13. 1.4358×1000
14. 5.271×700
15. 25.1345×3000
16. 4.50204×5000

17. Multiply thirty-one, and forty-two thousandths by seven, and six hundredths.

18. Multiply one thousand two hundred fifty by twenty, and two ten-thousandths.

19. A farmer sold 41.36 bushels of wheat at $\$1.12\frac{1}{2}$ per bushel. How much did he receive for the wheat?

20. Mr. C. sold 10.48 cords of wood at $\$3.37\frac{1}{2}$ per cord. What was the amount of the sale?

21. A grocer sold 6 pounds of rice at $7\frac{1}{2}$ cents a pound, $24\frac{1}{4}$ pounds of lard at $5\frac{1}{2}$ cents a pound, $8\frac{3}{4}$ pounds of butter at 20 cents a pound, and 25 pounds of sugar at $6\frac{1}{2}$ cents a pound. How much did he receive for all?

DIVISION.

205. 1. Divide 34.144 by 1.6.

1.6) 34.144 (21.34
32
21
16
54
48
64
64

1ST EXPLANATION.—34144 divided by 16 is 2134; but thousandths divided by tenths give hundredths. Hence, the quotient is hundredths, or 21.34.

2D EXPLANATION.—Dividing as in whole numbers, we obtain the quotient 2134. Since the dividend is the product of the divisor and quotient, it must contain as many decimal places as both of them. The number of decimal places in the quotient must therefore be 3 minus 1, or 2 decimal places. Pointing off two decimal places from the right, we have as the quotient, 21.34.

2. Divide .01612 by 1.24.

1.24) .01612 (.013
124
372
372

1ST EXPLANATION.—1612 divided by 124 is 13; but hundred-thousandths divided by hundredths give thousandths ($\frac{1}{100000} \div \frac{1}{100} = \frac{1}{100000} \times \frac{100}{1} = \frac{1}{1000}$). Hence, the quotient is 13 thousandths, or .013.

2D EXPLANATION.—Dividing as in whole numbers, we obtain the quotient 13. Since the dividend is the product of the divisor and quotient, it must contain as many decimal places as both of them. The number in the quotient must therefore be 5 minus 2, or 3 decimal places. Since there are only two figures in the quotient, we must *prefix* a cipher to make the required number of decimal places. Hence, the quotient is .013.

RULE.—*Divide as in whole numbers, and from the right of the quotient point off as many figures as the number of decimal places in the dividend exceeds the number in the divisor.*

1. If the number of figures in the quotient is less than the number of decimal places to be pointed off, prefix ciphers to make as many.

2. Before dividing, see that the dividend contains *at least as many* decimal places as the divisor. When it does not, *annex* ciphers to make the decimal places equal. To continue the division, annex more ciphers.

3. In business, the division is seldom carried farther than to obtain three decimal places in the quotient; for this number of places shows the money value in cents and mills.

Find the quotients:

3. 16.21 ÷ 4.5
4. 308.2 ÷ .68
5. 57.78 ÷ 18
6. 95.91 ÷ .023
7. 9.672 ÷ 2.6
8. 17.28 ÷ 14.4
9. 1728 ÷ .144
10. 1.728 ÷ 144
11. .1728 ÷ 1.44
12. 81.098 ÷ .46
13. 317.646 ÷ 126
14. 172.929 ÷ 293.1
15. 38.1951 ÷ 4.107
16. 7.40070 ÷ .05482
17. 3367.77 ÷ .0087
18. $79.46 ÷ $2.90
19. $13.872 ÷ $.34
20. $1830 ÷ $.05½

21. Divide 745.36 by 200.

200) 745.36
3.7268

EXPLANATION.—Since moving the decimal point two places to the left divides the decimal by 100 (Prin. 4, Art. 190), we need only divide 745 36 by 2, and place the decimal point two places to the left. Hence, $745.36 \div 200 = 3.7268$.

22. $183 \div 30$
23. $2445 \div 50$
24. $276.3 \div 300$
25. $44.675 \div 500$
26. $1.4938 \div 700$
27. $354.671 \div 1000$
28. $6354.28 \div 2000$
29. $76.8975 \div 5000$
30. $913.212 \div 6000$
31. $567891 \div 10000$

32. Divide fifty-one hundredths, by fifteen thousandths.

33. Divide four hundred forty-eight, and five tenths by thirty-seven, and three hundred seventy-five thousandths.

34. Divide three thousand seven hundred sixty-four, and fifty six hundredths by two thousand.

35. Divide seventy-two thousand three hundred eighty-two, and three hundred five ten-thousandths by three thousand.

36. Divide eighty-seven thousand six hundred seventy, and four thousand two hundred six hundred-thousandths by four thousand.

37. At \$4.75 a barrel, how many barrels of flour can be bought for \$760?

38. Mr. Late received \$30.45 for eggs which he sold for 21¢ per dozen. How many dozen did he sell?

39. A man paid \$22.68¾ for cloth at \$2.75 a yard. How many yards did he buy?

40. Mr. Wood bought a quantity of coal for which he paid \$54.95. How many tons at the rate of \$3.50 a ton did he purchase?

41. A farmer sold 5 tons of hay at \$11.25 a ton. He received his pay in lime for which he was charged 7½¢ per bushel. How many bushels of lime did he get?

42. How many cords of wood, at \$3.37½ per cord, can be bought for \$1333.12½?

43. There are 31.5 gallons in a barrel. How many barrels are there in 992.25 gallons?

44. Mr. Crops sold his farm containing 72.85 acres for \$6607.49½. How much did *he receive for it* per acre?

SHORT PROCESSES.

206. The various processes of shortening calculations are here explained and rules given where deemed advisable. The student should make his own examples under each process and practice upon them until the process is thoroughly mastered.

207. To multiply by the aliquot parts of 10, 100, or 1000.

208. An **Aliquot Part** of a number is the whole or mixed number which will exactly divide that number.

Thus, 5, $3\frac{1}{3}$, $2\frac{1}{2}$, etc., are aliquot parts of 10; 25, 20, $33\frac{1}{3}$, etc., are aliquot parts of 100.

209. The following are aliquot parts of 10:

$5 = \frac{1}{2}$ of 10	$2 = \frac{1}{5}$ of 10	$1\frac{1}{4} = \frac{1}{8}$ of 10
$3\frac{1}{3} = \frac{1}{3}$ of 10	$1\frac{2}{3} = \frac{1}{6}$ of 10	$1\frac{1}{9} = \frac{1}{9}$ of 10
$2\frac{1}{2} = \frac{1}{4}$ of 10	$1\frac{3}{7} = \frac{1}{7}$ of 10	

210. The aliquot parts of 100 often used are:

$50 = \frac{1}{2}$ of 100	$12\frac{1}{2} = \frac{1}{8}$ of 100	$6\frac{2}{3} = \frac{1}{15}$ of 100
$33\frac{1}{3} = \frac{1}{3}$ of 100	$11\frac{1}{9} = \frac{1}{9}$ of 00	$6\frac{1}{4} = \frac{1}{16}$ of 100
$25 = \frac{1}{4}$ of 100	$10 = \frac{1}{10}$ of 100	$5 = \frac{1}{20}$ of 100
$20 = \frac{1}{5}$ of 100	$9\frac{1}{11} = \frac{1}{11}$ of 100	$4 = \frac{1}{25}$ of 100
$16\frac{2}{3} = \frac{1}{6}$ of 100	$8\frac{1}{3} = \frac{1}{12}$ of 100	$2\frac{1}{2} = \frac{1}{40}$ of 100
$14\frac{2}{7} = \frac{1}{7}$ of 100	$7\frac{1}{7} = \frac{1}{14}$ of 100	$2 = \frac{1}{50}$ of 100

211. Other parts of 100 are:

$30 = \frac{3}{10}$ of 100	$60 = \frac{3}{5}$ of 100	$75 = \frac{3}{4}$ of 100
$37\frac{1}{2} = \frac{3}{8}$ of 100	$62\frac{1}{2} = \frac{5}{8}$ of 100	$80 = \frac{4}{5}$ of 100
$40 = \frac{2}{5}$ of 100	$66\frac{2}{3} = \frac{2}{3}$ of 100	$87\frac{1}{2} = \frac{7}{8}$ of 100
$41\frac{2}{3} = \frac{5}{12}$ of 100	$70 = \frac{7}{10}$ of 100	$90 = \frac{9}{10}$ of 100

212. The following are aliquot parts of 1000:

$500 = \frac{1}{2}$ of 1000	$125 = \frac{1}{8}$ of 1000	$33\frac{1}{3} = \frac{1}{30}$ of 1000
$333\frac{1}{3} = \frac{1}{3}$ of 1000	$100 = \frac{1}{10}$ of 1000	$25 = \frac{1}{40}$ of 1000
$250 = \frac{1}{4}$ of 1000	$83\frac{1}{3} = \frac{1}{12}$ of 1000	$20 = \frac{1}{50}$ of 1000
$200 = \frac{1}{5}$ of 1000	$66\frac{2}{3} = \frac{1}{15}$ of 1000	$10 = \frac{1}{100}$ of 1000
$166\frac{2}{3} = \frac{1}{6}$ of 1000	$50 = \frac{1}{20}$ of 1000	$5 = \frac{1}{200}$ of 1000

213. 1. Multiply 3528 by $16\frac{2}{3}$.

6) 352800
58800

EXPLANATION.—Since $16\frac{2}{3}$ is $\frac{1}{6}$ of 100, we annex two ciphers and take $\frac{1}{6}$ of the result.

Multiply:

2. 2735 by 20	11. 4356 by $8\frac{1}{3}$	20. 5348 by 75
3. 4136 by 25	12. 5744 by $37\frac{1}{2}$	21. 7256 by $87\frac{1}{2}$
4. 3724 by 50	13. 4708 by 25	22. 8652 by $41\frac{2}{3}$
5. 6372 by $33\frac{1}{3}$	14. 9432 by $66\frac{2}{3}$	23. 9640 by 70
6. 5424 by $12\frac{1}{2}$	15. 6720 by $16\frac{2}{3}$	24. 9056 by $62\frac{1}{2}$
7. 7623 by $3\frac{1}{3}$	16. 4262 by 500	25. 7142 by 125
8. 4250 by $2\frac{1}{2}$	17. 3456 by 250	26. 9416 by $83\frac{1}{3}$
9. 3726 by $1\frac{2}{3}$	18. 2578 by $66\frac{2}{3}$	27. 5967 by $333\frac{1}{3}$
10. 5430 by $16\frac{2}{3}$	19. 6435 by $166\frac{2}{3}$	28. 6954 by 200

214. To multiply any number by 11.

1. Multiply 24635 by 11.

24635
11
270985

EXPLANATION.—Write the right-hand figure 5, and then the sums of 5 and 3, 3 and 6, 6 and 4, 4 and 2 with 1 carried, and finally write the left-hand figure 2.

RULE.— *Write the units figure of the number for the units figure of the product, the sum of the units and tens figures for the tens figure of the product, the sum of the tens and hundreds figure for the hundreds figure of the product, and so on, carrying when necessary; and finally write the left-hand figure.*

215. To multiply by any number of two digits, whose units figure is 1.

1. Multiply 42735 by 41.

42735 × 41
170940
1752135

EXPLANATION.—We multiply the multiplicand by the tens figure, 4, of the multiplier, placing the first right-hand figure of the partial product under the tens figure of the multiplicand; and then add the partial product to the multiplicand. Their *sum is the* product required.

The same process may be used in multiplying by such numbers as 701, 3001, etc., but care must be taken when multiplying by the hundreds, thousands, etc., to place the first right-hand figure respectively under the hundreds or thousands figure of the multiplicand.

216. To multiply by any number of two digits, whose first figure is 1.

1. Multiply 37628 by 19.

37628 × 19
338652
714932

EXPLANATION.—We multiply the multiplicand by the units figure, 9, of the multiplier, placing the first right-hand figure of the partial product under the multiplicand, and in place to the right of the units figure. Then adding the partial product to the multiplicand, we have the product required.

217. To multiply when the sum of the units figures is 10 and the left-hand figures are the same.

1. Multiply 96 by 94.

96
94
9024

EXPLANATION.—Multiply the two right-hand figures, 6 and 4, and place the product, 24, as the units and tens figures of the product. Then multiply one of the left-hand figures by the number next higher, 9 by 10 (9 + 1), and prefix the product, 90, to the result. The result, 9024, is the product required.

RULE.—*Write the product of the units. Add 1 to the number expressed by the left-hand figure or figures and multiply this sum by the number expressed by the left-hand figure or figures, and prefix this product to the product of the units.*

When the units figures are 9 and 1, prefix a cipher to the 9 in the multiplier before multiplying the left-hand numbers.

218. To multiply when the units figures are the same and the tens figures add 10.

1. Multiply 78 by 38.

78
38
2964

EXPLANATION.—Multiply the units, 8 and 8, and place the product, 64, as the units and tens figures of the product. Then take the product of the tens figures, 7 and 3, and increase their result by the number expressed by the units figure, 8, which gives 29 which we prefix to the 64, and the result, 2964, is the product required.

When the product of the units does not occupy two places prefix a cipher before taking the product of the tens. Thus, 62 × 42 = 2604.

219. To multiply when the units and tens figures of the multiplicand are the same, and the two figures of the multiplier add 10.

1. Multiply 77 by 64.

77
64
4928

EXPLANATION.—Take the product of the units for the two right-hand figures of the product. Then increase the tens figure of the multiplier by 1 and take the product of the tens figures, thus changed, for the left-hand figures of the product.

220. To multiply two numbers of two figures, in each of which the units figure is 5.

1. Multiply 85 by 65.

85
65
5525

EXPLANATION.—Write 25 (the product of the units) as the two right-hand figures of the product. Then take the product of the tens (8×6) to which add half their sum ($\frac{1}{2}$ of $\overline{8 + 6}$), making $48 + 7$, or 55. This we prefix to the 25, and the result, 5525, is the product required.

When the sum of the tens figures is an odd number, in taking one-half of it the remainder is 5 tens, which must be added to the tens figure of the product.

221. To multiply by using the complements of the numbers.

222. The **Complement** of a number is the difference between the number and the unit of the next higher order.

Thus, the complement of 7 is 3; of 19 is 81; of 96 is 4 of 985 is 15, etc.

1. Multiply 97 by 96.

97 .. 3
96 .. 4
9312

EXPLANATION.—The complement of 97 is 3, and of 96 is 4. The product of the complements is 12, which gives the units and tens figures of the product. For the remaining two figures of the product, subtract either the lower complement from the upper number or the upper complement from the lower number. 4 from 97, or 3 from 96, leaves 93, which we write to the left of the 12, making 9312, the product required. There should be the same number of figures in the product as in both the multiplier and multiplicand.

The method given above is an excellent one when the compliments of the numbers are small, so that the work may be done mentally.

223. To multiply when both numbers are between 100 and 125 or between 1000 and 1025.

1. Multiply 112 by 108.

112
108
12096

EXPLANATION.—This is a modification of the preceding method. We take the product of the excess of 100 in each number (12×8) for the units and tens figures of the product. Then we add the excess of either number over 100 to the other number ($112 + 8$ or $108 + 12$), making 120, which prefixed to the 96, gives us 12096, the product required.

224. To multiply by a number a little less or a little greater than 100, 1000, etc.

1. Multiply 5432 by 98.

5432
98
543200
10864
532336

EXPLANATION.—98 times a number is 100 times the number *minus* 2 times the number. Hence, annex two ciphers to the number and from the result subtract 2 times the number.

2. Multiply 573 by 103.

57300
1719
59019

EXPLANATION.—In this example, we add 3 times the multiplicand to the multiplicand with the ciphers annexed, because the multiplier is 3 more than 100.

When the multiplier is a little greater or a little less than 1000, three ciphers should be annexed to the multiplicand (Art. 82.)

Find the products:

3. 4673×99
4. 7562×98
5. 8356×97
6. 6748×999
7. 5876×998
8. 634157×96
9. 473658×99
10. 765436×997
11. 512344×995
12. 896432×999

225. To multiply by 15, 150 and 1500.

1. Multiply 394 by 15.

3940
1970
5910

EXPLANATION.—By annexing a cipher to the multiplicand, it is multiplied by 10; and 15 is half as much more than 10. Hence, we add to the multiplicand with the cipher annexed its half.

RULE.—*To multiply by 15, annex a cipher to the multiplicand and add to the result its half. To multiply by 150, annex two ciphers, and by 1500, annex three ciphers to the multiplicand and proceed as before.*

226. To multiply by first making convenient changes in the multiplicand and multiplier without affecting the product.

1. Multiply 25 by 16.

$25 \times 16 =$
$100 \times 4 = 400$
Or
$50 \times 8 = 400$

EXPLANATION.—Multiplying the multiplicand and dividing the multiplier by the same number do not affect the product.

Hence, we may mentally take the product of 25×4 and of $16 \div 4$, which equal 100×4 or 400.

We may also say $25 \times 16 = 50 \times 8$; or $25 \times 16 = 1600 \div 4$.

The student by exercising his ingenuity may very soon learn to see relations in numbers which will enable him to perform many of the operations mentally.

227. To multiply by any number of 9's.

1. Multiply 763 by 99.

76300
763
75537

EXPLANATION.—By annexing two ciphers to the multiplicand it is multiplied one time too often. Hence, we subtract the multiplicand (see Art. 217).

RULE.—*Annex to the multiplicand as many ciphers as there are nines in the multiplier, and from the result subtract the multiplicand.*

228. To multiply when one part of the multiplier is a factor of another part.

1. Multiply 6374 by 248.

6374
248
50992
152976
1580752

EXPLANATION.—8, the number of units, is a factor of 24, which may be regarded as tens. We first multiply by the 8 units. The 24 tens are 3 times as many tens as there are units, hence the product obtained by multiplying by 8 is multiplied by 3. The sum of the partial products is the entire product.

2. Multiply 3465 by 412.

3465
412
13860
41580
1427580

EXPLANATION.—4, the number of hundreds, is a factor of 12, which may be regarded as units. We first multiply the number by the 4 hundreds, writing the first figure of the product under hundreds. The 12 units are 3 times as many units as there are hundreds, hence the product obtained by multiplying by 4 is multiplied by 3, and the first figure is written in units' place. The sum of the partial products is the entire product.

Find the products:

3. 2643×63
4. 4235×84
5. 5432×217
6. 7356×369
7. 6423×497
8. 23425×318
9. 15743×424
10. 28372×515
11. 36482×612
12. 46054×816

229. To find the product of numbers of two or more figures by cross multiplication.

230. Cross Multiplication is a process by which the product only is *written, the partial products* being combined mentally.

1. Multiply 38 by 27.

38
27
1026

EXPLANATION.—First, multiply units by units; second, t units and units by tens; third, tens by tens. Thus, 7×8 The 6 is written in units' place. 5 is carried to 7×3 and making 42. The 2 is written in tens' place. 4 is carried to making 10, which is written in the next two places of the product.

2. Multiply 346 by 53.

346
53
18338

EXPLANATION.—1st, multiply units by units; 2d, tens by and units by tens; 3d, hundreds by units, and tens by tens hundreds by tens. Thus, $3 \times 6 = 18$. The 8 is written in place. 1 is carried to 3×4 and 5×6, making 43. The 3 i ten in tens' place. 4 is carried to 3×3 and 5×4, making 33. The hand 3 is written in hundreds' place. The left-hand 3 is carried to making 18, which is written in the next two places of the product.

3. Multiply 537 by 426.

537
426
228762

EXPLANATION.—6 times 7 are 42. The 2 is written in place. 4 is carried to 6×3 and 2×7, making 36. Th written in tens' place. 3 is carried to 6×5, 2×3 and making 67. The 7 is written in hundreds' place. 6 is car 2×5 and 4×3, making 28. The 8 is written in thousands' place carried to 4×5, making 22, which is written in the next two places product.

If the student will solve the above examples by the ordinary meth then compare the partial products with the above process, he will fin the only difference is that by the latter method we begin to use each of the multiplier at the point where we find it begins to affect th product, while by the common method the entire multiplicand is firs tiplied by the units figure of the multiplier, then by the tens figur and then the partial products are added for the entire product. C attention to the order of procedure and a reasonable degree of practi enable the student to multiply readily by at least two or three places

231. To multiply any number containing $\frac{1}{2}$ by itself.

1. Multiply $7\frac{1}{2}$ by $7\frac{1}{2}$.

$7\frac{1}{2}$
$7\frac{1}{2}$
$56\frac{1}{4}$

EXPLANATION.—Multiply the integer by the next higher i and annex $\frac{1}{4}$ to the product. Thus, $7 \times 8 = 56$, to which is an $\frac{1}{4}$ (the product of the two fractions).

232. To multiply mixed numbers when the integers alike and the sum of the fractions is 1.

$8\frac{2}{5}$
$8\frac{2}{5}$
$72\frac{6}{25}$

EXPLANATION.—To the product of the integer by the next *integer* annex the product of the fractions. Thus $8 \times 9 =$ *which is* annexed $\frac{2}{5} \times \frac{3}{5}$ or $\frac{6}{25}$, making $72\frac{6}{25}$.

233. To multiply any two mixed numbers having like fractions.

1. Multiply $8\frac{2}{3}$ by $7\frac{2}{3}$.

$8\frac{2}{3}$
$7\frac{2}{3}$
$66\frac{4}{9}$

EXPLANATION.—Add the product of the fractions, the product of the integers, and $\frac{2}{3}$ of the sum of the integers. Thus, $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, which is written under the fractions. Then, $7 \times 8 + \frac{2}{3}(7 + 8) = 56 + 10 = 66$, which is written as the integral part of the product, making the entire product $66\frac{4}{9}$.

2. Multiply $11\frac{1}{4}$ by $8\frac{1}{4}$.

$11\frac{1}{4}$
$8\frac{1}{4}$
$88\frac{1}{16}$
$4\frac{1}{4}$
$92\frac{1}{16}$

EXPLANATION.—In this example the sum of the integers is 19, and $\frac{1}{4}$ of 19 is $4\frac{3}{4}$. This is placed under the partial product $88\frac{1}{16}$, to which it is added for the complete result.

234. Business method for multiplying mixed numbers.

1. Multiply $17\frac{1}{4}$ by $9\frac{1}{3}$.

$17\frac{1}{4}$
$9\frac{1}{3}$
153
8
161

EXPLANATION.—9 times 1 are 153. $\frac{1}{3}$ of 17 to the nearest unit is 6, and $\frac{1}{4}$ of 9 to the nearest unit is 2. 6 and 2 are 8, which added to 153 = 161.

RULE.—*Multiply the integers. Then multiply each integer by the fraction in the other number to its nearest unit, and add the partial products.*

TO ESTIMATE THE WEIGHT OF LIVE CATTLE.

235. The weight of cattle by measurement can only be ascertained approximately.

236. To find the superficial feet:

Measure in inches the girth just back of the shoulder-blade and behind the front legs, and the length of the back from the root of the tail to the forepart of the shoulder-blade. Multiply the girth by the length, and divide by 144. The quotient will be the number of superficial feet.

For a girth of from 5 to 7 feet, allow 23 lbs. to the superficial foot.

For a girth of from 7 to 9 feet allow 31 lbs. to the superficial foot.

For small cattle and calves of a girth of from 3 to 5 feet, allow 16 lbs. to the superficial foot.

For hogs, sheep, and all cattle of a girth less than 3 feet, allow 11 lbs. to the superficial foot.

237. To find the number of pounds of beef, veal, etc., in an animal.

1. What is the estimated weight of beef in a steer whose girth is 6 ft. 8 in., and length 5 ft. 4 in.?

SOLUTION.—6 ft. 8 in. = 80 in.; 5 ft. 4 in. = 64 in. 80 in. × 64 in. = 5120 sq. in. This divided by 144 = $35\frac{5}{9}$ square, or superficial, feet. $35\frac{5}{9} \times 23$ lbs. = $817\frac{7}{9}$ lbs.

RULE.—*Multiply the number of superficial feet by the number of pounds allowed for animals of different girths, and the product will be the estimated weight.*

When the animal is very fat $\frac{1}{10}$ of the weight as found above should be added, and if only half fattened $\frac{1}{10}$ of the weight should be deducted for greater accuracy.

238. To find the cost when the price per 100 or 1000 is given.

1. What will 245 rails cost at $4.75 per hundred?

$4.75
2.45
$11.6375

EXPLANATION.—Since 100 rails cost $4.75, 245 rails, which are equal to 2.45 times 100 rails, will cost 2.45 times $4.75, or $11.64

RULE.—*Reduce the quantity to hundreds by pointing off two places from the right, or to thousands by pointing off three places. Multiply the price by this result and point off the product as in multiplication of decimals.*

1. In business, the mills in final results are dropped when less than five. When five mills or more, they are usually regarded as a cent.

2. The price per hundred may be expressed *per C*, and per thousand, *per M*. The character ℔ is often used for *per*.

2. What will be the cost of 500 bushels of lime at $6.35 per C?
3. What will 1240 pickets cost, at $1.12½ ℔ C?
4. How much will 9850 shingles cost, at $5.35 ℔ M?
5. What will be the cost of 3471 feet of boards at $15.25 ℔ M?

6. A grocer sold one month 8640 pounds of flour at $2.65 ℔ C. What was the amount of the sale?

7. Mr. Lake bought 25750 laths at $2.85 ℔ M. How much did they cost him?

8. How much must be paid for 635 feet of boards at $15.50 ℔ M, 2540 feet of scantling at $1.35 ℔ C, and 3764 feet of flooring at $17.50 ℔ M?

239. To find the cost of hay.

1. Find the cost of 879 pounds of hay at $12 per ton.

879
$6
$5.274

EXPLANATION.—At $12 a ton is $6 per 1000 pounds. Hence, by multiplying 879 by 6 and removing the decimal three places from the right, the cost is found to be $5.27.

RULE.—*Multiply the number of pounds by half the price per ton, and point off three places from the right.*

2. How much must be paid for 4680 pounds of hay at $8.50 per ton (2000 pounds)?

SUGGESTION.—Dividing the price of a ton by 2, gives the price per 1000 pounds. Or dividing the quantity expressed in thousands (the quantity with the decimal point moved three places toward the left) by 2 will give the number of tons.

3. What will be the cost of 5157 pounds of fertilizer at $18 a ton?

4. A farmer sold 16750 pounds of hay for $15.60 a ton. How much did he receive for it?

5. What must be paid for the shipment of 17587 pounds of merchandise at the rate of $13 a ton?

6. If a ton of bone dust is worth $25.75, what is the value of 7240 pounds?

7. A farmer received $56.76 for 9460 pounds of hay. What was the price per ton?

8. What is the value of 13760 pounds of hay when the price is $15 per ton?

9. A farmer sold 25 loads of hay each weighing 1700 pounds, at $13.50 per *ton*. *How much* did he receive for the hay?

240. To find the cost of a given number of pounds when the price per bushel is given.

1. How much should be given for 476 pounds of corn, at 40¢ per bushel of 56 pounds?

476
$.40
56) $190.40 ($3.40
169
224
224
0

EXPLANATION.—At $.40 per lb. the cost would be 476 times $.40, or $190.40. But since the price is $.40 per bu. of 56 lb, the cost will be $\frac{1}{56}$ of $190.40, or $3.40.

RULE.—*Multiply the weight in pounds by the price per bushel, and divide the product by the number of pounds in a bushel.*

Parts of bushels are sometimes written without the denominator expressed, thus, 8^{28} bu. of corn $= 8\frac{28}{56}$ bu. $= 476$ lb.

For table of weights of produce see page 132.

2. How much will 1560 lbs. of oats cost, at $.25 per bu. of 32 lbs.?

3. Find the cost of 2150 lbs. of wheat, at $.80 per bu. of 60 lbs.

4. What will 1728 lbs. of barley cost, at $.65 per bu. of 48 lbs.?

5. How much must be paid for 2347 lbs. of corn, in the ear, at $.40 per bu. of 70 lbs.?

6. I bought 360 lbs. of timothy seed at $2.25 per bu. of 45 lbs. How much did I pay for it?

It is well always to observe the relation of numbers for exact multiples or aliquot parts. In problem 6, 360 lbs. is just 8 times 45 lbs. or 8 bu.

7. How much will 1400 pounds of shelled corn cost at $.65 per bu. of 56 lbs.?

8. A load of potatoes which weighed 1860 pounds was sold at $.75 per bushel of 60 lbs. How much was received for the load?

9. Find the cost of 3500 pounds of turnips selling at 25¢ per bu. of 55 lbs.

10. How much will 336 lbs. of buckwheat cost at $.70 per bushel of 48 lbs.

11. A farmer sold 340 lbs. of timothy seed at $2.35 per bu. of *45. lb.* How much did he get for it?

PROOFS OF FUNDAMENTAL PROCESSES.

241. The proofs given under addition, subtraction, multiplication, and division are the most reliable that can be given. Other methods are employed, among which perhaps the one given below is as reliable a test as any of them.

242. Method by casting out the nines.

It was probably the Arabians who discovered centuries ago that when the number of 9's in a number is found, the remainder is equal to the sum of the digits of the number, or to the sum with the 9's cast out.

Thus, take any number, as 65, and dividing by 9, we have 7 (nines) and 2 remaining. Adding the digits $6 + 5 = 11$, and dividing by 9, we have 1 (nine) and 2 remaining as in dividing the number. Similarly $426 \div 9 = 47$ with 3 remaining, and adding the digits $4 + 2 + 6 = 12$, which divided by 9 gives a remainder of 3. Similarly $7135 \div 9 = 792$ with 7 remaining, and adding the digits $7 + 1 + 3 + 5 = 16$, which divided by 9 gives a remainder of 7.

PROOF OF ADDITION.

243. 1. Prove that $283 + 462 + 375 + 859 = 1979$.

$283 = 4$
$462 = 3$
$375 = 6$
$859 = 4$
$\overline{1979 = 8}$

EXPLANATION.—The remainder or excess of 9's, in the first addend is 4 units; in the second, 3 units; in the third, 6 units; in the fourth, 4 units. The sum of the units remaining is 17, which divided by 9 gives a remainder of 8. The remainder or excess of 9's in the sum 1979 is also 8. Hence, the result is probably correct.

It should be remembered that this method of proof is not infallible, for the figures might be transposed and yet the same excess of nines would appear.

2. Prove that $276 + 420 + 158 + 335 = 1189$.

3. Prove that $536 + 324 + 275 + 427 = 1562$.

4. Prove that $342 + 536 + 427 + 163 + 625 = 2093$.

5. Prove that $943 + 827 + 389 + 675 + 781 = 3615$.

6. *Prove that* $876 + 597 + 949 + 762 + 685 = 3869$.

PROOF OF SUBTRACTION.

244. 1. Prove that 8352 — 3416 = 4936.

8352 = 0
3416 = 5
4936 = 4

EXPLANATION.—The excess of 9's in the minuend is 0. The excess of 9's in the subtrahend is 5. 5 subtracted from the radix, 9, leaves a remainder of 4. The excess of 9's in the remainder 4936 is also 4. Hence, the result is likely to be correct.

2. Prove that 76598 — 53124 = 23474.

3. Prove that 56348 — 34687 = 21661.

PROOF OF MULTIPLICATION.

245. 1. Prove that 526 × 74 = 38924.

526 = 4
74 = 2
38924 = 8

EXPLANATION.—The excess of 9's in the multiplicand is 4; in the multiplier 2. The product of these two remainders is 8, and the excess of 9's in the product 38924 is also 8. Hence, the result is believed to be correct.

2. Prove that 635 × 86 = 54610.

3. Prove that 742 × 95 = 70490.

PROOF OF DIVISION.

246. 1. Prove that 9350 ÷ 34 = 275.

7 8 5
34) 9350 (275

EXPLANATION.—The excess of 9's in the divisor and quotient are respectively 7 and 5. This product is 35 or an excess of 8, which corresponds to the excess of 9's in the dividend. The work is therefore presumed to be correct, since the product of the divisor and quotient equals the dividend.

2. Prove that 516725 ÷ 357 = 1447, and rem. 146.

6 8 7 2
357) 516725 (1447 Rem. 146

EXPLANATION.—The product of the divisor and quotient, plus the remainder, equals the dividend. The excess of 9's in the divisor, quotient, and remainder are respectively 6, 7, and 2. The product of 6 and 7 is 42, to which is added the excess of the remainder 2, making 44, or an excess of 8. The excess of 9's in the dividend is also 8. Hence, the result is presumed to be correct.

3. Prove that 19152 ÷ 56 = 342.

4. Prove that 875342 ÷ 426 = 2054, and rem. 338.

BILLS AND ACCOUNTS.

247. A Bill is an invoice or written statement showing in d charges for services rendered or items of goods sold to a custor

248. Invoice is the technical name for a bill of merchan sold by a merchant to his customer.

249. The Bill or Invoice should state clearly the place date of sale, the names of buyer and seller, the terms, articles sold, the name, quantity, number of packages, and p of each item; the amount of each item extended to the first mo column, and the total amount extended to the second mo column.

1. The terms *Invoice* and *Bill* are now used interchangeably. Form the term Invoice was applied only to written statements of goods sold by merchant (importing or wholesale) to another.

2. A bill is receipted when the person to whom it is due, or his ag writes at the bottom of the bill "Received Payment," or "Paid," and s his name.

250. A **Debt** is that which is due from one person to anotl A **Debtor** is a party owing a debt. To **Debit** is to charge or e in an account as a debt.

251. A **Credit** is what is due to a person or account, or it i be a sum paid toward discharging a debt. To **Credit** is to e in an account as an offset to a debt. A **Creditor** is a part whom a debt is due.

252. An Account is a record showing by debits or credits by both debits and credits, the result in value of the dealings business with any person, property or thing.

253. The Ledger is the principal book which contains the entries, systematically arranged under the proper accounts, of transactions recorded.

254. The Debit, or **Debtor** (Dr.) side of an account shows amounts of the items due *from* the account for goods, mo services, etc.

255. The Credit (Cr.) side of an account shows the amount the items due *to the account* for goods, money, services, etc.

256. The **Balance** of an account is the difference between the amounts of the debit and credit sides of the account.

257. To **Balance an Account,** we find the difference of the footings of the two sides and add it to the smaller side so as to cause the two amounts to be equal.

This is done, among other reasons, to prove the correctness of the balance and to keep a record of it.

258. A **Statement** is a written exhibit based upon the debits and credits previously existing in an account.

It is customary in business establishments to send out statements of each personal account once a month. These statements should show the balance due for previous months, and under their proper dates, the amount of each itemized bill debited for the current month, the amount of each credit item for the current month, and the final balance due at the end of the current month.

259. An Inventory is a list, in detail, showing the property of which a business or estate is found to be possessed.

An inventory is taken whenever it is desired to know the exact financial condition of a business. The inventory together with all other resources and liabilities as shown by the regular books of account enable us to find the net value in money of the capital employed.

260. The following abbreviations are in common use in business

A1.,	First Quality.	**C. O. D.,**	Collect on Delivery.	**F. O. B.,**	Free on board.
A/c or Acct.,	Account.	**Com.,**	Commission.	**Frt.,**	Freight.
Agt.,	Agent.	**Cr.,**	Creditor.	**Hhd.,**	Hogshead.
Amt.,	Amount.	**Cwt.,**	Hundred weight.	**i. e.,**	That is.
Bal.,	Balance.	**Dft.,**	Draft.	**Ins.,**	Insurance.
Bbl. or Bar.,	Barrel.	**Dis.,**	Discount.	**Lb.,**	Pound.
Bdl.,	Bundle.	**Do. or ditto,**	The same.	**Mdse.,**	Merchandise.
Blk.,	Black.	**Doz.,**	Dozen.	**Messrs.,**	Gentlemen or Sirs.
B/L.,	Bill of Lading.	**Dr.,**	Debtor.	**N. B.,**	Take Notice.
Bot.,	Bought.	**Ea.,**	Each.	**No., #**	Number.
Bu.,	Bushel.	**E. & O. E.**	Errors and Omissions Excepted.	**O. L.,**	Old Ledger.
Bx.,	Box.	**Etc., &c.,**	And so forth.	**P.,**	Page.
Ct. or ¢,	Cent.	**Exp.,**	Expense.	**Pp.,**	Pages.
Ctg.,	Cartage.	**Exch.,**	Exchange.	**Pay't,**	Payment.
Ck.,	Check.	**Fo., fol.,**	Folio.	**Pd.,**	Paid.
Cks.,	Casks.			**Per,**	By, or by the.
Cs.,	Cases.			**Pkg.,**	Package.
Chgd.,	Charged.			**Pc.,**	Piece.
Co.,	*Company, County.*			**Rec'd, Recd.**	Received.

261. *Time* Abbreviations:

an., or an'y,	January.	Oct.,	October.	Cent.,	Century.
		Nov.,	November.	D.,	Day.
eb., or eb'y,	February.	Dec.,	December.	Ds.,	Days.
		Mo.,	Month.	Hr.,	Hour.
Mar. & Mch,	March.	Yr.,	Year.	M.,	Minute.
Apr.,	April.	Inst.,	Present month.	Sec.,	Second.
Aug.,	August.	Prox.,	Next month.	Wk.,	Week.
Sept.,	September.	Ult.,	Last month.		

In abbreviating measures of capacity, weight, distance or time, it is not necessary to add an *s* for the plural. Capitals are often used in abbreviations for weights, measures, etc.

262. Commercial Characters:

@	At.	"	Ditto, the same.	o/c	Old account.
%	Account.	%	Per cent.	℔	By, or by the.
✓	Check mark.	c/o	Care of.	£	Pounds Sterling.
	Number.	n/c	New account.	×	By, as 4 × 6 inches
	One and one-fourth.	1^2	One and two-fourths.	1^3	One and three-fourths.

BILLS.

263. Copy, extend, and foot:

1.

Orrstown, Pa., Aug. 9, 1901.

H. Dole,

Bought of JOHN MACE & CO.

Terms Cash.

2	Plows	$7.75	15	50		
3	Harrows	6.50				
1	Mower		35			
4	Buckets	.35				
3	Rakes	.18			**	**
	Rec'd Payment,					
	John Mace & Co.					

2.

Philadelphia, June 24, 1902.

D. B. Wood & Co.,
Strasburg, Pa.,

Bought of HOWARD & GATES.

2	Doz. Men's Blk. Fur Hats @ $15.		
3	" Boys' " " " 10.		
2	" " Fancy Caps " 6.50		
1½	" Children's " " " 4.75		
	Rec'd Pay't,		
	Howard & Gates.		
	Per M.		

3.

New York, July 27, 1902.

Hale & King,
Erie, Pa.,

To A. B. HUGHES & SON, Dr.

Terms: 30 Days.

Case.			No. Yd.	Price	Items.		Amt.	
#731	10	Pcs. Muslin, 34 32¹ 31³ 32³ 36 35 36³ 34³ 31¹ 38³		6¢	* *	* *		
#504	6	Pcs. Lowell Prints, 26 28³ 24 27¹ 32³ 31³		4½¢	*	* *		
#317	8	Pcs. Exeter Prints, 24 23¹ 26³ 28 31³ 29¹ 32³ 34¹,		5¢	* *	* *	* *	* *

STATEMENTS.

4.

Reading, Pa., Sept. 1, 1902.

Lapp & Free,

To DIX MFG. CO, Dr.

1902.				
Aug.	2	To Bill rendered	230	85
	5	" " "	176	
	11	" " "	415	40
	23	" " "	1634	75
	26	" " "	380	

5.

Pittsburgh, Pa., Apr. 1, 1904.

Race, Law & Co.,
Richmond, Va.

In account with OLEAN IRON CO., Dr.

1904.						
Jan.	*4*	*To Invoice rendered,*	*1306*	*75*		
	12	" " "	*780*	*50*		
Feb.	*25*	" " "	*1830*		* * * *	* *
		Cr.				
Feb.	*14*	*By Cash,*	*960*			
Mar.	*17*	" "	*700*		* * * *	
		Balance due,				

6. John Hale bought of Harp & Leed, Albany, N. Y., 7 yd. black cassimere at $1.15, 4 yd. broadcloth at $3.25, and a vest pattern for $2.50. Make out the bill.

7. W. Warren & Co., New York, sold to Mrs. H. C. Kay, July 7, 1903, 2 soup tureens at $1.35, 4 sauce tureens at 65¢, 2 glass pitchers at 55¢, and four covered dishes at 35¢. Make out and receipt the bill.

8. Rice & Brown bought of J. Hold & Co., Aug. 3, 1903, terms 30 da., 160 lb. Java coffee at 25¢, 140 lb. Mocha coffee at 23¢, 5 bar. granulated sugar at $9, 36 gal. syrup at 35¢, 70 lb. black tea at 35¢, and 40 lb. green tea at 45¢. What was the amount of the bill?

9. Fare & Co., bought of Hoar & Co., Boston, May 18, 1903,
5 pieces A. cotton, 40, 42, 43, 44^1, 45^3, at 5¢.
4 pieces L. gingham, 46^1, 48^2, 43^3, 50, at 7¢.
6 pieces G. shirting, 38, 36, 34, 35, 32, 30, at 6¢.

Find the amount of the bill.

10. J. H. Book & Co., bought of J. Boyd & Bro., Philadelphia, for cash, July 28, 1903, 8 pc. R. gingham, 48, 50, 46, 44^2, 43^3, 44, 42, 47, at 8¢.
12 pc. M. gingham, 32, 34^1, 33, 35, 36^2, 38, 37^3, 39^2, 40^3, 42, 44^1, 46, at 9¢.

Make out a receipted bill.

DENOMINATE NUMBERS.

264. A **Denominate Number** is a concrete number in which the unit is a measure established by law or custom.

Thus, 2 feet, 3 pounds, 5 quarts, are denominate numbers.

265. A **Measure** is a unit of value, length, area, volume, time, etc., by which the quantity of anything is estimated by the number of times it will contain the unit of measure.

Thus, a dollar, a yard, an acre, a quart, a pound, are measures.

266. A **Simple Denominate Number** is a denominate number composed of units of only one denomination.

Thus, 3 feet, 4 hours, 7 pounds, 5 pints, are simple denominate numbers.

267. A **Compound Denominate Number** is a denominate number composed of units of two or more related denominations.

Thus, 2 pounds 5 ounces is a compound denominate number. So also is 5 yards 1 foot 3 inches.

268. A **Standard Unit** is a unit of measure from which other units of the same kind may be derived.

Thus, the yard is the standard unit of length because the other units are derived from it.

269. Reduction of denominate numbers is the process of changing them from one denomination to another without altering their value.

270. Reduction Descending is the process of changing a denominate number to an equivalent number, or measure, or amount, of a *lower* denomination.

Thus, the change of gallons to an equivalent in quarts, pints, or gills is reduction descending.

271. Reduction Ascending is the process of changing a denominate number to an equivalent number of a *higher* denomination.

Thus, the change of gills to an equivalent in pints, quarts, gallons, or *rrels is reduction* ascending.

272. Denominate Numbers may be embraced under four distinct classes: *Value*, *Weight*, *Extension*, and *Time*.

Some of these classes contain subdivisions of such importance that eight classes may be recognized: *Value*, *Weight*, *Length*, *Surface*, *Volume*, *Capacity*, *Time*, and *Angles*.

MEASURES OF VALUE.

273. The **Value** of anything is its worth, or that property by virtue of which it is useful or estimable.

274. Value depends mainly upon cost of production, utility, and difficulty of attainment.

275. Money is the common measure of value. It is of two kinds, *coin* and *paper money*.

276. Coin or **Specie** is stamped pieces of metal having a value fixed by law.

277. Paper Money consists of notes or bills (promises to pay money) issued by Governments and banks, and authorized to be used as money.

Currency (from *currere*, to run) is a term applied to money or to that which circulates as money.

278. Legal Tender is a term applied to all moneys which are required by law to be accepted in payment of debts.

UNITED STATES MONEY.

279. United States Money is the lawful currency of the United States.

280. The denominations and scale of United States money are shown in the following

TABLE.

10 Mills (m.) = 1 Cent . . . ct.	10 Dimes = 1 Dollar . . . $
10 Cents = 1 Dime . . . d.	10 Dollars = 1 Eagle . . . E.

Scale.—Decimal.

1. The *unit* of value is the dollar. Its standard weight in gold and alloy is 25.8 grains ($\frac{9}{10}$ pure gold, $\frac{1}{10}$ alloy). The dollar mark is probably a combination of *U. S.*, the initials of United States.

2. The **Coins** of the United States are of *gold*, *silver*, *nickel*, and *bronze*. The *gold* coins are the double eagle, eagle, half-eagle, and the quarter-eagle piece. The *silver* coins are the dollar, half-dollar, quarter-dollar, and dime. The *nickel* coin is the five-cent piece. The *bronze* coin is the cent. There are various other coins of the United States in circulation but they are no longer coined.

3. The gold and silver coins consist of 9 parts by weight of pure metal and 1 part alloy. The alloy is used to toughen the metal so as to reduce the loss from use. The alloy of gold coins consists of $\frac{1}{10}$ part silver and $\frac{9}{10}$ copper. The alloy of silver coins is pure copper. The nickel coins consist of $\frac{1}{4}$ nickel and $\frac{3}{4}$ copper. The cent consists of $\frac{19}{20}$ copper and $\frac{1}{20}$ tin and zinc.

4. All gold coins are a *legal tender* for any amount. The currency silver dollars are also a full *legal tender* for all sums not otherwise provided for by contract. The smaller silver coins are *legal tender* for all sums not exceeding $10 in any one payment; and the nickel and bronze coins, for any amount not exceeding 25 cents in any one payment. No foreign coins are a legal tender in the United States.

281. All **operations** in United States money are performed the same as with common decimal expressions, and hence were considered under decimals in their proper connection.

CANADA MONEY.

282. The **Currency of Canada** is nominally the same as that of the United States, the table and the denominations being the same.

1. The **Coins** consist of silver and copper. The *silver coins* are the fifty, twenty-five, and twenty cent pieces; also the dime and half dime. The *copper coin* is the cent.

2. Canadian coins are not received at their full face value in all parts of the United States. They are usually taken at $\frac{4}{5}$ of their face value.

ENGLISH OR STERLING MONEY.

283. English or Sterling Money is the legal currency of England.

TABLE.

4 Farthings (*far.* or *qr.*)	= 1 Penny	*d.*
12 Pence	= 1 Shilling	*s.*
20 Shillings	= 1 Pound, or Sovereign	£

£ *s.* *d.* *far.*
1 = 20 = 240 = 960

Scale.—20, 12, 4.

1. *The unit is* the pound, represented by the sovereign and £1 bank note. *Par value in U. S. money* is $4.8665.

2. The **Coins** of Great Britain are of *gold*, *silver*, and *copper*. The *gold* coins are the sovereign and the half-sovereign. The *silver* coins are the crown (equal to 5 shillings), half-crown, florin (equal to 2 shillings), shilling, six-penny, and three-penny pieces. The *copper* coins are the penny and the half-penny. The guinea (equal to 21 shillings), and the half-guinea are in circulation but no longer coined.

3. Shillings and pence are sometimes expressed by writing them on opposite sides of an oblique line, and farthings are commonly expressed as fractions of a penny. Thus, 4*s.* 6*d.* 3 *far.* may be written 4/6¾.

REDUCTION OF ENGLISH MONEY.

284. To reduce to lower denominations.

1. How many pence in £5 8*s.* 7*d.*?

	20	12
£	*s.*	*d.*
5	8	7
20		
108		
12		
1303		

EXPLANATION.—In one pound there are 20 shillings, and in £5 there are 5 times 20 shillings, which increased by 8*s.*, are 108 shillings.

In one shilling there are 12 pence, and in 108*s.* there are 108 times 12 pence, which increased by 7*d.* are 1303 pence.

Hence, £5 8*s.* 7*d* = 1303*d.*

RULE.—*Multiply by the units in the scale from the given to the required denomination, being careful to add the given units of each denomination.*

EXAMPLES.

Reduce to pence:

2. 6*s.* 4*d.*
3. 9*s.* 7*d.*
4. 11*s.* 6*d.*
5. £2 4*s.* 6*d.*
6. £15 7*s.* 8*d.*
7. £24 16*s.* 11*d.*

Reduce to farthings:

8. 7*s.* 6*d.* 3 *far.*
9. 12*s.* 5*d.* 2 *far.*
10. 17*s.* 4*d.* 3 *far.*
11. 6£ 7*s.* 6*d.* 2 *far.*
12. 11£ 14*s.* 7*d.* 3 *far.*
13. 21£ 15*s.* 11*d.* 3 *far.*

14. Reduce ⅘ of a pound to units of lower denominations.

SOLUTION.

⅘ of a £ = ⅘ of 20*s.* = $\frac{80}{7}$*s.* 11⅐*s.*
⅘ of a *s.* = ⅘ of 12*d.* = $\frac{48}{7}$*d.* = 5⅐*d.*
⅐ of a *d.* = ¼ of 4 *far.* = ⅘ *far.*
∴ ⅘ of a £ = 11*s.* 5*d.* ⅘ *far.*

Reduce to units of lower denominations:

15. $\frac{3}{4}d.$	17. $\frac{2}{3}s.$	19. $£\frac{3}{4}$	21. $£\frac{7}{12}$
16. $\frac{5}{6}d.$	18. $\frac{3}{7}s.$	20. $£\frac{5}{6}$	22. $£\frac{14}{15}$

23. Reduce .76 of a pound to units of lower denominations.

SOLUTION.

.76 of a £ = .76 of 20*s.* = 15.20*s.*
.2 of a *s.* = .2 of 12*d.* = 2.4*d.*
.4 of a *d.* = .4 of 4 *far.* = 1.6 *far.*
∴ .76 of a £ = 15*s.* 2*d.* 1.6 *far.*

Reduce to units of lower denominations:

24. .75*d.*	26. .24*s.*	28. .375*s.*	30. £.625
25. .86*d.*	27. .56*s.*	29. £.95	31. £.875

285. To reduce to higher denominations.

1. Reduce 3415 pence to units of higher denominations.

12) 3415*d.*
20) 284*s.* + 7*d.*
£14 + 4*s.*

EXPLANATION.—Since there are 12 pence in one shilling, in 3415*d* there are as many shillings as 12 is contained times in 3415, which are 284*s.* and 7*d.*

Since there are 20 shillings in one pound, in 284*s.* there are as many pounds as 20 is contained times in 284, which are £14 and 4*s.*

Hence, 3415*d.* = £14 4*s.* 7*d.*

RULE.—*Divide by the units in the scale from the given to the required denomination.*

EXAMPLES.

Reduce to units of higher denominations:

2. 4760*d.*	5. 6847*s.*	8. 17643 *far.*
3. 5896*d.*	6. 7432*d.*	9. 26475 *far.*
4. 7135*a.*	7. 6345s.	10. 43598 *far.*

11. Reduce $\frac{2}{3}$ of a pence to a fraction of a pound.

SOLUTION.

1*d.* = $\frac{1}{12}$ of a *s.* ∴ $\frac{2}{3}d.$ = $\frac{2}{3}$ of $\frac{1}{12}s.$ = $\frac{1}{18}s.$
1*s.* = $\frac{1}{20}$ of a £ ∴ $\frac{1}{18}s.$ = $\frac{1}{18}$ of $£\frac{1}{20}$ = $£\frac{1}{360}$

Or,

1*d.* = $\frac{1}{240}$ of a £ ∴ $\frac{2}{3}$d. = $\frac{2}{3}$ of $£\frac{1}{240}$ = $£\frac{1}{360}$

Reduce to the fraction of a pound:

12. $\frac{3}{4}s.$	15. $\frac{5}{8}d.$	18. $\frac{1}{4}$ *far.*
13. $\frac{7}{8}s.$	16. $\frac{7}{11}d.$	19. $\frac{1}{2}$ *far.*
14. $\frac{5}{6}s.$	17. $\frac{11}{12}d.$	20. $\frac{3}{4}$ *far.*

21. Reduce 7*s.* 6*d.* 3 *far.* to the decimal of a pound.

4 | 3. *far.*
12 | 6.75 *d.*
20 | 7.5625*s.*
£.37817

EXPLANATION.—There are 4 far. in 1*d.*, hence, $\frac{1}{4}$ of the number of farthings equals the number of pence. $\frac{1}{4}$ of 3 equals .75, which with 6*d.* equals 6.75*d.* Since there are 12*d.* in 1*s.*, $\frac{1}{12}$ of the number of pence equals the number of shillings. $\frac{1}{12}$ of 6.75 equals .5625, which with 7*s.*, equals 7.5625*s.* Since there are 20*s.* in £1, $\frac{1}{20}$ of the number of shillings equals the number of pounds. $\frac{1}{20}$ of 7.5624 equals £.3782.

Reduce to the decimal of a pound:

22. 5*s.* 4*d.* 2 *far.* 23. 12*s.* 7*d.* 1 *far.* 24. 18*s.* 11*d.* 3 *far.*

Express as pounds and decimals of a pound:

25. £3 4*s.* 5*d.* 2 *far.*	27. £20 14*s.* 7*d.* 1 *far.*
26. £7 8*s.* 6*d.* 3 *far.*	28. £35 19*s.* 11*d.* 3 *far.*

286. To reduce English money to United States money.

1. Reduce £4 5*s.* 6*d.* 1 *far.* to dollars and cents.

SOLUTION.—£4 5*s.* 6*d.* 1 *far.* = £4.278+
£1 = $4.8665
∴ £4 278+ = 4.278 × $4.8665 = $20.82.

Find the value in U. S. Money:

2. £6 5*s.* 6*d.*	4. £30 8*s.* 3*d.*	6. £12 4*s.* 7*d.* 3 *far.*
3. £20 6*s.* 9*d.*	5. £45 9*s.* 6*d.*	7. £20 15*s.* 10*d.* 2 *far.*

287. To reduce United States Money to English money.

1. Find the value of $135.75 in English money.

SOLUTION.—$135.75 ÷ $4.8665 = £27.89479+
£27.89479+ = £27 17*s.* 10*d.* 3 *far.*

Find the values in English money:

2. $270.42	4. $475.60	6. $537.65	8. $1834.80
3. $315.16	5. $620.10	7. $742.40	9. $4972.40

FRENCH MONEY.

288. The Legal Currency of France is decimal.

TABLE.

10 Millimes (m) = 1 Centime . . . ct.
10 Centimes = 1 Decime . . . dc.
10 Decimes = 1 Franc . . . . fr.

Scale.—Decimal.

1. The *unit* is the franc. Its value in United States money is $0.193.

2. The **Coins** of France are of gold, silver, bronze, and copper. The *gold coins* are the hundred, forty, twenty, ten, and five franc pieces. The *silver coins* are the five, two, and one franc pieces. The *bronze* coins are the ten, five, two, and one centime pieces. There are also copper coins in ten and five centime pieces.

GERMAN MONEY.

289. German Money is the legal currency of the German Empire.

TABLE.

100 Pfennigs (Pennies) = 1 Mark.

Scale.—Decimal.

1. The *unit* is the mark. Its value in United States money is $0.238.

2. The **Coins** of the German Empire are of gold, silver, nickel, and copper. The *gold coins* are the twenty, ten, and five mark pieces. The *silver coins* are the two and one mark pieces. The *nickel coins* are the ten and five pfennig pieces. The *copper coins* are the two and one pfennig pieces.

NOTE.—In effecting exchanges, money accomplishes two purposes. It is useful

1. As a means of payment. In the discharge of this function it takes the place of the barter equivalent formerly given by both parties to any trade. The invention of money greatly facilitated trade by affording a recognized medium of exchange.

2. As a measure of value. In this function money is simply a yard-stick, or accepted unit of measure. When a government stamps or coins a piece of metal, it thereby indicates the weight and fineness of the metal coined.

The most interesting fact as to coin or real money (as distinguished from notes and bills or representatives of money) is that it has two values: 1st, Its nominal or face value in local exchanges or home trade—or that value authorized by the Government which issues the coin. This value is intended to remain stationary, regardless of the fluctuations in the value of the metal or metals coined. 2d, Its real or barter value in the exchanges of the world or international trade. This value has no regard for the value placed on the coin by the people among whom it is coined, but is determined by the quantity and quality of the metal contained in the coin.

It is on these principles that the values of foreign coins, as declared in *the table (on p. 12)*, are from time to time determined.

290. The following table shows the value of the standard coins of the world, in U. S. money, as proclaimed by the Secretary of the Treasury, in the latest official report:

Countries.	Standard.	Monetary unit.	Value in U. S. gold.	Coins.
Argentine Republic..	Gold.....	Peso.....	$0.965	Gold—argentine ($4.824) and ½ argentine; silver—peso and divisions.
Austria-Hungary.....	Gold.....	Crown.....	.203	Gold—20 crowns ($4.052) and 10 crowns.
Belgium.....	Gold.....	Franc.....	.193	Gold—10 and 20 franc pieces; silver—5 francs.
Brazil.....	Gold.....	Milreis.....	.546	Gold—5, 10, and 20 milreis; silver—½, 1, and 2 milreis.
British North America (except Newfoundland).	Gold.....	Dollar.....	1.00	
British Honduras.....	Gold.....	..do.....	1.00	
Chile.....	Gold.....	Peso.....	.428	Gold—escudo ($1.825), doubloon ($3.65), and condor ($7 30); silver—peso and divisions.
Costa Rica.....	Gold.....	Colon.....	.465	Gold—2, 5, 10, and 20 colons; silver—5, 10, 25, and 50 centisimos.
Cuba.....	Gold.....	Peso.....	.926	Gold—doubloon ($5.017); silver—peso (60 cents).
Denmark.....	Gold.....	Crown.....	.268	Gold—10 and 20 crowns.
Egypt.....	Gold.....	Pound (100 piasters).	4.943	Gold—10, 20, 50, and 100 piasters; silver—1, 2, 10, and 20 piasters.
Finland.....	Gold.....	Mark.....	.193	Gold—10 and 20 marks ($1.93 and $3.859).
France.....	Gold.....	Franc.....	.193	Gold—5, 10, 20, 50, and 100 francs; silver—5 francs.
Germany.....	Gold.....	Mark.....	.238	Gold—5, 10, and 20 marks.
Great Britain.....	Gold.....	Pound sterling.	4.866⅔	Gold—sovereign (pound sterling) and half sovereign.
Greece.....	Gold.....	Drachma.....	.193	Gold—5, 10, 20, 50, and 100 drachmas; silver—5 drachmas.
Hayti.....	Gold.....	Gourde.....	.965	Gold—1, 2, 5, and 10 gourdes; silver—gourde and divisions.
India.....	Gold.....	Rupee.....	.324	Gold—sovereign ($4.8665); silver—rupee and divisions.
Italy.....	Gold.....	Lira.....	.193	Gold—5, 10. 20, 50, and 100 lire; silver—5 lire.
Japan.....	Gold.....	Yen.....	.498	Gold—1, 2, 5, 10, and 20 yen.
Liberia.....	Gold.....	Dollar.....	1.00	
Netherlands.....	Gold.....	Florin.....	.402	Gold—10 florins; silver—½, 1, and 2½ florins.
Newfoundland.....	Gold.....	Dollar.....	1.014	Gold—$2 ($2.027).
Peru.....	Gold.....	Sol.....	.487	Gold—libra ($4.8665); silver—sol and divisions.
Portugal.....	Gold.....	Milreis.....	1.08	Gold—1, 2, 5, and 10 milreis.
Russia.....	Gold.....	Ruble.....	.515	Gold—imperial ($7.718) and ½ imperial ($3.80); silver—¼, ½, and 1 ruble.
Spain.....	Gold.....	Peseta.....	.193	Gold—25 pesetas; silver—5 pesetas.
Sweden and Norway..	Gold.....	Crown.....	.268	Gold—10 and 20 crowns.
Switzerland.....	Gold.....	Franc.....	.193	Gold—5, 10, 20, 50, and 100 francs; silver—5 francs.
Turkey.....	Gold.....	Piaster.....	.044	Gold—25, 50, 100, 200, and 500 piasters.
Uruguay.....	Gold.....	Peso.....	1.034	Gold—peso; silver—peso and divisions.
Venezuela.....	Gold.....	Bolivar.....	.193	Gold—5, 10, 20, 50, and 100 bolivars; silver—5 bolivars.

EXAMPLES.

291. 1. How many dollars in 200 francs?

2. How many francs in $337.75?

3. How many dollars in 485.57 francs?

4. How many francs would be received in exchange for $500?

5. How many dollars in 150 marks?

6. How many marks should be given in exchange for $241.32?

7. How many crowns of Denmark are worth $40.20?

8. How many gold roubles of Russia are worth $20.60?

9. How many piasters of Turkey are worth $44?

10. What is the value of 5000 florins of the Netherlands? Of 5000 English florins?

11. How many dollars in 264 marks, 25 pfennigs?

12. A lady bought 10 yards of alpaca, in London, for which she paid 5*s*. 3*d*. a yard. How much did it cost in U. S. money?

13. What is the value in U. S. money of 56 yards of merino at 7*s*. 6*d*. a yard?

14. How many dollars in 1000 drachmas of Greece?

15. A merchant returning from Europe, had 20 sovereigns, 5 crowns (Gt. Britain), 40 francs, and 10 marks (Germany). What was their total value in U. S. money?

MEASURES OF WEIGHT.

292. Weight is the measure of the force by which bodies are naturally drawn toward the earth.

293. The kinds of weight in common use are *Troy Weight*, *Apothecaries' Weight*, and *Avoirdupois Weight*.

TROY WEIGHT.

294. Troy Weight is used in weighing gold, silver, jewels, and also in philosophical experiments.

TABLE.

24 Grains (gr.)	= 1 Pennyweight . . .	pwt.
20 Pennyweights	= 1 Ounce	oz.
12 Ounces	= 1 Pound	lb.

lb. oz. pwt. gr.
1 = 12 = 240 = 5760

Scale.—12, 20, 24.

1. The *standard unit* of weight is the *Troy pound.* It is equal to the weight of 22.794377 cubic inches of pure (distilled) water, at its greatest density (temperature 39.83°F., barometer at 30 inches), and is identical with the imperial Troy pound of Great Britain.

2. The grains of the other weights are the same as the Troy grains.

APOTHECARIES' WEIGHT.

295. Apothecaries' Weight is used by apothecaries and physicians in weighing medicines for prescriptions.

TABLE.

20 Grains (gr. xx) = 1 Scruple . . . ℈
3 Scruples (℈ iij) = 1 Dram . . . . ʒ
8 Drams (ʒ viij) = 1 Ounce . . . ℥
12 Ounces (℥ xij) = 1 pound . . . . ℔

℔	℥	ʒ	℈	gr.
1	= 12	= 96	= 288	= 5760

Scale.—12, 8, 3, 20.

1. The *unit* is the pound, which is identical with the Troy pound, as are also the ounce and the grain, though the ounce is differently divided.

2. Drugs and medicines are bought and sold in large quantities by Avoirdupois weight.

AVOIRDUPOIS WEIGHT.

296. Avoirdupois Weight is the common weight of commerce, and is used for weighing almost everything except jewels and precious metals.

TABLE.

16 Ounces (oz.) = 1 Pound . . . . lb.
100 Pounds = 1 Hundred-weight. . cwt.
20 Hundred-weight = 1 Ton . . . . . T.

T.	cwt.	lb.	oz.
1 =	20	= 2000	= 32000

Scale.—20, 100, 16.

1. The *unit* is the pound. It is derived from the Troy pound, and contains 7000 grains.

2. In great Britain 28 lb. equal 1 qr., 112 lb. equal 1 cwt., and 2240 lb. equal 1 ton. These are called the *long hundred* and the *long ton*, or *shipping ton* of the United States. It is used at the U. S. Custom House in invoices of English goods, and by wholesale dealers in iron and coal. Retail dealers in coal *usually sell by the* hundred.

297. The following denominations are in use:

56 lb. Butter	= 1 Firkin.
100 lb. Grain or Flour	= 1 Cental.
100 lb. Dried Fish	= 1 Quintal.
100 lb. Nails	= 1 Keg.
196 lb. Flour	= 1 Barrel.
200 lb. Beef or Pork	= 1 Barrel.
200 lb. Fish	= 1 Barrel.
230 lb. Lime	= 1 Barrel.
280 lb. Salt at N. Y. Works	= 1 Barrel.

298. Gross Weight is the weight of the goods together with the box or whatever contains them.

299. Net Weight is the weight of the goods only.

MINIMUM WEIGHTS OF PRODUCE.

The following are the minimum weights of certain articles of produce, bought and sold by the bushel, according to the laws of the United States:

	Per Bushel.		Per Bushel.		Per Bushel.
Barley . . .	48 lbs.	Corn Meal . .	48 lbs.	Peas	60 lbs.
Beans, Castor .	46 "	Dried Apples .	26 "	Peas, Ground .	24 "
Beans, White .	60 "	Dried Peaches .	33 "	Potatoes, White	60 "
Blue Grass Seed	44 "	Flax Seed . .	56 "	Potatoes, Sweet	55 "
Bran	20 "	Hemp Seed . .	44 "	Rye	56 "
Buckwheat . .	48 "	Hungarian Grass Seed	50 "	Salt*	50 "
Clover Seed . .	60 "	Millet Seed . .	50 "	Timothy Seed .	45 "
Corn, in the ear	70 "	Oats	32 "	Turnips . . .	55 "
Corn, shelled .	56 "	Onions . . .	57 "	Wheat . . .	60 "

*The weight of salt per bushel as adopted by different States varies from 50 to 80 pounds. Coarse salt in Illinois is reckoned at 50 pounds, and in Pennsylvania at 80 pounds per bushel. Fine salt in Illinois and Kentucky is reckoned at 55 pounds, and in Pennsylvania at 62 pounds per bushel.

300. Comparative Table of Weights.

	Troy.	Apothecaries.	Avoirdupois.
1 lb.	= 5760 gr.	= 5760 gr.	= 7000 gr.
1 oz.	= 480 gr.	= 480 gr.	= 437½ gr.
	175 lb.	= 175 lb.	= 144 lb.
	175 oz.	= 175 oz.	= 192 oz.

DIAMOND WEIGHT.

301. Diamond Weight is used in weighing diamonds and other precious stones.

TABLE.

16 Parts = 1 Carat Grain.
4 Carat Grains = 1 Carat.

1. The *unit* is the carat, which equals 3.168 Troy grains. The carat grain equals .792 of a Troy grain.

2. The *carat of weight* should be carefully distinguished from the *assay carat*. The former denotes absolute weight, while the latter is used to denote the fineness, or purity, of gold in coins and jewelry, and means $\frac{1}{24}$ part. Thus, gold 14 carats fine contains 14 parts pure gold and 10 parts alloy.

EXAMPLES.

302. Reduce to units of lower denominations:

1. 4 oz. 16 pwt. 15 gr.
2. 9 oz. 17 pwt. 12 gr.
3. 12 lb. 6 oz. 13 pwt. 20 gr.
4. 35 lb. 10 oz. 8 pwt. 5 gr.
5. $\frac{7}{8}$ of an oz., Troy.
6. $\frac{4}{5}$ of a lb., Troy.
7. .365 of a lb., Troy.
8. 5 ʒ 1 ℈ 13 gr.
9. 8 ℥ 3 ʒ 2 ℈ 15 gr.
10. 11 ℔ 7 ℥ 1 ℈ 12 gr.
11. 25 ℔ 5 ʒ 2 ℈ 10 gr.
12. $\frac{2}{3}$ of a ℔, Apoth.
13. 5 cwt. 20 lb. 12 oz.
14. 7 cwt. 25 lb. 15 oz.
15. 12 cwt. 14 lb. 7 oz.
16. 3 T. 17 cwt. 40 lb. 5 oz.
17. 8 T. 8 cwt. 8 lb. 8 oz.
18. $\frac{3}{5}$ of a ton.
19. .84 of a cwt.
20. .675 of a T.

Reduce to units of higher denominations:

21. 1440 gr., Troy.
22. 7650 pwt.
23. 1080 gr., Apoth.
24. 4320 ℈.
25. 9860 gr., Troy.
26. 5740 ʒ.
27. 8350 gr., Apoth.
28. 2160 oz., Avoir.
29. 9780 oz., Avoir.
30. 5640 lb.
31. 64130 oz., Avoir.
32. 8232 lb. flour to barrels.
33. 9780 lb. pork to barrels.
34. 9860 lb. salt to barrels.

35. Reduce $\frac{3}{5}$ of a pwt. to the fraction of a pound.
36. Reduce $\frac{5}{16}$ of a ℈ to the fraction of a pound.
37. Reduce $\frac{7}{8}$ of an oz. to the fraction of a ton.
38. Reduce .275 gr., Troy, to the fraction of an ounce.
39. Reduce 5 ℥ 3 ʒ 2 ℈ 10 gr. to the decimal of a pound.

MEASURES OF EXTENSION.

303. Measures of Extension are used in measuring lengths, distances, surfaces, and solids.

304. Measures of Length are used in measuring distances in any direction.

305. A Line is that which has length but not breadth or thickness. Its extent is estimated by ascertaining how many times it contains some definite length, regarded as a unit of measure.

LINEAR MEASURE.

306. Linear, or Long, Measure is used in measuring lines, or distances.

TABLE.

12 Inches (in.)	= 1 Foot	ft.
3 Feet	= 1 Yard	yd.
5½ Yards, or 16½ Feet	= 1 Rod	rd.
320 Rods	= 1 Mile	mi.

mi.	rd.	yd.	ft.	in.
1 =	320 =	1760 =	5280 =	63360

Scale.—320, 5½, 3, 12.

307. The following denominations are also used:

3 Sizes	= 1 Inch.	Used by shoemakers.
4 Inches	= 1 Hand.	Used to measure the height of horses.
6 Feet	= 1 Fathom.	Used to measure depths at sea.
3.3 Feet	= 1 Pace.	Used in pacing distances.
5 Paces	= 1 Rod.	
40 Rods	= 1 Furlong.	
8 Furlongs	= 1 Mile, or Statute Mile.	
1.15 Statute Miles	= 1 Geographical, or Nautical, Mile or Knot.	
3 Geographical Miles	= 1 League.	
60 Geographical Miles, or 69.16 Statute Miles	= 1 Degree	of Lat. on a Meridian, or of Long. on the Equator.

1. The *standard unit* of length is the yard, from which all other measures of length are derived.

2. The yard is divided into halves, quarters, eighths, and sixteenths, for measuring goods sold by the yard.

3. The length of a degree of latitude varies. The average length, 69.1 miles, is the standard adopted by the United States Coast Survey.

4. *The following* denominations are used: The span = 9 inches; the *common cubit = 18 inches*; the sacred cubit = 21.888 inches.

SURVEYORS' LINEAR MEASURE.

308. Surveyors' Linear Measure is used by surveyors and engineers in measuring the dimensions of land, length of roads, etc.

TABLE.

7.92 Inches	= 1 Link	l.
25 Links	= 1 Rod	rd.
4 Rods or 100 Links	= 1 Chain	ch.
80 Chains	= 1 Mile	mi.

mi.	ch.	rd.	l.	in.
1 =	80 =	320 =	8000 =	63360

Scale.—80, 4, 25, 7.92.

1. The *unit* employed by surveyors is the *Gunter's Chain*, which is 4 rods, or 66 feet long.

2. An Engineer's chain, used by civil engineers, is 100 feet long, and consists of 100 links. In computing, links are written as so many hundredths of a chain.

EXAMPLES.

309. Reduce to feet:

1. 4 rd. 1 yd. 2 ft.
2. 8 rd. 2 yd. 1 ft.
3. 10 rd. 4 yd. 2 ft.
4. 36 rd. 10 ft.
5. 7 mi. 32 rd. 12 ft.
6. 15 mi. 80 rd. 14 ft.

Reduce to inches:

7. 3 yd. 2 ft. 5 in.
8. 4 yd. 1 ft. 8 in.
9. 5 yd. 2 ft. 8 in.
10. 12 rd. 10 ft. 5 in.
11. 4 mi. 50 rd. 4 yd. 2 ft. 3 in.
12. 9 mi. 180 rd. 3 yd. 2 ft. 4 in.

Reduce to links:

13. 4 rd. 20 l.
14. 15 rd. 12 l.
15. 32 rd. 15 l.
16. 8 ch. 2 rd. 10 l.
17. 20 ch. 3 rd. 18 l.
18. 15 mi. 30 ch. 75 l.

Reduce to units of lower denominations:

19. $\frac{2}{5}$ rd.
20. $\frac{5}{8}$ rd.
21. $\frac{4}{7}$ rd.
22. $\frac{2}{3}$ mi.
23. $\frac{4}{9}$ mi.
24. $\frac{7}{15}$ mi.
25. $\frac{5}{6}$ ch.
26. $\frac{3}{5}$ ch.
27. $\frac{7}{10}$ ch.
28. .65 yd.
29. .475 mi.
30. .625 ch.

Reduce to units of higher denominations:

31. 1440 in.	34. 3280 yd.	37. 10230 ft.
32. 2060 l.	35. 7456 ft.	38. 24172 yd.
33. 5132 ft.	36. 8375 l.	39. 76548 in.

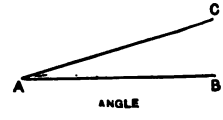
40. Reduce $\frac{4}{5}$ of a foot to the fraction of a rod.
41. Reduce .475 of an inch to the fraction of a yard.
42. Reduce $\frac{7}{8}$ of a foot to the fraction of a mile.
43. Reduce $\frac{3}{4}$ of a chain to the fraction of a mile.
44. Reduce 2 ft. 6 in. to the decimal of a yard.
45. Reduce 4 yd. 1 ft. 8 in. to the decimal of a rod.

SURFACE MEASURES.

310. A **Surface** is anything that has only length and breadth.

Thus, the floor, a face of a wall, or the outside of anything is a surface.

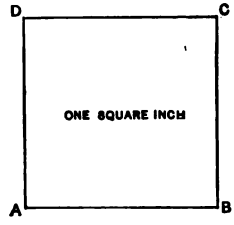
311. An **Angle** is the difference in the direction of two lines that meet.



ANGLE

The point at which the lines meet is called the *Vertex*, and the lines are called the *sides* of the angle.

312. A **Square** is a plane (flat) surface that has four equal sides and four equal angles.

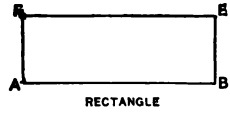


SQUARE

A *square inch* is a square, each side of which is *one inch* long. A *square yard* is a square, each side of which is *one yard* long.

The angles of a square are called *right angles*.

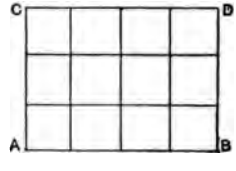
313. A **Rectangle** is a plane surface that has four straight sides and four right angles.



RECTANGLE

A square is a rectangle which has four equal sides.

314. The **Area** of a surface is the number of square units it contains.



Thus, if a rectangle is four units long and 3 units wide, it may be divided into 3 rows of squares with 4 square units in each row, making 12 square units *in all.*

Hence, the area is 12 square units.

SURFACE, OR SQUARE MEASURE.

315. Surface, or Square, Measure is used in measuring surfaces, such as land, boards, paving, plastering, etc.

TABLE.

144 Square Inches (sq. in.)	= 1 Square Foot	sq. ft.
9 Square Feet	= 1 Square Yard	sq. yd.
30¼ Square Yards, or 272¼ Square Feet	= 1 Square Rod	sq. rd.
160 Square Rods	= 1 Acre	A.
640 Acres	= 1 Square Mile	sq. mi.

A. sq. rd. sq. yd. sq. ft. sq. in
1 = 160 = 4840 = 43560 = 6272640

Scale.—640, 160, 30¼, 9, 144.

1. The *unit* for land is the *acre;* for other surfaces it is usually the *square yard.*

2. A square rod is sometimes called a *pole* or *perch.* The *rood* is found in old title deeds and surveys. It equals 40 perches, or square rods.

3. Plastering, ceiling, etc., are commonly estimated by the *square yard;* paving, stone-cutting, and glazing, by the *square foot.*

4. Roofing and slating are usually estimated by the 100 square feet, sometimes called a *square.*

5. A square piece of land, measuring 209 feet, or about 70 paces on each side, contains very nearly one acre. A rectangular piece 10 rods by 16 rods contains exactly an acre.

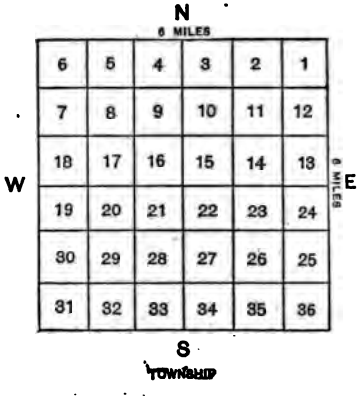
SURVEYORS' SQUARE MEASURE.

316. Surveyors' Square Measure is used in measuring land.

TABLE.

625 Square Links (sq. l.)	= 1 Square Rod	sq. rd.
16 Square Rods	= 1 Square Chain	sq. ch.
160 Square Rods	= 1 Acre	A.
	AND,	
10000 Square Links	= 1 Square Chain.	
10 Square Chains	= 1 Acre.	
640 Acres	= 1 Square Mile	sq. mi.
36 Square *Miles*	= 1 Township	Tp.

317. In some portions of the United States, 36 square miles (6 miles square) equal a *Township*.



1. Each township (Tp.) is divided into 36 equal squares of 1 square mile each, and numbered as in the diagram. These squares are called sections (Sec.). Each section is divided into half-section (320 acres) and quarter-sections (160 acres). The corners of all quarter-sections are permanently marked by monuments of stone or wood, when the township is formed, and a description of each monument and its location (surroundings) is made in the field notes of the surveyor.

2. The laying out of townships in surveying United States lands is easily understood. A selected *North* and *South* line is surveyed as a *Principal Meridian*, and an *East* and *West* line, intersecting it, is surveyed as a *Base Line*. From these, other lines are run at right angles, six miles apart, which divide the territory into *Townships* six miles square. A row of townships running north and south is called a *Range*. The townships in each range are numbered north and south from the base line, and the ranges are numbered east and west from the principal meridian.

EXAMPLES.

318. Reduce to square inches:

1. 4 sq. ft. 75 sq. in.
2. 7 sq. ft. 108 sq. in.
3. 5 sq. yd. 4 sq. ft.
4. 10 sq. rd. 26 sq. yd.
5. 120 sq. rd. 6 sq. ft.
6. 5 A. 16 sq. yd. 7 sq. ft.

Reduce to units of higher denominations:

7. 4350 sq. in.
8. 5480 sq. in.
9. 2760 sq. ft.
10. 6740 sq. ft.
11. 4384 sq. yd.
12. 7680 sq. rd.
13. 1200 sq. ch.
14. 3420 sq. l.
15. 6250 sq. l.

Reduce to units of lower denominations:

16. $\frac{3}{4}$ sq. rd.
17. $\frac{11}{15}$ sq. yd.
18. $\frac{3}{5}$ sq. ch.
19. $\frac{7}{8}$ sq. ch.
20. .345 sq. rd.
21. .675 sq. ch.

22. Reduce $\frac{4}{5}$ of a sq. ft. to the fraction of a sq. rd.

SOLUTION.—1 sq. rd. $= 272\frac{1}{4}$ sq. ft., or $\frac{1089}{4}$ sq. ft.

$\therefore$ 1 sq. ft. $= \frac{4}{1089}$ sq. rd.

$\frac{4}{5}$ sq. ft. $= \frac{4}{5}$ of $\frac{4}{1089}$ sq. rd. $= \frac{16}{5445}$ sq. rd.

23. Reduce $\frac{3}{4}$ of a sq. ft. to the fraction of a sq. rd.

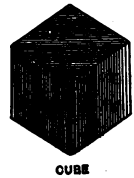
24. Reduce $\frac{5}{8}$ of a sq. yd. to the fraction of a sq. rd.

25. Reduce $\frac{2}{3}$ of a sq. in. to the fraction of a sq. yd.
26. Reduce .85 of a sq. rd. to the fraction of an A.
27. Reduce .75 of a sq. ch. to the fraction of a sq. mi.
28. Reduce 5 sq. yd. 3.6 sq. ft. to the decimal of a sq. rd.
29. Reduce 4 sq. rd. to the decimal of a sq. ch.
30. Reduce 6 sq. ch. 8 sq. rd. to the decimal of an A.

MEASURES OF VOLUME.

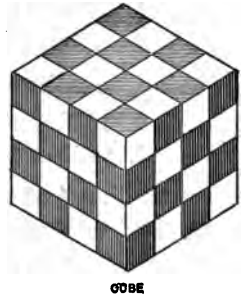
319. A **Solid** is anything that has length, breadth, and thickness, or height.

320. A **Cube** is a solid whose six sides are equal squares.


CUBE

A cube whose sides are each a square foot is called a *cubic foot*. One whose sides are each a square yard is called a *cubic yard*.

321. The Solid Contents, or Volume of any body is the number of solid units it contains.


CUBE

Thus, if a solid is 4 units long, 4 units wide, and 4 units thick, there are 4 times 4, or 16 cubes upon one surface, and since there are four such layers, there are 4 times 16, or 64 cubic units in all.

Hence, the volume is 64 cubic units.

CUBIC MEASURE.

322. Cubic, or Solid, Measure is used in measuring things that have length, breadth, and thickness.

TABLE.

1728 Cubic Inches (cu. in.)	= 1 Cubic Foot	cu. ft.
27 Cubic feet	= 1 Cubic Yard	cu. yd.
16 Cubic feet	= 1 Cord Foot	cd. ft.
8 Cord Feet, or 128 Cubic Feet	= 1 Cord	cd.

cu. yd. cu. ft. cu. in.
1 = 27 = 46656

Scale.—27, 1728.

1. The units, except the cord, are derived from the corresponding units of linear measure. Thus, the cubic foot is 12 in. long, 12 in. wide, and 12 in. thick. Hence, it contains $12 \times 12 \times 12$, or 1728 cubic inches. The cubic yard contains $3 \times 3 \times 3$, or 27 cubic feet.

2. A cubic yard of earth is called a *load*.

3. A cord of wood or stone is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. A cord foot is a part of this pile 1 ft. long, containing 16 cu. ft.

4. A *perch* of stone or masonry is $16\frac{1}{2}$ ft. long, 1 ft. wide, and 1 ft. high, and contains $16\frac{1}{2}$ cubic ft. Formerly a perch of masonry was considered $1\frac{1}{2}$ ft. wide, and contained $24\frac{3}{4}$ cu. ft. The perch of $16\frac{1}{2}$ cu. ft. is now in general use.

5. In measuring cargoes in the United States, a ton is estimated to each 40 cubic feet of occupied space. The tonnage of vessels is expressed in tons of 100 cu. ft. each, called the *register ton*.

6. In measuring timber $\frac{1}{4}$ of the contents of round timber is deducted for waste in hewing or sawing it. Hence, 40 cu. ft. of round timber or 50 cu. ft. of hewed timber is estimated as equal to *one ton* or *load*. Round timber is given in terms of the number of cubic feet to the ton a log would contain when "squared." "40 cu. ft. of round timber" is really a little more than 50 cu. ft. in the log.

EXAMPLES.

323. Reduce to cubic inches:

1. 12 cu. ft. 240 cu. in.
2. 35 cu. yd. 25 cu. ft.
3. 5 cd. ft. 7 cu. ft.
4. 2 cd. ft. 8 cu. ft. 12 cu. in.

Reduce to units of higher denominations:

5. 134620 cu. in.
6. 213542 cu. in.
7. 473400 cu. ft.
8. 648732 cu. ft.

9. Reduce $\frac{4}{5}$ of a cu. ft. to the fraction of a cubic yard.
10. Reduce 6 cd. ft. 4 cu. ft. to the decimal of a cord.

MEASURES OF CAPACITY.

324. Measures of Capacity are measures used to determine the quantity of fluids and of many dry substances.

There are three kinds of measures of capacity: *Liquid Measure*, *Apothecaries' Fluid Measure*, and *Dry Measure*.

LIQUID MEASURE.

325. Liquid Measure is used in measuring liquids.

TABLE.

4 Gills (gi.)	= 1 Pint	pt.
2 Pints	= 1 Quart	qt.
4 Quarts	= 1 Gallon	gal.

gal. qt. pt. gi.
1 = 4 = 8 = 32

Scale.—4, 2, 4.

1. The *standard unit* of liquid measure in the United States is the *gallon*, which contains 231 cubic inches, and will hold a little over 8⅓ lb. Av. of distilled water. The *imperial gallon* of Great Britain is a little more than one-fifth larger, and contains 277.274 cu. in. or 10 lb. Av. of distilled water, temperature 62° F., the barometer at 30 inches.

2. In estimating the capacity of cisterns, tanks, etc., 31½ gallons are considered a barrel (bar.), and 2 barrels, or 63 gallons a hogshead (hhd.). In commerce, barrels and hogsheads are of variable capacity.

3. The following denominations are sometimes used: 42 gallons = 1 tierce; 2 tierces = 1 puncheon; 2 hhd. (126 gal.) = 1 pipe, or butt; 2 pipes = 1 tun. These are not fixed measures. When used in business, the denominations merely indicate the *names* of the casks or their approximate size, without reference to their actual capacity. Casks are usually gauged and have their capacity marked upon them.

APOTHECARIES' FLUID MEASURE.

326. Apothecaries' Fluid Measure is used in prescribing and compounding liquid medicines.

TABLE.

60 Minims (♏)	= 1 Fluid drachm . . .	ƒ3
8 Fluid drachms	= 1 Fluid ounce	ƒ℥
16 Fluid ounces	= 1 pint	O.
8 Pints	= 1 Gallon	Cong.

Cong. O. ƒ℥ ƒ3 ♏
1 = 8 = 128 = 1024 = 61440

Scale.—8, 16, 8, 60.

1. Cong. is the abbreviation for *congius*, the Latin for gallon. O. is for the Latin *octarius*, one-eighth, the pint being one eighth of a gallon.

2. A common teaspoon holds about one fluid drachm; a common tablespoon, about ½ a fluid ounce; the common teacup, about 4 fluid ounces. The minim is the equivalent of a drop of water.

DRY MEASURE.

327. Dry Measure is used in measuring dry substances, such as grain, fruit, vegetables, salt, etc.

TABLE.

2 Pints (pt.)	= 1 Quart	qt.
8 Quarts	= 1 Peck	pk.
4 Pecks	= 1 Bushel . . .	bu.

bu. pk. qt. pt.
1 = 4 = 32 = 64

Scale.—4, 8, 2.

1. The standard unit in the United States is the *Winchester bushel.* I form it is a cylinder 18½ in. in diameter, and 8 in. deep. It contains 2150.4 cu. in., and is equal in volume to 77.627413 lb. Av. of distilled water, at it greatest density.

2. The Imperial bushel of Great Britain contains 2218.192 cu. in., o 1.03152 standard bushels of the United States. The English Quarter c grain is equal to a quarter of the long ton. Hence, it is equivalent to 8 bu of 70 lbs. to the bushel, or 9⅛ bu. of 60 lbs. to the bushel.

3. One-half of a peck, or 4 quarts, is called a dry gallon.

4. A pint, quart, or gallon, dry measure is more than the same quantity liquid measure. A quart, dry measure, is $\frac{1}{32}$ of a bushel, or $\frac{1}{32}$ of 2150.42 cu in., which is about 67⅕ cu. in., while a quart liquid measure is ¼ of 231 cu in., or 57¾ cu. in.

	Cu. In. In One Gal.	Cu. In. In One Qt.	Cu. In. In One Pt.	Cu. In. In One Gi.
Dry Meas. . . .	268⅘	67⅕	33⅗	8⅘
Liquid Meas. . .	231	57¾	28⅞	7$\frac{7}{32}$

328. In buying and selling the principal kinds of grain an seeds, the bushel is regarded as a certain number of pounds. Th weight of a bushel of the various grains as used by the Boards o Trade in the principal cities of the United States is as follows:

Barley	48 lb.	Oats	32 lb
Beans	60 "	Peas	60 "
Buckwheat	48 "	Rye	56 "
Clover Seed	60 "	Timothy Seed	45 "
Corn, shelled	56 "	Wheat	60 "
Corn, in the ear	70 "	Wheat Bran	20 "

EXAMPLES.

329. Reduce to gills.

1. 2 qt. 1 pt. 3 gi.
2. 3 qt. 1 pt. 2 gi.
3. 12 gal. 2 qt. 1 pt. 2 gi.
4. 17 gal. 3 qt. 1 pt. 3 gi.

Reduce to minims, or drops:

5. 6 ƒ3 30 ♏.
6. 7 ƒ3 45 ♏.
7. 5 ƒ℥ 5 ƒ3 5 ♏.
8. 3 O. 10 ƒ℥ 6 ƒ3 20 ♏.
9. 6 O. 3 ƒ℥ 4 ƒ3 45 ♏.
10. 7 Cong. 12 O. 5 ƒ℥ 35 ♏

Reduce to pints:

11. 2 pk. 3 qt. 1 pt.
12. 3 pk. 2 qt. 1 pt.
13. 3 pk. 7 qt. 1 pt.
14. 2 bu. 3 pk. 5 qt.
15. 5 bu. 6 qt. 1 pt.
16. 8 bu. 3 pk. 4 qt. 1 pt.

Reduce to units of higher denominations:

17. 216 gi.	20. 423 $f\mathfrak{Z}$	23. 1430 pk.
18. 785 gi.	21. 1000 $f\mathfrak{Z}$	24. 2150 qt.
19. 913 gi.	22. 1860 $\mathfrak{m}$.	25. 8742 pt.

26. Reduce $\frac{3}{4}$ of a pint to the fraction of a gallon.
27. Reduce $\frac{5}{8}$ of a quart to the fraction of a bushel.
28. Reduce 2 pk. 6 qt. to the decimal of a bushel.
29. Reduce 4 bu. 3 pk. 4 qt. to bushels.
30. Reduce 5 bu. 2 pk. 3 qt. 1 pt. to bushels.

MEASURES OF TIME.

330. Measures of Time are certain natural divisions of time fixed by the revolution of the earth on its axis and around the sun, and also other divisions which are artificial.

TABLE.

60 Seconds (sec.)	= 1 Minute	min.
60 Minutes	= 1 Hour	hr.
24 Hours	= 1 Day	da.
365 Days	= 1 Year	yr.
366 Days	= 1 Leap Year	yr.
100 Years	= 1 Century	cen.

ALSO,

7 Days	= 1 Week	wk.
4 Weeks	= 1 Lunar Month	mo.
30 Days	= 1 Commercial Month . .	mo.
12 Calendar Months	= 1 Year	yr.

cen.	yr.	mo.	da.	hr.	min.	sec.
1 =	100 =	1200 =	36500 =	876000 =	52560000 =	3153600000

Scale.—100, 365, 24, 60, 60.

1. The *unit of time* is the day. It is determined by the revolution of the earth on its axis. The *Sidereal Day* is the exact time of the revolution of the earth on its axis. The *Solar Day* is the time of the apparent revolution of the sun around the earth. The *Astronomical Day* is the solar day beginning and ending at noon. The *Civil Day* begins and ends at 12 o'clock, midnight.

2. In banking business, the law fixes the end of the day at the hour appointed for closing the bank.

3. In most business transactions 30 days are considered a month, and 12 such months a year. For many purposes 4 weeks constitute a month.

4. The common year contains 52 weeks and 1 day, the leap year 52 weeks and 2 days. Hence, each year begins one day later in the week than did the preceding year, except the year following leap year, which is two days later.

5. The *Solar Year* is the exact time required by the earth to mak complete revolution around the sun. Its length is 365 da. 5 hr. 48 49.7 sec., or nearly 365¼ days. Instead of reckoning this part of a day year, it is disregarded, and an addition is made when this amounts *day*, which is nearly every fourth year. This one day is added to the r of February. Since the part of a day of the *true* or *solar year* is a *lit than one fourth of a day*, the addition of one day every fourth year is a too much, and to correct this excess, addition is made to only every centennial year. With this correction the error amounts to only a more than a day in 4000 years. Hence,

RULE FOR LEAP YEARS.—*Centennial years exactly divisil 400, and other years exactly divisible by 4, are Leap Years.*

6. The reckoning of time by the ancients was very inaccurate. Th owing largely to their ignorance of astronomy; but changes were alsc for political reasons. The calendar was reformed by direction of Caesar, 46 B. C., who made the year to consist of 365¼ days, adding o every fourth year. This was known as the Julian calendar and ga world a year more than ten minutes too long. In 1582 Pope Gregor corrected the error of 10 days which resulted from adding one day fourth year, by striking out 10 days from the calendar, calling the October the 15th, and ordaining that henceforth only those centennia should be leap years which are exactly divisible by 400. This calen known as the Gregorian calendar, and is used by nearly all ci countries except Russia, which still clings to the Julian calendar. difference of the two calendars is now 12 days, the current date in is 12 days behind that of other civilized countries of the world.

7. The Julian and Gregorian calendars are sometimes designated expressions Old Style (O. S.), and New Style (N. S.)

331. The year begins with the month of January, and ends the month of December.

The months, their names, and the number of days in eac as follows:

January,	31 da.	Jan.	July, 31 da. . . . .
February,	28 or 29 da. . .	Feb.	August, 31 da. . . . .
March,	31 da.	Mar.	September, 30 da. . . . .
April,	30 da.	Apr.	October, 31 da. . . . .
May,	31 da.	May.	November, 30 da. . . . .
June,	30 da.	June.	December, 31 da. . . . .

1. The following rhyme is helpful in remembering the nu of days in each month:

"Thirty days hath November,
April, June, and September;
February twenty-eight alone,
All the rest have thirty-one,
Except when Leap Year comes in fine,
When February has twenty-nine"

332. The **Seasons** are *Winter*, *Spring*, *Summer*, and *Autumn*, or *Fall*.

The Winter months are *December*, *January*, and *February;* the Spring months, *March*, *April*, and *May;* the summer months, *June*, *July*, and *August;* and the Autumn months are *September*, *October*, and *November*.

333. The number of days from any day of one month to the same day of any other month in the same year is readily found by the following table:

TABLE.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Apr.	May	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February . . .	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September . .	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November . . .	61	92	120	151	181	212	242	273	304	334	365	30
December . . .	31	62	90	121	151	182	212	243	274	304	335	365

EXPLANATION.—Suppose we wish to find the number of days from April 5th to October 25th. We find April in the vertical column and looking for the number in the same line under October column we find 183, to which we add 20 days (since the time is not to October 5th but to October 25th), making 203, the number of days required.

The table gives February 28 days. Hence, allowance must be made for leap year.

EXAMPLES

334. Reduce to units of lower denominations:

1. 3 hr. 10 min. 15 sec.
2. 5 hr. 24 min. 32 sec.
3. 12 hr. 45 min. 55 sec.
4. 6 wk. 4 da. 17 hr.
5. 7 wk. 5 da. 12 hr.
6. 9 wk. 4 da. 20 hr.
7. $\frac{4}{5}$ of a day.
8. .625 of a day.

Reduce to units of higher denominations:

9. 1220 min.
10. 5472 min.
11. *2560 hr.*
12. 7000 hr.
13. 8379 hr.
14. 9990 hr.
15. 574168 sec.
16. 813742 sec.
17. 1000000 sec.

18. Reduce $\frac{2}{3}$ of a minute to the fraction of a day.

19. Reduce $\frac{5}{8}$ of a day to the fraction of a week.

20. Reduce .85 of a minute to the decimal of a day.

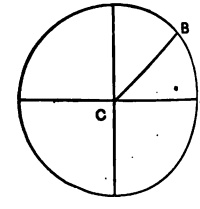
21. Reduce .325 of an hour to the decimal of a week.

22. Reduce 4 min. 25 sec. to the decimal of a day.

CIRCULAR MEASURE.

335. Circular Measure is used to measure arcs of circles and angles.

This measure is used by surveyors in determining the direction in which to take their linear measurements; by navigators in finding the latitude and longitude of places and vessels at sea; and by astronomers in making observations.



336. A Circle is a plain figure bounded by a curved line, every part of which is equally distant from a point within called the *centre*.

337. The **Circumference** of a circle is the bounding line. Any part of the circumference, as A B, is called an *arc*. Any angle having its vertex at the centre, is measured by the arc included between its sides. Thus, the angle A C B is measured by the arc A B.

TABLE.

60 Seconds (″)	= 1 Minute	′
60 Minutes	= 1 Degree	°
30 Degrees	= 1 Sign	S.
12 Signs, or 360°	= 1 Circumference	C.

C. S. ° ′ ″

1 = 12 = 360 = 21600 = 1296000.

Scale.—12, 30, 60, 60; or 360, 60, 60.

1. The unit is the degree, which is $\frac{1}{360}$ of a circle. All circles, great and small, contain the same number of degrees. Therefore, a degree is not a measure of space, but a measure of an angle whose vertex is the centre of the circle.

2. It will be seen that a quadrant measures a right angle whether the circle be large or small.

3. A minute of the earth's circumference is called by geographers a *geographic mile*, and by sailors a knot or nautical mile. It is equal to a *small fraction more* than 1.15 common miles.

EXAMPLES.

338. 1. Reduce 15° 20′ 10″ to seconds.

2. Reduce 37° 18′ 32″ to seconds.

3. How many seconds are there in 40° 28′ 21″?

4. Reduce 24350″ to higher denominations.

5. Reduce 74680″ to higher denominations.

COUNTING.

339. In counting certain classes of articles, the following denominations are used:

TABLE.

12 Things	= 1 Dozen	doz.
12 Dozen	= 1 Gross	gr.
12 Gross	= 1 Great Gross	G. gr.

Scale.—Duodecimal.

Two things of a kind are frequently called a *pair*, six a *set*, and twenty a *score*; as, a *pair* of gloves, a *set* of chairs, a *score* of years.

STATIONERS' TABLE.

340. Paper in the stationery trade is sold by the following

TABLE.

24 Sheets (sh.)	= 1 Quire	qu.
20 Quires	= 1 Ream	rm.
2 Reams	= 1 Bundle	bdl.
5 Bundles	= 1 Bale	bl.

bl. bdl. rm. qu. sh.
1 = 5 = 10 = 200 = 4800

Scale.—5, 2, 20, 24.

Wholesale dealers usually sell paper by the pound.

BOOKS.

341. In printing books large sheets of paper are used which are folded into leaves according to the size of the book. The terms *folio*, *quarto*, *octavo*, etc., applied to printed books, are based on sheets about 17 × 22 in., and indicate the number of leaves into which a ***sheet is folded.***

1. The following table shows the number of leaves and pages to the sheet (size 17 inches by 22 inches called *Folio*), and the sizes of the pages in standard books, the pages of course being always double the number of leaves:

TABLE.

Size of Book.	Leaves to each sheet.	Pages to each sheet.	Length of page.	Width of page.
Folio	2	4	17	11
Quarto	4	8	11	8½
Octavo	8	16	8½	5½
12 mo.	12	24	7½	4½
16 mo.	16	32	5½	4½

2. Printing paper is made of many sizes, to suit the requirements of the printer. In book printing, paper about 24 × 38 inches, called *Double Royal*, is extensively used; but the size of the book is, nevertheless, based on the size of the paper called *Medium*, which is 18 × 23 inches.

3. A careful examination of the upper or lower inside edge of an open book will determine the number of leaves in a sheet, as each full-sized sheet is folded separately, and placed together side by side with the packs of other sheets, or "collated" before a book is bound.

342. Copyists are often paid by the *folio* for making copies of legal papers, records, and documents. In copying usually 100 words are considered the unit of measure or *folio*.

343. In type-setting, an *em*, the portion of a line formerly occupied by the letter *m*, then a square type, is used as a unit by which to measure and estimate the amount of printed matter.

PRACTICAL PROBLEMS.

344. 1. How much must be paid for 4 gross of pencils at 3 cents apiece?

2. How many sheets of paper in 5 reams?

3. What will 1240 sheets of foolscap cost at 15 cents a quire?

4. How much must be paid for a quarter gross of pens at the rate of $16.80 for a great gross?

5. How much paper will be required to issue an edition of 3000 copies of a 12 mo. book of 300 pages, allowing a quire to a ream for waste?

MEASURES OF HEAT.

345. Temperature, or **Intensity of Heat,** is measured by instruments called Thermometers. The principal kinds in common use are Fahrenheit's, the Thermométre Centigrade, and Réaumur's. The first named is the kind mostly used in England and America, though the Thermométre Centigrade, owing to its convenient decimal scale, is coming into general use for scientific purposes throughout the world. A comparison of the three different scales is shown in the following

TABLE.

Standard Points.	Degrees on the Scale.		
	Fahrenheit.	*Centigrade.*	*Réaumur.*
Boiling point of water at sea level, . .	212°	100°	80°
Melting point of ice at sea level . . .	32°	0°	0°

346. To reduce from one Thermometer to another.

Let F°, C°, R°, represent any number of degrees, Fahrenheit, Centigrade or Réaumur. It will be seen from the table that the difference between the melting point of ice and the boiling point of water is 180° F, 100° C, and 80° R.

Hence, $C° = \frac{5}{9}(F° - 32°)$. $\therefore F° = \frac{9}{5} C° + 32°$.

$R° = \frac{4}{9}(F° - 32°)$. $\therefore F° = \frac{9}{4} R° + 32°$.

$C° = \frac{5}{4} R°$. $\therefore R° = \frac{4}{5} C°$.

1. On the Fahrenheit thermometer, the *freezing point* of water is 32°; seawater, 28°; strong wine, 20°; mercury, 39° below zero. The *boiling point* of alcohol is 167°; water, 212°; oil of turpentine, 560°; linseed oil, 600°; quicksilver, 660°. The *melting point* of butter, lard, and tallow is about 92°; sperm-aceti, 112°; beeswax, 142°; sulphur, 239°; tin, 442°; bismuth, 507°; lead, 617°; zinc, 773°; copper, 2200°; silver, 1832°; gold, 2518°; cast iron, 2800°.

2. The mean temperature of the blood is 99.5°. Iron red-heat in the dark is 752°; in twilight, 884°; in daylight, red-heat fully visible, 1077°. The heat of common fire is 790°.

EXAMPLES.

347. 1. Reduce 77° F. to Centigrade.

Solution: $\frac{5}{9} \times (77° - 32°) = 25°$ C.

2. Reduce 80° C. to Fahrenheit.

Solution: $\frac{9}{5} \times 80 + 32 = 176°$ F.

3. Reduce 50° F. to Réaumur.

4. Reduce 240° C. to Réaumur.

5. Mercury freezes at 39° below zero, or — 39° F. At what does it freeze *Centigrade?*

ADDITION.

348. Addition of Denominate Numbers is the process of finding the sum of two or more similar compound denominate numbers.

349. Denominate numbers are added, subtracted, multiplied, and divided by the same general methods as are employed for corresponding operations in abstract numbers, except that in the latter we have the uniform scale of ten, while in compound numbers we have a varying scale.

350. 1. Add £3 5*s.* 7*d.*, £4 17*s.* 8*d.*, £13 11*s.* 5*d.*, and £9 6*s.* 10*d.*

£	*s.*	*d.*
3	5	7
4	17	8
13	11	5
9	6	10
31	1	6

EXPLANATION.—We write the numbers so that similar units shall stand in the same column, and begin at the right to add.

The sum of the pence is 30*d.* = 2*s.* 6*d.* We write the 6*d.* in the pence column, and adding the 2*s.* to the column of shillings we obtain 41*s.* = £2 1*s.* We write the 1*s.* in the column of shillings, and add the £2 to the column of pounds, obtaining £31 pounds which we write in the pounds column. Hence, the entire sum is £31 1*s.* 6*d.*

EXAMPLES.

2. Add £18 16*s.* 5*d.*, £23 17*s.* 9*d.*, £47 13*s.* 8*d.*, and £62 11*s.* 11*d.*

3. Add 18 lb. 9 oz. 18 pwt. 12 gr., 13 lb. 7 oz. 15 pwt. 21 gr., 19 lb. 11 oz. 16 pwt. 15 gr.

4. Add 13 T. 15 cwt. 54 lb. 15 oz., 24 T. 12 cwt. 35 lb. 13 oz., 68 T. 17 cwt. 84 lb. 9 oz.

5. Add 5 yr. 7 mo. 12 da., 4 yr. 8 mo. 21 da., 3 yr. 7 mo. 25 da.

6. Add 14 cd. 121 cu. ft. 235 cu. in., 24 cd. 104 cu. ft. 670 cu. in., 35 cd. 94 cu. ft. 537 cu. in.

7. Find the sum of 12 mi. 132 rd. 3 yd. 2 ft. 5 in., 12 mi. 76 rd. 4 yd. 8 in., 5 mi. 92 rd. 2 yd. 1 ft. 7 in., 21 mi. 83 rd. 5 yd. 2 ft. 11 in.

8. Find the sum of 146 mi. 140 rd. 2 yd. 1 ft. 3 in., 150 mi. 80 rd. 3 yd. 1 ft. 10 in., 167 mi. 120 rd, 1 yd. 2 ft. 7 in., 187 mi. 200 rd. 5 yd. 1 ft. 4 in., 185 mi. 85 rd. 2 yd. 2 ft. 10 in., 147 mi. 175 rd. 4 yd. 1 ft. 9 in., 275 mi. 190 rd. 2 ft. 8 in.

SUBTRACTION.

351. Subtraction of Denominate Numbers is the process of finding the difference between two similar compound denominate numbers.

352. 1. From 11 oz. 9 pwt. 18 gr. take 6 oz. 14 pwt. 13 gr.

oz.	pwt.	gr.
11	9	18
6	14	13
4	15	5

EXPLANATION.—We write the subtrahend under the minuend, placing similar units in the same column and begin at the right to subtract.

13 gr. from 18 gr. leave 5 gr., which we write under the grains. 14 pwt. cannot be subtracted from 9 pwt.; we will therefore take 1 oz. from 11 oz., leaving 10 oz., and add it to the 9 pwt. 1 oz. equals 20 pwt., which added to the 9 pwt. equal 29 pwt. Then 14 pwt. taken from 29 pwt. leave 15 pwt., which we write under the pwt. column. 6 oz. from 10 oz. leave 4 oz., which we write. Hence, the remainder is 4 oz. 15 pwt. 5 gr.

EXAMPLES.

2. From £124 11*s.* 7*d.*, take £87 19*s.* 5*d.*

3. From £144 7*s.* 9*d.*, take £68 12*s.* 11*d.*

4. From 84 lb. 10 oz. 14 pwt. 17 gr., take 38 lb. 11 oz. 17 pwt. 20 gr.

5. From 240 T. 12 cwt. 70 lb. 13 oz., take 136 T. 9 cwt. 85 lb. 15 oz.

6. From 94 mi. 186 rd. 1 yd. 2 ft. 7 in., take 68 mi. 259 rd. 4 yd. 1 ft. 10 in.

353. To find the difference in time between two dates.

1. What is the difference in time from June 9, 1898, to Dec. 5, 1903?

yr.	mo.	da.
1903	12	5
1898	6	9
5	5	26

EXPLANATION.—We express dates by the number of the year, the month, and the day.

Since the later date expresses the greater period of time, we write it as the minuend, and the earlier date as the subtrahend. We then subtract as in denominate numbers, considering 30 days 1 month, and 12 months 1 year. The remainder will be as correct as can be expressed in months and days.

EXAMPLES.

2. Find the time from Feb. 6, 1884, to Sept. 14, 1891.

3. Find the time from July 18, 1890, to Mar. 14, 1895.

4. *How long was it from* May 15, 1889, to April 10, 1898?

5. Franklin was born Jan. 6, 1706, and died Apr. 17, 1790. How old was he?

6. Washington was born Feb. 22, 1732, and died Dec. 14, 1799 What was his age?

7. Find your age by compound subtraction as above.

8. What is the exact number of days from June 24, to Sept. 10?

Suggestion.—When the exact time is required, we reckon the actual number of days in each month, counting the last day of the later date, but not the first day of the earlier.

9. What is the exact number of days from Apr. 16, 1892, to Nov. 21, 1893?

10. What is the exact number of days from Jan. 15, 1896, to Jan. 3, 1897?

MULTIPLICATION.

354. Multiplication of Denominate Numbers is the process of finding the product when the multiplicand is a compound denominate number.

355. 1. Multiply £10 14*s.* 7*d.* by 9.

£	*s.*	*d.*
10	14	7
		9
96	11	3

EXPLANATION.—We write the multiplier under the lowest denomination of the multiplicand, and begin at the right to multip'y.

9 times 7*d.* are 63*d.* = 5*s.* 3*d.* We write the 3*d.* under the pence, and add the 5*s.* to the product of shillings. 9 times 14*s.* are 126*s.*, plus 5*s.* equal 131*s.*, or £6 11*s.* We write the 11*s.* under the shillings and add the £6 to the product of pounds. 9 times £10 are £90, plus £6 equal £96, which we write under pounds. Hence, the product is £96 11*s.* 3*d.*

EXAMPLES.

2. Multiply £16 9*s.* 10*d.* by 8.

3. Multiply £175 7*s.* 8*d.* 3 *far.* by 7.

4. Multiply 18 lb. 6 oz. 14 pwt. 15 gr. by 5.

5. Multiply 15 bu. 2 pk. 5 qt. 1 pt. by 12.

6. Multiply 8 hhd. 45 gal. 3 qt. 1 pt. by 35.

7. Multiply 9 bu. 3 pk. 5 qt. 1 pt. by 40.

8. A farmer sold 8 loads of hay, each containing 17 cwt. 38 lb. How much did he sell?

9. If a pipe discharges 5 hhd. 30 gal. 2 qt. of water in an hour, how much will it discharge in a day?

10. If one silver spoon weighs 2 oz. 18 pwt. 20 gr., what will be *the weight of 3* sets of 6 spoons each?

DIVISION.

56. Division of Denominate Numbers is the process of find- the quotient when the dividend is a compound denominate ıber.

57. 1. Divide £92 7*s.* 6*d.* by 5.

£	*s.*	*d.*
)2	7	6
8	9	6

EXPLANATION.—We write the divisor at the left of the dividend, and begin at the highest denomination to divide. One fifth of £92 is £18, with a remainder of £2. We write the £18 in the quotient and add the £2 ıining to the next lower denomination, making 47*s.*
ne fifth of 47*s.* is 9*s.* with 2*s.* remaining. We write the 9*s.* in the quo-, and the 2*s.* remaining to the next lower denomination, making 30*d.*
ıe fifth of 30*d.* is 6*d.*, which we write in the quotient. Hence the entire ient is £18 9*s.* 6*d.*

ıggestion.—When the divisor is a compound denominate number, reduce dividend and divisor to the lowest denomination mentioned in either, then divide as in whole numbers.

EXAMPLES.

2. Divide £53 13*s.* 11*d.* by 7.

3. Divide £123 16*s.* 8*d.* by 8.

4. Divide 117 hhd. 42 gal. 2 qt. 1 pt. 2 gi. by 9.

5. Divide 208 lb. 3 oz. 17 pwt. 12 gr. by 12.

6. If 8 farmers together raise 1165 bu. 2 pk. 5 qt. 1 pt. of ın, what is the average amount raised by each?

7. Mr. B. traveled 500 mi. 300 rd. in 16 days. What was the rage distance for each day?

8. A ship sailed 75° 40′ 20″ in 24 days. How far did she sail ın average per day?

9. An Englishman earned £2 3*s.* 5*d.* in a week. In how many ıks could he earn £26 1*s.*?

Solution.—£26 1*s.* = 6252*d.*
£2 3*s.* 5*d.* = 521*d.*
6252*d.* ÷ 521*d.* = 12, the number of weeks.

10. Divide 50 oz. 10 pwt. 22 gr. by 3 oz. 12 pwt. 5 gr.

11. If a sack holds 2 bu. 2 pk. 3 qt. of grain, how many sacks it require to hold 51 bu. 3 pk. 4 qt.?

12. If a bale of hay weighs 3 cwt. 15 lb., how many bales will it ı to weigh 7 T. 1 cwt. 75 lb.?

LATITUDE, LONGITUDE, AND TIME.

358. Latitude is distance north or south from the equator.

A place is said to be in *north latitude* if *north* of the equator, and in *south latitude* if *south* of the equator.

359. Longitude is distance east or west from any given starting point or meridian.

A place is said to be in *east longitude* if *east* of the given meridian, and in *west longitude* if *west* of the given meridian.

A *meridian* is an imaginary line passing from the North Pole to the South Pole, through any place. Longitude is reckoned from a meridian which is taken arbitrarily. The English and Americans commonly reckon from that of the Royal Observatory at Greenwich, though the meridian of Washington is sometimes used in this country. In other countries different meridians are chosen, but chiefly those which pass through their capitals. The meridian selected is called the *prime* or *first* meridian, and its longitude is marked 0°.

360. The diurnal revolution of the earth on its axis from west to east, causes the apparent motion of the sun from east to west, round the earth, or through 360° of longitude, every 24 hours. Hence, the difference of time between any two places may be determined from their difference of longitude; or, their difference of time being known, their difference of longitude may be readily ascertained from the following

TABLE.

360°	of	longitude	= 24 hours or 1 day of time.
15°	"	"	= 1 hour of time.
15′	"	"	= 1 minute of time.
15″	"	"	= 1 second of time.
1°	"	"	= 4 minutes of time.
1′	"	"	= 4 seconds of time.

STANDARD TIME.—For the convenience of the railroads and the traveling public, a standard of time was established by mutual agreement in 1883, by which trains are run and local time regulated. This system, called the "Standard Time System," divides the United States and Canada into four sections of time-belts, each covering 15° of longitude, 7½° of which are east and 7½° west of the governing, or standard meridian, and the time throughout each belt is the same as the astronomical or local time of the governing meridian of that belt.

The standard meridians are the 75th, the 90th, the 105th, and the 120th, west of Greenwich. Since these meridians are exactly 15° apart there is a difference in time of precisely one hour between each succeeding section.

The first section, Eastern Time, includes all territory between the Atlantic coast and an irregular line drawn from Buffalo to Atlanta, Ga.

The second section, Central Time, includes all the territory between the last-named line and an irregular line from Bismarck, N. D., to the mouth of the Rio Grande.

The third section, Mountain Time, includes all territory between the last-named line and nearly the western borders of Idaho, Utah, and Arizona.

The fourth section, Pacific Time, includes the rest of the country to the Pacific coast.

When it is 12 o'clock noon at Philadelphia, Eastern Time, the time at Chicago, Central Time, is 11 o'clock A. M.; at Denver, Mountain Time, 10 o'clock A. M.; and at San Francisco, Pacific Time, 9 o'clock A. M. The Standard Time at Philadelphia is about 1 minute faster than true local time, 9 minutes slower at Chicago, and 10 minutes faster at San Francisco. Denver being located about on the 105th meridian the Standard and the local time are the same.

The changes from one time standard to another, in traveling, are made at the ends of roads, or at well-known points of departure, and where they are attended with the least danger and inconvenience. This system is fast coming into general use and, before long, the whole business of the country may be regulated by Standard time.

361. To find the difference of latitude or longitude between two places.

1. The latitude of New Orleans is 29° 57′ 46″ north, and of Chicago 41° 50′ 1″ north. What is their difference of latitude?

41°	50′	1″
29	57	46
11	52	15

EXPLANATION.—Since New Orleans and Chicago are both in north latitude, their difference of latitude must be the distance which Chicago is further north than New Orleans; and this distance is found by subtracting the less latitude of New Orleans from the greater latitude of Chicago.

2. The latitude of Baltimore is 39° 17′ 48″ north, and of Rio Janeiro 22° 54′ 24″ south. What is their difference of latitude?

39°	17′	48″
22	54	24
62	12	12

EXPLANATION.—Since the distance from Baltimore to the equator is 39° 17′ 48″, and from the equator to Rio Janeiro is 22° 54′ 24″ farther, the total distance or difference of latitude must be the sum of these two distances.

RULE.—*If the latitudes or the longitudes of the two given places are both of the same kind, subtract the less from the greater; if of different kinds, take their sum.*

Longitude, east or west, cannot exceed 180°, or a semi-circumference. **Hence, in adding** two longitudes, if their sum exceed 180°, such sum should be **subtracted** *from 360° to obtain* the correct difference of longitude.

EXAMPLES.

3. The latitude of Boston is 42° 21′ 28″ north, and of Liverpool 53° 24′ 4″ north. What is their difference of latitude?

4. The latitude of Philadelphia is 39° 57′ 7″ north, and of Denver 39° 40′ 36″ north. What is their difference of latitude?

5. The longitude of Washington is 77° 2′ 48″ west, and of San Francisco 122° 26′ 15″ west. What is their difference of longitude?

6. The latitude of Montreal is 45° 30′ 17″ north, and of Bahia, Brazil, 13° 0′ 37″ south. What is their difference of latitude?

7. The longitude of New York is 74° 3′ west, and of Berlin 13° 23′ 45″ east. What is their difference of longitude?

8. The longitude of San Francisco is 122° 26′ 15″ west, and of Canton, China, 113° 26′ 30″ east. What is their difference of longitude?

362. To find the difference in time between two places when the difference in longitude is given.

1. The difference in longitude between two places is 33° 2′ 30″. What is their difference in time?

15)	33°	2′	30″
	2	12	10

EXPLANATION.—Since 15° of longitude equal 1 hr. of time, 15′ of longitude 1 min. of time, and 15″ of longitude 1 sec. of time, $\frac{1}{15}$ of 33° 2′ 30″, is the difference in hours, minutes, and seconds of time. Hence, the difference in time is 2 hr. 12 min. 10 sec.

RULE.—*Divide the difference of longitude expressed in degrees, minutes, and seconds, by 15. The result will be the difference of time respectively in hours, minutes, and seconds.*

EXAMPLES.

2. The difference in longitude between two places is 64° 5′ 20″. What is their difference in time?

3. The longitude of Boston is 70° 3′ 58″ west, and that of St. Louis 90° 15′ 16″ west. Find their difference in time.

4. Washington is 77° 2′ 48″ west, and Rome 12° 27′ 14″ east from Greenwich. What is the difference in time?

5. The longitude of Bangor, Me., is 68° 47′ west, and of Cincinnati 84° 24′ west. When it is noon at Bangor, what is the solar time at Cincinnati?

6. Baltimore is 76° 37′ west, and Berlin is 13° 23′ 45″ east longitude. When it is 9 o'clock A. M. at Baltimore, what time is it at Berlin?

363. To find the difference in longitude between two places when the difference in time is given.

1. The difference in time between two places is 4 hr. 15 min. 25 sec. What is the difference in longitude?

hr.	min.	sec.
4	15	25
		15
63°	51′	15″

EXPLANATION.—Since there are 15 times as many degrees, minutes, and seconds of longitude as there are hours, minutes, and seconds of time, 15 times 4 hr. 15 min. 25 sec., is the difference in degrees, minutes, and seconds of longitude. Hence, the difference in longitude is 63° 51′ 15″.

RULE.—*Multiply the difference of time expressed in hours, minutes, and seconds, by 15. The result will be the difference of longitude respectively in degrees, minutes, and seconds.*

EXAMPLES.

2. The difference in time between two places is 5 hr. 13 min. 17 sec. What is their difference in longitude?

3. The difference of time between Denver, Colo., and Greenwich is 6 hr. 59 min. 47.6 sec. What is the longitude of Denver?

4. If the exact difference in time between Chicago and Washington is 42 min. 15 sec., what is their difference in longitude?

5. When the solar time in San Francisco is 10 min. past 6 o'clock P. M., the solar time in Calcutta is 13 min. 3 sec. past 8 o'clock A. M. If the longitude of San Francisco is 122° 26′ 15″ west, what is the longitude of Calcutta?

6. The difference in time between Greenwich and Havana, Cuba, is 5 hr. 29 min. 26 sec. What is the longitude of Cuba?

7. The difference in time between New York and Astoria, Ore., is about 3 hr. 19 min. 25 sec. The longitude of New York is 74° 3′ west. What is the longitude of Astoria?

8. Yokohama, Japan, time is about 7 hr. 17 min. 26 sec. faster than St. Petersburg time. The longitude of the latter city is 30° 19′ east. What is the longitude of Yokohama?

9. A captain of a vessel finds on observation that by solar time it is 9 A. M., but by his chronometer, Greenwich time, it is 10 min. past 12 o'clock P. M. What is the longitude of his ship?

METRIC SYSTEM OF WEIGHTS AND MEASURES.

364. The Metric System is a decimal system of denominate numbers, which has for its basis a certain unit of measure called the *meter*.

1. The length of the meter was originally intended to be one ten-millionth part of a quadrant of the earth's circumference, or of the distance from either pole to the equator; but later calculations have shown the meter to be a very little less than that.

2. The system derives its name from the *meter*, because all the units of measure in the system are derived from it.

365. The Primary Units of the metric system are the following:

Kind of Measure.	*Primary Unit.*	*Pronunciation.*
Length	Meter	(mē'-ter).
Surface	Square Meter	(square mē'ter).
Volume	Cubic Meter	(cubic mē'-ter).
Capacity	Liter	(lē'-ter).
Weight	Gram	(grăm)

366. Lower Denominations than the primary units are denoted by prefixing the following from the Latin: *milli* ($\frac{1}{1000}$ of a primary unit); *centi* ($\frac{1}{100}$ of a primary unit); *deci* ($\frac{1}{10}$ of a primary unit).

367. Higher Denominations than the primary units are denoted by prefixing the following from the Greek: *deka* (10 primary units); *hekto* (100 primary units); *kilo* (1000 primary units); *myria* (10000 primary units).

1. In the prefixes *deka* and *hekto*, the letter *k* is usually changed to *c*; as *decameter*, *hectometer*, *decaliter*, etc.

2. The Metric System of weights and measures is based wholly upon the decimal method of notation. All the divisions and multiples are by 10. This is so great an improvement upon the old methods that it has been adopted or legalized by most of the civilized nations of the world. It is authorized by law in the United States.

368. The following tables give all the denominations, but many of them are not in common use. Those usually employed in business or science are printed in bold-faced type.

369. Abbreviations beginning with *small letters* indicate divisions of the unit; those beginning with capital letters denote *multiples of the unit.*

METRIC TABLES.

MEASUREMENTS OF LENGTH.

370. The unit of length is the *meter*. It equals 39.37079 inches, or about 3.28 feet.

TABLE.

10 **Millimeters** (mm)	= 1 **Centimeter** (cm)	=	.3937079 in.
10 Centimeters	= 1 Decimeter (dm)	=	3.937079 in.
10 Decimeters	= 1 **Meter** (m)	=	39.37079 in.
10 Meters	= 1 Decameter (Dm)	=	32.80899 ft.
10 Decameters	= 1 Hectometer (Hm)	=	19.92781 rd.
10 Hectometers	= 1 **Kilometer** (Km)	=	.621382 mi.
10 Kilometer	= 1 Myriameter (Mm)	=	6.21382 mi.

1. The meter is a little over 3 feet 3 inches and 3-tenths of an inch in length.

2. The 5-cent piece is about $\frac{1}{50}$ of a meter in diameter, or 2 centimeters.

MEASURES OF SURFACE.

371. The unit of surface is the *square meter*. It equals about 1.196 square yards.

TABLE.

100 Sq. Millimeters	= 1 **Sq. Centimeter**	= .155 + sq. in.
100 Sq. Centimeters	= 1 Sq. Decimeter	= 15.5 + sq. in.
100 Sq. Decimeters	= 1 **Sq. Meter**	= 1.196 + sq. yd.
100 Sq. Meters	= 1 Sq. Decameters	= 119.6034 sq. yd.
100 Sq. Decameters	= 1 Sq. Hectometer	= 2.47114 A.
100 Sq. Hectometers	= 1 **Sq. Kilometer**	= 247.114 A.

In measuring land, the square meter is called a *centâre* (ca), the square decameter an *âre* (a) and the square hectometer a *hectare* (Ha). The hectare equals very nearly 2½ acres, and is the metric denomination commonly employed in expressing the quantity of land in farms.

MEASURES OF VOLUME.

372. The unit of volume is the *cubic meter*. It equals 35.317 cubic feet, or 1.308 cubic yards, or .2759 cord.

TABLE.

1000 Cu. Millimeters (cu mm)	= 1 **Cu. Centimeter** (cu cm).
1000 Cu. Centimeters	= 1 Cu. Decimeter (cu dm).
1000 Cu. Decimeters	= 1 **Cu. Meter** (cu m).

In measuring wood the cubic meter is called a *stêre* (s), $\frac{1}{10}$ of a cubic meter a *decistere, etc.*

MEASURES OF CAPACITY.

373. The unit of capacity in both liquid and dry measure is the *liter*. It is a cylinder whose volume is equal to that of a cube whose edge is a decimeter. The liter equals .9081 qt., dry measure, or 1.05673 qt., liquid measure.

TABLE.

10 Milliliters (ml)	= 1 **Centiliter** (cl)	= .6102 cu. in.
10 Centiliters	= 1 Deciliter (dl)	= .845 gi.
10 Deciliters	= 1 **Liter** (l)	= 1.05625 qt.
10 Liters	= 1 Decaliter (Dl)	= 10.5625 qt.
10 Decaliters	= 1 **Hectoliter** (Hl)	= 26.406 gal.
10 Hectoliters	= 1 Kiloliter (Kl)	= 264.06 gal.

In measuring grains, fruits, vegetables, etc., the hectoliter, like the bushel, is used for measuring large quantities, and the decaliter, liter, or deciliter for small quantities.

MEASURES OF WEIGHT.

374. The unit of weight is the *gram*. It is the weight of a cubic centimeter of distilled water in a vacuum, and at its greatest density. The gram equals 15.4324874 Troy grains.

TABLE.

10 **Milligrams** (mg)	= 1 **Centigram** (cg)	=	.15432 + gr.
10 Centigrams	= 1 Decigram (dg)	=	1.54324 + gr.
10 Decigrams	= 1 **Gram** (g)	=	15.43248 + gr.
10 Grams	= 1 Decagram (Dg)	=	.35273 + oz.
10 Decagrams	= 1 Hectogram (Hg)	=	3.52739 + oz.
10 Hectograms	= 1 **Kilogram** (Kg)	=	2.20462 + lb.
10 Kilograms	= 1 Myriagram (Mg)	=	22.04621 + lb.
10 Myriagrams	= 1 Quintal (Q)	=	220.46212 + lb.
10 Quintals	= 1 **Tonneau** (T)	=	2204.62125 + lb.

1. The equivalents in ounces and pounds are Avoirdupois.

2. The tonneau, like our ton, is used to express the weight of very large quantities; the kilogram (generally abbreviated to kilo) like our pound, to express the weight of moderate quantities; and the gram, and centigram, like our pennyweight and grain, to express the weight of minute quantities, such as precious jewels, medicines, etc.; the milligram for exceedingly minute quantities.

3. The silver dollar weighs about $26\frac{7}{16}$ grams; the dime about $2\frac{1}{2}$ grams; *and the five-cent piece*, called the nickel, 5 grams.

NOTATION AND NUMERATION.

375. In the Metric System any quantity, except surfaces and solids, is written according to the decimal system by placing the decimal point between the unit and its divisions, making each denomination occupy that order of the decimal scale which is denoted by its prefix.

Thus, $5^{Hg}\ 7^{Dg}\ 6^{g}\ 4^{dg}\ 8^{mg}$ are written 576.408^{g}, or 576.408 grams.

376. A metric quantity is commonly read like any other decimal; but the fractional part may be read as an integer of the denomination of its right-hand order.

Thus 576.408^{g}, instead of being read $5^{Hg}\ 7^{Dg}\ 6^{g}\ 4^{dg}\ 8^{mg}$, is commonly read 576 and 408 thousandths grams. It may also be read 576 grams and 408 milligrams.

377. In the Metric System surfaces are based upon the scale of 100, and solids upon the scale of 1000. Hence each denomination of these measures must occupy as many decimal places as there are 0's in the scale of its measure.

Thus, $15^{sq\ m}\ 6^{sq\ dm}$ are written $15.06^{sq\ m}$; and $87^{cu\ m}\ 9^{cu\ dm}\ 15^{cu\ cm}\ 135^{cu\ mm}$ are written $87.009015135^{cu\ m}$.

EXERCISES.

378. 1. Copy and read the expression 325.164^{m}.

EXPLANATION.—The expression is read 325 and 164 thousandths meters; or 325 meters and 164 millimeters; or $3^{Hm}\ 2^{Dm}\ 5^{m}\ 1^{dm}\ 6^{cm}\ 4^{mm}$.

Copy and read the following metric quantities:

2. 140.25^{m}.	6. $254.14^{sq\ m}$.	10. $14.3026^{sq\ m}$.
3. 204.07^{l}.	7. $361.435^{cu\ m}$.	11. $17.1481^{sq\ m}$.
4. 375.143^{g}.	8. 420.138^{l}.	12. $35.314254^{cu\ m}$.
5. 612.015^{m}.	9. 157.005^{g}.	13. $23.256143285^{cu\ m}$.

14. Write $7^{Dm}\ 6^{m}\ 13^{cm}$ as a mixed decimal of meters.

15. Write $4^{Hl}\ 5^{Dl}\ 2^{l}\ 3^{dl}$ as a mixed decimal of liters.

16. Write $7^{Kg}\ 8^{Hg}\ 6^{Dg}\ 5^{g}\ 4^{mg}$ as a mixed decimal of grams.

17. Write $15^{sq\ m}\ 3^{sq\ dm}\ 2^{sq\ cm}$ as a mixed decimal of square meters.

18. Write $4^{cu\ m}\ 15^{cu\ dm}\ 43^{cu\ cm}\ 5^{cu\ mm}$ as a mixed decimal of cubic meters.

19. Write $7^{Mg}\ 8^{Kg}\ 6^{Dg}\ 4^{cg}\ 5^{mg}$ as a mixed decimal of grams.

20. Write $75^{sq\ m}\ 13^{sq\ dm}\ 24^{sq\ cm}\ 16^{sq\ mm}$ as a mixed decimal of square decimeters.

REDUCTION.

379. In the Metric System a denominate number can be reduced to higher or lower denominations by changing the place of the decimal point.

1. *In all measures, except square and cubic measures, move the decimal point one place to the right for each lower denomination; one place to the left for each higher denomination.*

2. *In square measure move the decimal point two places for each lower or higher denomination; and in cubic measure move three places.*

EXAMPLES.

380. 1. Reduce 713456^{cg} to hectograms.

$713456^{cg} = 71.3456^{Hg}$

EXPLANATION.—Since the required denomination is hectograms, we move the decimal point to the left to the order of hectograms, and annex the proper abbreviation.

Reduce:

2. 12.4235^{Dm} to dm.
3. $5\ 13628^{Hg}$ to cg.
4. 3215.46^{ml} to l.
5. 412637^{cl} to Kl.
6. 241.673^{m} to Hm.
7. 341267.8^{cg} to Mg.
8. $234.56^{sq\ cm}$ to sq. m.
9. $5.134264^{sq\ Dm}$ to sq. dm.
10. $4134563^{cu\ cm}$ to cu. m.
11. $4.186745^{cu\ m}$ to cu. cm.

TABLES OF EQUIVALENTS.

381. The Metric System was legalized in the United States on July 28, 1866, when Congress enacted that all reductions from the metric to the common system, or the reverse, to be legal, must be made in accordance with the equivalents here given.

LINEAR MEASURE.

1 Inch	= 2.54 Centimeters.	1 Centimeter	= .3937 of an inch.
1 Foot	= .3048 of a Meter.	1 Decimeter	= .328 of a foot.
1 Yard	= .9144 of a Meter.	1 Meter	= 1.0936 yards.
1 Rod	= 5.029 Meters.	1 Decameter	= 1.9884 rods.
1 Mile	= *1.6093* Kilometers.	1 Kilometer	= .62137 of a mile.

Square Measure.

.ch = 6.452 Sq. Centimeters	1 Sq. Centimeter = .155 of a sq. in.
ot = .0929 of a Sq. Meter.	1 Sq. Decimeter = .1076 of a sq. ft.
rd = .8361 of a Sq. Meter.	1 Sq. Meter = 1.196 sq. yards.
od = 25 293 Sq. Meters.	1 Are = 3.954 sq. rods.
= 40.47 Ares.	1 Hectare = 2.471 acres.
ile = 259 Hectares.	1 Sq. Kilometer = .3861 of a sq. mi.

Cubic Measure.

1. = 16.387 Cu. Centimeters.	1 Cu. Centimeter = .061 of a cu. in.
t. = 28.317 Cu. Decimeters.	1 Cu. Decimeter = .0353 of a cu. ft.
l. = .7645 of a Cu. Meter.	1 Cu. Meter = 1.308 cu. yards.
= 3.624 Steres.	1 Stere = .2759 of a cord.

Measures of Capacity.

. qt. = .9463 of a Liter.	1 Liter = 1.0567 liquid qts.
t. = 1.101 Liters.	1 Liter = .908 of a dry qt.
gal. = .3785 of a Decaliter.	1 Decaliter = 2.6417 liquid gals.
= .881 of a Decaliter.	1 Decaliter = 1 135 pecks.
l = .3524 of a Hectoliter.	1 Hectoliter = 2.8375 bushels.

Measures of Weight.

1 Grain, Troy	= .0648 of a Gram.
1 Ounce, Troy	= 31.104 Grams.
1 Ounce, Avoir.	= 28.35 Grams.
1 Pound, Troy	= .3732 of a Kilogram.
1 Pound, Avoir.	= .4536 of a Kilogram.
1 Ton (2000 lb.)	= .9072 of a Tonneau.
1 Gram	= 15.432 grains, Troy.
1 Gram	= .03215 of an ounce, Troy.
1 Gram	= .03527 of an ounce, Avoir.
1 Kilogram	= 2.679 pounds, Troy.
1 Kilogram	= 2.2046 pounds, Avoir.
1 Tonneau	= 1.1023 tons (of 2000 lb.)

To reduce from the metric system to the common n, or the reverse.

EXAMPLES.

uce:

235.14^{g} to lb., Avoir.
173.5^{m} to yards.
1500^{Km} to miles.
48.75$^{sq\ m}$ to sq. yd.
70.25^{Ha} to acres.
125$^{cu\ m}$ to cu. yd.

7. 25 C. of wood to steres.
8. 40 yd. of cloth to meters.
9. 100 bu. of wheat to hectoliters.
10. 150 bu. of grain to kilos.
11. 35 gal. of vinegar to liters.
12. 60 qt. of berries to liters.

PRACTICAL PROBLEMS.

383. 1. How much must be paid for 10.5 meters of cloth at $3.75 a meter?

2. What will be the cost of 15^{Dl} of potatoes at the rate of 20¢ per peck?

3. What will 16.25^{s} of wood cost at $1.12\frac{1}{2}$ a stere?

4. How much must I pay for 5^{Dl} of oil at the rate of 4¢ per quart?

5. What will 18.75^{m} of cloth cost, if 3.125^{m} cost $10.60?

6. What will be the cost of 18^{Kg} 25^{g} of sugar at 12¢ a kilo?

7. Mr. B. bought 500^{a} of land at $2.50 an are, and sold it for $290 a hectare. What was the gain?

8. Which is the cheaper and how much, to buy cloth at $4.25 per meter, or at $4 per yd?

9. What must be paid in U. S. money for 125^{m} of cloth, invoiced at 12.4 francs per meter?

10. A Boston merchant bought 300^{m} of silk in Paris, at 7.5 francs a meter. After paying 70¢ a yard duty and freight, he sold the silk for $3.25 a yard. What was his profit?

11. What part of a square meter is a square yard? Express the result in a decimal of *four* places.

12. A train travels 495^{Km} in 9 hours at a uniform rate of speed. How many meters in a minute does it travel?

13. How much railway iron weighing 48^{Kg} to the meter will be required for a double-track railway 150^{Km} in length?

14. A copper wire weighs 75 grams to the meter in length. Find the weight in kilograms of 26 kilometers of such wire.

15. Give the English unit corresponding most nearly to the (*a*) centimeter, (*b*) kilogram, (*c*) meter, (*d*) liter, (*e*) kilometer.

INVOLUTION AND EVOLUTION.

INVOLUTION.

384. Involution is the process of raising a number to any desired power.

385. A **Power** of a number is the product obtained by using the number one or more times as a factor—i. e., by multiplying it one or more times by itself.

The power may be indicated by a small figure called an *Exponent*, placed a little above and at the right of the number. Thus $4^1 = 4 \times 1 = 4$; $4^2 = 4 \times 4 = 16$; $4^3 = 4 \times 4 \times 4 = 64$; $4^4 = 4 \times 4 \times 4 \times 4 = 256$.

386. The **Square** of a number is its second power.

387. The **Cube** of a number is its third power.

EXAMPLES.

388. 1. Find the fifth power of 6.

Solution: $6^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7776$.

2. Find the third power of 8, 11, 14, 24.

3. Find the square of 12, 15, 18, 3.5, .42.

4. Find the cube of 9, 13, 17, 3.1, .55.

5. Find the value $(\frac{3}{4})^3$.

Solution: $(\frac{3}{4})^3 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$.

6. Find the value of $(\frac{2}{3})^3$, $(\frac{4}{5})^2$, $(\frac{6}{7})^2$, $(\frac{11}{21})^2$

7. Find the value of $4^2 \times 4^3$.

Solution: $4^2 \times 4^3 = 4 \times 4 \times 4$ times $4 \times 4 = 4^5$.

8. Find the value of $3^2 \times 3^3 \times 3^4$.

9. What power is $(5^2)^3$.

Solution: $(5^2)^3 = 5^3 \times 5^3 = 5^6$.

10. What power is $(7^3)^4$? $(8^3)^2$? $(12^4)^2$? $(15^2)^4$?

389. Find the square of 65 in terms of its tens and units.

$65 =$	$60 + 5$
$65 =$	$60 + 5$
$325 =$	$60 \times 5 + 5^2$
$390 =$	$60^2 + 60 \times 5$
$4225 =$	$60^2 + 2\,(60 \times 5) + 5^2$

390. PRINCIPLE.—*The square of any number consisting of two figures is equal to the square of the tens, plus twice the product of the tens into the units, plus the square of the units.*

Thus, $23 = 20 + 3$, and $23^2 = 20^2 + 2\,(20 \times 3) + 3^2$.

Find in terms of their tens and units the square of the following numbers:

2. 36.	4. 58.	6. 71.	8. 95.
3. 42.	5. 67.	7. 83.	9. 87.

391. 1. Find the cube of 65 in terms of its tens and units.

$65^2 = 4225 =$ $60^2 + 2\,(60 \times 5) + 5^2$

$65 = 65 =$ $60 + 5$

$21125 =$ $(60^2 \times 5) + 2\,(60 \times 5^2) + 5^2$

$25350 =$ $60^3 + 2\,(60^2 \times 5) + (60 \times 5^2)$

$65^3 = 274625 =$ $60^3 + 3\,(60^2 \times 5) + 3\,(60 \times 5^2) + 5^3$

392. PRINCIPLE.—*The cube of any number consisting of two figures is equal to the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units.*

Thus, $29 = 20 + 9$, and $29^3 = 20^3 + 3\,(20^2 \times 9) + 3\,(20 \times 9^2) + 9^3 = 24389$.

Find in terms of their tens and units the cube of the following numbers.

2. 15.	4. 25.	6. 45.	8. 78.
3. 18.	5. 34.	7. 58.	9. 85.

393. If we let t represent the tens of a number and u the units, then the square of a number consisting of tens and units will be the square of $(t + u)$ or $t^2 + 2\,tu + u^2$ (Art. 390), and the cube of a number consisting of tens and units will be the cube of $(t + u)$ or $t^3 + 3\,t^2u + 3\,tu^2 + u^3$ (Art. 392).

EVOLUTION.

394. Evolution is the process of finding a desired root of a number.

Evolution is the reverse of Involution.

395. A **Root** of a number is one of the equal factors of the *number.*

The root of a number may be indicated by the symbol $\surd$, called the *dical Sign*, and a small figure, called the *Index*, placed in the angle of the nbol. Thus, $\surd 16$, or $\sqrt[2]{16}$ denotes the square root of 16; $\sqrt[3]{27}$ denotes) cube root of 27. Roots may also be indicated by the denominator of a ctional exponent. Thus $16^{\frac{1}{2}}$ denotes $\surd 16$; $27^{\frac{1}{3}}$ denotes $\sqrt[3]{27}$.

396. The **Square Root** of a number is one of the two equal tors of the number.

Thus, 4 is the square root of 16, since 4×4 are 16.

397. The **Cube Root** of a number is one of the three equal tors of the number.

Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$ are 27.

SQUARE ROOT.

398. The process of finding one of the two equal factors of a mber is called *extracting the square root.*

399. Numbers and their squares compared.

$1^2 = 1$	$10^2 = 100$	$100^2 = 10000$
$9^2 = 81$	$99^2 = 9801$	$999^2 = 998001$

he square of 1, 10, or 100, the smallest number of *one*, *two*, or *ee* figures respectively, contains *twice* the figures of the number *one.*

The square of 9, 99, or 999, the largest number of *one*, *two*, or *ee* figures respectively, contains *twice* the figures of the number. The same may be shown for the square of a number consisting any number of figures. Hence,

400. PRINCIPLE.—*The square of a number is expressed by twice many figures as the number itself, or by twice as many less one.*

WRITTEN EXERCISES.

401. 1. What is the square root of 729?

Full Form.		In Practice.
$+ u)^2 = t^2 + 2tu + u^2 =$	7'29 (20	7'29 | 27
$t^2 =$ $20^2 =$	4 00 7	4
$2tu + u^2 = (2t + u)u =$	3 29 27	47)3 29
rial divisor, $2t = 40$		3 29
$(2t + u)u = (40 + 7)7 =$	3 29	

EXPLANATION.—Pointing the number off into periods of two figures each (Art. 400), we find its root will consist of two places—tens and units. Hence 729 consists of $t^2 + 2tu + u^2$ (Art. 393).

The greatest number of tens whose square is contained in 729 is 2 tens. Squaring the tens and subtracting, we have 329, which equals $2tu + u^2$.

Since $2tu$ is much more than u^2, 329 must consist principally of $2tu$. Hence, if we divide 329 by $2t$, or 40, we can at once, at least approximately, ascertain the units. Dividing, allowing some for u^2, we find the units to be 7.

We now find $2tu + u^2$, or, its equal, $(2t + u)u$, equals 329; and subtracting, nothing remains.

Therefore the square root of 729 is 2 tens and 7 units, or 27.

In practice, omit the ciphers and condense the process as indicated.

402. Any number may be regarded as composed of tens and units, and therefore its square root may be found in like manner.

Thus 625 = 62 tens + 5 units; 4096 = 409 tens + 6 units.

2. Extract the square root of 256036.

```
25'60'36 | 506
25
1006) 60 36
      60 36
```

EXPLANATION.—The first trial divisor is 10, and the remainder is 60. Since 10 is not contained in 6, we place a 0 in the root and annex a 0 to the trial divisor, and then bring down the next period, 36.

100 is contained in 603, 6 times. Annexing the 6 to the trial divisor, we have the true divisor which multiplied by the unit equals 6036.

Hence, the square root of 256036 is 506.

RULE.—*Begin at the units' place and separate the given number into periods of two figures each.*

Find the greatest number whose square is contained in the left-hand period, and write it as the first figure of the root. Square this root and subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

Double the root found for a **trial divisor**; *divide the dividend, omitting the right-hand figure, by this divisor. The quotient or the quotient diminished will be the second figure of the root.*

Annex to the trial divisor for a **complete divisor** *the second figure of the root; multiply this divisor by the second figure of the root, subtract the product from the dividend, and to the remainder annex the next period for the next dividend.*

Double the root now found for a second **trial divisor**, *find the third figure of the root as before, and thus proceed until all the periods have been used.*

1. When a number is an imperfect square, annex periods of decimal ciphers and continue the process as far as desired.

2. Decimals are pointed off into periods of two figures each, beginning at the decimal point.

3. The square root of a common fraction may be found by extracting the square root of each term when both are perfect squares. When imperfect squares, reduce the fraction to a decimal and then extract its root.

Extract the square root of the following :

3. 1296.	7. 14641.	11. 1532.7225.	15. $\frac{256}{441}$
4. 5776.	8. 41616.	12. 324.720400.	16. $\frac{625}{1024}$.
5. 7056.	9. 765625.	13. .0000015625.	17. $\frac{1369}{2401}$.
6. 9604.	10. 998001.	14. 508169.3796.	18. $\frac{46656}{65536}$.

19. What is the square root of 12, carried to four decimal places?

20. What is the square root of $37\frac{1}{2}$ carried to four decimal places?

CUBE ROOT.

403. The process of finding one of the three equal factors of a number is called *extracting the cube root.*

404. **Numbers and their cubes compared.**

$1^3 = 1$	$10^3 = 1000$	$100^3 = 1000000$
$4^3 = 64$	$40^3 = 64000$	$400^3 = 64000000$
$9^3 = 729$	$99^3 = 970299$	$999^3 = 997002999$

The cube of 1, 10 or 100, the smallest numbers of *one*, *two*, or *three* figures respectively, contains *three* times the figures of the number *less two.*

The cube of 4, 40, or 400, contains *three* times the figures of the number *less one.*

The cube of 9, 99, or 999, the largest number of *one*, *two*, or *three* figures respectively, contains *three* times the figures of the number. Hence the cube of a number of *one*, *two*, or *three* figures contains *three times as many figures as the number* or *three times as many less one or two.*

The same may be shown for the cube of a number consisting of any number of figures. Hence,

405. PRINCIPLE.—*The cube of a number is expressed by three times as many figures as the number itself, or by three times as many less one or two.*

WRITTEN EXERCISES.

406. 1. What is the cube root of 42875?

Full Form.

$t^3+3t^2u+3tu^2+u^3$ = 42'875(30

$t^3=30^3$ = 27 000 | 5

$3t^2u+3tu^2+u^3$ = 15 875 | 35

$3t^2 = 3 \times 30^2 = 2700$

$3tu = 3\times30\times5 = 450$

$u^2=5^2$ = 25

$(3t^2+3tu+u^2)u = 3175\times5$ = 15 875

In Practice.

42'875 | 35

27

15 875

$3^2\times3$ = 27

$3\times3\times5$ = 45

5^2 = 25

3175 | 15 875

EXPLANATION.—Pointing the number off into periods of three figures each (Art. 405), we find its root will consist of two places—tens and units. Hence 42875 consists of $t^3 + 3t^2u + 3tu^2 + u^3$.

The greatest number of tens whose cube is contained in 42875 is 3 tens. Cubing the tens, and subtracting, we have 15875, which equals $3t^2u + 3tu^2 + u^3$.

Since $3t^2u$ is much greater than $3tu^2 + u^3$, 15875 must consist principally of $3t^2u$. Hence, if we divide 15875 by $3t^2$, or 2,700 we can at once, at least approximately, ascertain the units. Dividing, we find the units to be 5.

We now find $3tu$ equals $3 \times 30 \times 5 = 450$, and u^2 equals $5^2 = 25$. Adding these to the trial divisor, 2700, and multiplying by the units, we have $(3t^2 + 3tu + u^2)u$, which equals $3175 \times 5 = 15875$; subtracting, nothing remains.

Therefore the cube root of 42875 is 3 tens and 5 units, or 35.

1. In practice, omit the ciphers and condense the process as indicated.

2. When the root consists of more than two orders, consider the entire root already found as *tens* and the next order of the root as *units* and proceed as before.

2. What is the cube root of 76765625 ?

76'765'625 | 425

4^3 = 64

Trial divisor, 3×4^2 = 48 | 12765

$3 \times 4 \times 2$ = 24

2^2 = 4

Complete divisor, 5044 | 10088

2677625

Trial divisor, 3×42^2 = 5292

$3 \times 42 \times 5$ = 630

5^2 = 25

Complete divisor, 535525 | 2677625

RULE.—*Begin at the units' place and separate the given number into periods of three figures each.*

Find the greatest number whose cube is contained in the left-hand period, and write it as the first figure of the root. Cube this root and subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

Take three times the square of the root already found, for a **trial divisor**; *divide the dividend, omitting the last two right-hand figures, by this divisor. The quotient or the quotient diminished will the second figure of the root.*

To the trial divisor add three times the product of the first and second figures of the root, **written one figure to the right,** *and also the square of the second figure of the root,* **written one figure to the right.** *Their sum will be the* **complete divisor.**

Multiply the complete divisor by the second figure of the root and subtract the product from the dividend, and to the remainder annex the next period for the next dividend.

Take three times the square of the root already found for a second **trial divisor,** *find the third figure of the root as before, and thus proceed until all the periods have been used.*

1. When a cipher occurs in the root, annex two ciphers to the trial divisor and annex the next period to the dividend.

2. When a number is an imperfect cube, annex periods of decimal ciphers and continue the process as far as desired.

3. Decimals are pointed off into periods of three figures each, beginning at the decimal point.

4. The cube root of a common fraction may be found by extracting the cube root of each term when both are perfect cubes. When imperfect cubes, reduce the fraction to a decimal and then extract its root.

Extract the cube root of the following:

3. 15625.	8. 438976.	13. 629.422793.
4. 32768.	9. 857375.	14. 890277.128.
5. 42875.	10. 48228544.	15. .074088.
6. 79507.	11. 71991296.	16. .000804357.
7. 175616.	12. 130323843.	17. .001953125.

18. What is the cube root of 6 to four decimal places?

19. What is the cube root of $\frac{512}{1728}$? $\frac{5}{8}$? $\frac{19683}{54872}$?

20. What is the cube root of $40\frac{5}{8}$ to four decimal places?

SHORT METHOD OF CUBE ROOT.

407. The following method of cube root is the shortest known. The shortness consists in obtaining the successive trial divisors by using the previous work.

21. What is the cube root of 12895213625 ?

		12'895'213'625	2345
		8	
T. d.,	12	4895	
	18		
	9		
C. d.,	1389	4167	
	9	728213	
T. d.,	1587		
	276		
	16		
C. d.,	161476	645904	
	16	82309625	
T. d.,	164268		
	3510		
	25		
C. d.,	16461925	82309625	

EXPLANATION.—The process is the same as in the previous method up to finding the second trial divisor.

To find the second trial divisor, 1587, we add the square of the last figure of the root (9), the complete divisor (1389), the second correction (9), and the first correction (18), as indicated by the bracket. In the same manner each succeeding trial divisor is obtained.

RULE.—*Find the first trial divisor by the usual method.*

Find each succeeding trial divisor by adding the square of the last figure of the root to the last complete divisor and the two corrections required to obtain it, and proceed as before.

This method is merely a modified form of the previous method, and is readily explained by the algebraic formula. It is the **labor saving** method of cube root, and should be thoroughly mastered by the student.

Extract the cube root of the following:

22. 41063625.	25. 633839779.	28. 196426.902797.
23. 347428927.	26. 1879080904.	29. 436.036824287.
24. 410172407.	27. 95256152263.	30. .000890277128.

PRACTICAL MEASUREMENTS.

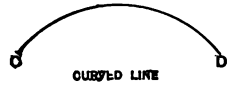
LINES.

408. A **Line** is that which has length only.

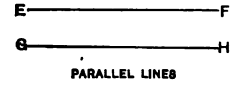
409. A **Straight Line** is a line that has the same direction at every point.

410. A **Curved Line** is a line that changes its direction at every point.

The word line is used for straight line, and curve for curved line.



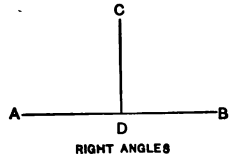
411. **Parallel Lines** are lines that have the same direction at every point and remain equidistant.



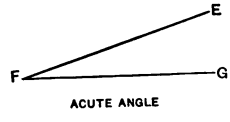
ANGLES.

412. An **Angle** is the difference in direction of two lines that meet.

The point in which the lines meet is called the *Vertex*, and the lines are called the *sides* of the angle.

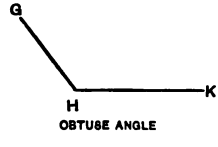


413. **A Right Angle** is an angle formed by one line perpendicular to another.



414. An **Acute Angle** is an angle smaller than a right angle.

415. An **Obtuse Angle** is an angle larger than a right angle.



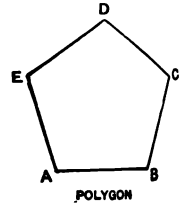
SURFACES.

416. A **Surface** is that which has length and breadth only.

417. **A Plane Surface**, or a **Plane**, is a surface in which if any two points are taken, the straight line joining these points will lie wholly in the surface.

418. A **Plane Figure** is a plane bounded by lines either straight or curved.

419. A **Polygon** is a plane figure bounded by straight lines.



POLYGON

The *Sides* of a polygon are the lines which bound it; and the *Perimeter* is the sum of the sides, or the distance around it.

420. A **Polygon** of three sides is called a *triangle;* of four sides, a *quadrilateral;* of five sides, a *pentagon;* of six sides, a *hexagon;* of seven sides, a *heptagon*, of eight sides, an *octagon*, etc.

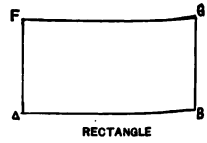
421. *a.* An **Equilateral Polygon** is one whose sides are equal.
b. An **Equiangular Polygon** is one whose angles are equal.

422. A **Diagonal** of a polygon is a line joining the vertices of any two angles not adjacent.

423. The **Area** of a polygon is the number of square units in its surface.

THE RECTANGLE.

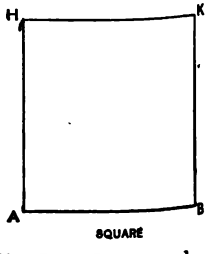
424. A **Rectangle** is a polygon having four sides and four right angles.



RECTANGLE

When the word side is used straight side is meant.

A *Square* is a rectangle whose sides are equal.



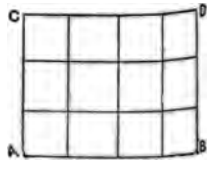
SQUARE

The *Dimensions* of a rectangle are its length and breadth.

The *Base* of a rectangle is the side upon which it seems to stand. The side opposite is called the *Upper Base.*

The *Altitude* of a rectangle is the perpendicular distance between its bases.

425. To find the area of a Rectangle.



In the rectangle *ABCD*, the area is the number of square units it contains, which is the number in each row multiplied by the number of rows; but this is equal to the number of linear units in the *length* multiplied by the number in the breadth. ***Hence,***

RULE.—*Multiply the length by the breadth.*

1. The dimensions must be expressed in uni s of the same denomination.

2. To find either side of a rectangle, divide the area by the other side.

EXAMPLES.

426. 1. How many square feet in a floor 25 feet long and 20 feet wide ?

2. How many square rods in a field 40 rods long and 18 rods wide ?

3. How many acres are there in a square farm each of whose sides is 120 rods ?

4. What is the width of a field 60 rods long which contains 15 acres ?

5. A field is 16 chains long and 30 rods wide. How many acres does it contain ?

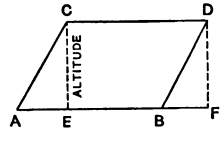
6. A certain garden is 150 feet long and 100 feet wide. Find its area in square meters.

7. Mr. G. has a rectangular field 40 decameters long and 35 dekameters wide. How many hectares does the field contain ?

427. To find the area of a Parallelogram.

428. A **Parallelogram** is a quadrilateral having its opposite sides parallel.

If the parallelogram *ABCD* be divided by the line *CE*, and the triangle *AEC* be placed at the right, we shall have the rectangle *EFCD*, whose base, altitude and area is the same as that of the parallelogram. Hence,



RULE.—*Multiply the base by the altitude.*

EXAMPLES.

429. 1. What is the area of a parallelogram whose base is 24 feet and altitude 15 feet ?

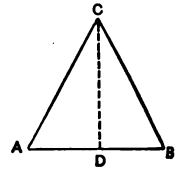
2. Find the area of a parallelogram whose base is 15 yards and altitude 18 feet.

3. A field in the form of a parallelogram contains 3 acres. Its length is 24 rods. *What is its* width, or altitude ?

THE TRIANGLE.

430. A **Triangle** is a polygon having three sides and three angles.

The *Altitude* of a triangle is the perpendicular distance from the vertex to the base, or the base produced or lengthened out, as *CD*.

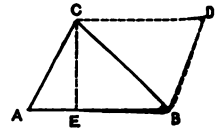


A *Scalene* triangle is one whose sides are all unequal; an *Isosceles* triangle is one two of whose sides are equal; an *Equilateral* triangle is one whose three sides are equal.

A *Right* triangle is one which has a right angle; an *Obtuse* triangle is one which has an obtuse angle; an *Acute* triangle is one whose angles are all acute; an *Equiangular* triangle is one whose three angles are equal.

431. To find the area of a Triangle when the Base and Altitude are given.

The triangle *ABC* is seen to be one-half of the parallelogram *ABCD*; but each has the same base *AB* and the same altitude *CE*. Hence,



RULE.—*Multiply the base by one-half of the altitude.*

EXAMPLES.

432. 1. What is the area of a triangle whose base is 40 feet and altitude 24 feet?

2. What is the area of a triangle whose base is 42 feet and altitude 19 feet?

3. How many acres in a triangular field whose base is 60 rods and altitude 48 rods?

4. How many square feet of boards will be required to cover the gables of a house that is 32 feet wide, the ridge of the roof being 11 feet above the square?

433. To find the area of a Triangle when the Three Sides are given.

The principles of practical measurements are derived from Geometry. The rules founded upon these principles cannot always be clearly illustrated or explained by arithmetic. In such cases only the rule with examples will be given.

RULE.—*From half the sum of the sides subtract each side separately. Multiply the half sum and these remainders together, and extract the square root of the product.*

The area of an equilateral triangle is equal to the square of one side multiplied by .433013.

EXAMPLES.

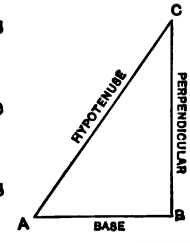
434. 1. Find the area of a triangle whose sides are, respectively, 12, 16, and 20 feet.

2. How many acres in a triangular field whose sides are, respectively, 50, 60, and 70 rods?

3. Find the area of a triangular field whose sides are each 60 rods.

THE RIGHT TRIANGLE.

435. A Right Triangle is a triangle which has one right angle.

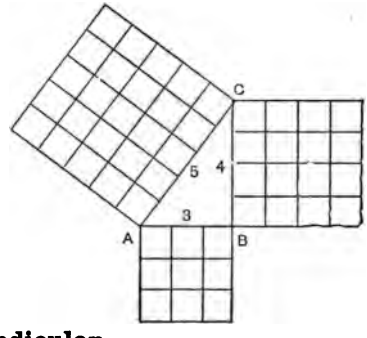


436. The **Hypotenuse** is the side opposite the right angle.

437. The **Perpendicular** is the side which forms a right angle with the base.

438. To find the Hypotenuse.

It is seen that the square described on the hypotenuse is equal to the sum of the squares described on the other two sides. Hence,



RULE.—*Add the square of the base to the square of the perpendicular, and extract the square root of their sum.*

439. To find the Base or Perpendicular.

RULE.—*From the square of the hypotenuse take the square of the given side, and extract the square root of the remainder.*

EXAMPLES.

1. The base of a right triangle is 15 inches, and the perpendicular is 8 inches. What is the hypotenuse?

Solution: Hypotenuse $= \sqrt{15^2 + 8^2} = \sqrt{225 + 64} = \sqrt{289} = 17$. Hence, the hypotenuse *is 17 in.*

2. The hypotenuse of a right triangle is 30 ft., and the perpendicular is 24 feet. Find the base.

Solution: $\sqrt{30^2 - 24^2} = \sqrt{900 - 576} = \sqrt{324} = 18$. Hence, the base is 18 feet.

3. The hypotenuse of a right triangle is 80 ft., and the base is 64 feet. What is the perpendicular?

4. A rectangular field is 70 rd. long and 45 rd. wide. What is the distance between the opposite corners?

5. How far apart are the opposite corners of a square farm containing 160 acres?

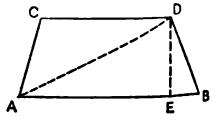
6. How far is it from a lower corner to the opposite upper corner of a room 18 ft. long, 16 ft. wide, and 10 ft. high?

7. I wish to build a house 36 ft. wide with the ridge of the roof 14 ft. above the body of the house. What must be the length of the rafters allowing 8 in. for extension?

440. To find the area of a Trapezoid.

441. A Trapezoid is a quadrilateral which has only two of its sides parallel.

The *Altitude* of a trapezoid is the perpendicular distance between its parallel sides.



It is seen that any trapezoid may be divided into two triangles by a diagonal; as DA. The area of triangle ABD is the length of the base AB multiplied by half the altitude DE (Art. 431), and the area of ACD is the length of the upper base CD multiplied by half the altitude DE. Hence,

RULE.—*Multiply the sum of the parallel sides by half the altitude.*

EXAMPLES.

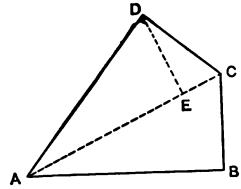
442. 1. What is the area of a trapezoid whose bases are 18 and 16 in., respectively, and altitude 14 in.?

2. A field in the form of a trapezoid, whose parallel sides are 30 rd. and 36 rd., respectively, and which has an altitude of 25 rd., contains how many acres?

3. A walk in the form of a trapezoid is 30 ft. long, 6 ft. wide at one end and 4 ft. at the other. How many square feet in the walk?

443. To find the area of a Trapezium.

444. A **Trapezium** is a quadrilateral which has no two sides parallel.



It will be seen that any trapezium may be divided by a diagonal, as AC, into two triangles whose areas may be found by Arts. 431 and 433. Hence,

RULE.—*Divide the trapezium into two triangles by a diagonal. Find the area of each triangle and take the sum.*

The diagonal may also be made the base of each triangle and perpendiculars may be drawn; as, DE. If one of the triangles is a right triangle, then either of its sides about the right angle may be regarded as the base and the other as the altitude; as, AB and BC.

EXAMPLES.

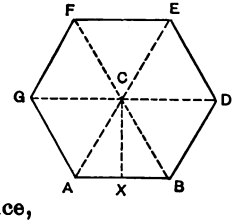
445. 1. Find the area of a trapezium whose diagonal is 75 ft., and the altitudes of the triangles, the diagonal being the base, 20 and 30 ft., respectively.

2. What is the area of a trapezium whose diagonal is 100 rd., and the perpendiculars, 36 and 44 rd., respectively?

3. How many acres in a farm in the form of a trapezium the length of whose sides are respectively 40, 50, 60, and 70 chains; and the length of the diagonal 80 chains?

446. To find the area of a Regular Polygon.

447. A **Regular Polygon** is a polygon which is equilateral and equiangular.



It will be seen that a regular polygon may be divided into a number of triangles equal to the number of sides of the polygon, having equal bases and altitudes. The area of each triangle is equal to the base multiplied by half the altitude, and the area of all the triangles, or the polygon, is equal to the sum of all the bases multiplied by half the altitude. Hence,

RULE.—*Multiply the perimeter by half the perpendicular falling from the center of the polygon upon one of its sides.*

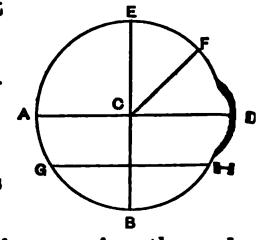
EXAMPLES.

448. 1. What is the area of a pentagon each of whose sides is 12 ft. and the perpendicular distance from the center to one of its sides is 9 ft.?

2. Find the area of a regular octagon whose side is 35 rd., if the perpendicular distance from its center to a side is 42 rd.

THE CIRCLE.

449. A **Circle** is a plane figure bounded by a curved line, every part of which is equally distant from a point within called the center.



The *Circumference* of a circle is the bounding line; as, *ABDE*.

An *Arc* is any part of the circumference; as, *EF*.
A *Chord* is any straight line having its extremities in the circumference; as, *AD*, or *GH*.

The *Diameter* of a circle is any straight line passing through the center and terminating in the circumference; as *AD*, or *EB*.

The *Radius* of a circle is any straight line drawn from the center to the circumference, as *CB*, *CD*, *CF*, etc.

The radius is half of the diameter of the circle.

450. To find the Circumference or Diameter of a Circle.

If we take a circle 4 inches in diameter and measure accurately the circumference, we shall find it to be about 12.5664 inches. Dividing the circumference, 12.5664 in. by the diameter, 4 in., we have for a quotient 3.1416. Hence,

RULE.—1. *To find the circumference, multiply the diameter by 3.1416.*

2. *To find the diameter, divide the circumference by 3.1416.*

This number, 3.1416, is often represented by the Greek letter π, called *pi*.

If we take a circle whose circumference is 1, its diameter will be $1 \div 3.1416$, or .3183. Hence,

3. *To find the diameter, multiply the circumference by .3183.*

If we let C represent the circumference, D the diameter, and R the radius, the rules may be briefly expressed as follows:

$$C = \pi D; \qquad D = \frac{C}{\pi}$$

$$C = 2 \pi R; \qquad D = C \times .3183.$$

EXAMPLES.

451. 1. What is the circumference of a circle whose diameter is 12 ft.?

2. What is the circumference of a wheel which has a diameter of 38 inches?

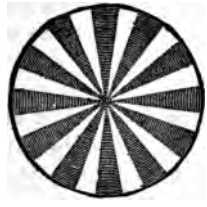
3. The circumference of a circle is 100 ft. What is its diameter?

4. What is the diameter of a circle whose circumference is 314.16 rd.?

5. Find the radius of a circle whose circumference is 94.248 ft.

6. The front wheel of a buggy is 41 inches in diameter. How many revolutions does it make in going a mile?

452. To find the area of a Circle.



A circle may be regarded as composed of a very large number of triangles, the sum of whose bases forms the circumference of the circle, and whose altitude is the radius of the circle. Hence,

RULE.—*Multiply the circumference by half the radius, or one-fourth of the diameter.*

The area of a circle is also equal to the *square of the radius multiplied by 3.1416*; or area $= \pi R^2$; or, $\frac{1}{4} \pi D^2 = .7854 \times D^2$.

EXAMPLES.

453. 1. What is the area of a circle whose diameter is 20 in. and circumference 62.832 in.?

2. What is the area of a circle whose diameter is 25 ft.?

3. Find the area of a circle whose radius is 18 ft.

4. Find the area of a circle whose radius is a rod.

5. The circumference of a circular field is 120 rd. How many acres in the field?

6. A has a field that is 40 rd. square, and B has a circular field 40 rd. in diameter. What is the distance around each field, and how many acres does each contain?

454. To find the Diameter or Circumference of a Circle from the area.

$C = \pi \times D$, or πD; and $C \times \frac{1}{4}D =$ Area.

$\therefore \pi D \times \frac{D}{4}$, or $\frac{\pi D^2}{4} =$ Area.

But $\frac{\pi D^2}{4} = \frac{3.1416 \times D^2}{4} = .7854 \times D^2$.

$\therefore .7854 \times D^2 =$ Area; or, $D^2 = \frac{\text{Area}}{.7854}$,

or, $D = \sqrt{\frac{\text{Area}}{.7854}}$. And $C = \pi D$, or $3.1416 \sqrt{\frac{\text{Area}}{.7854}}$

$= \sqrt{\frac{\text{Area}}{.07958}}$. Hence,

RULE.—1. *Divide the area by .7854, and the square root of the quotient will be the diameter.*

2. *Divide the area by .07958, and the square root of the quotient will be the circumference.*

To derive some of the rules in practical measurements, as has just been seen, involves a considerable knowledge of mathematics. Hence a number of practical rules will be given without any attempt to derive them.

EXAMPLES.

455. 1. The area of a circle is 350 sq. ft. What is its diameter?

2. Find the circumference of a circle whose area is 400 sq. rd.

3. What is the diameter of a circle which contains one acre?

456. To find the Side of a Square equal in area to a given Circle.

RULE.—*Multiply the diameter of the circle by .8862, or the circumference by .2821.*

The decimal .8862, or, approximately, the square root of .7854 is the side of a square equal in area to a circle whose diameter is 1; and .2821, the square root of .07958+, is the side of a square equal in area to a circle whose circumference is 1.

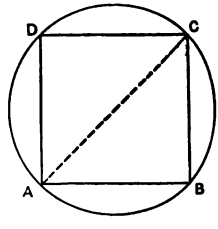
EXAMPLES.

1. The diameter of a circle is 20 ft. What is the side of a square of equal area?

2. A circular flower bed is 60 ft. in circumference. What must be the side of a square plot of the same area?

457. To find the Side of a Square inscribed in a given Circle.

458. A square is inscribed in a circle when the vertices of its angles are in the circumference.



It is seen that any inscribed square may be divided into two equal right triangles, having the diameter of the circle as a common hypotenuse.

In the triangle ABC, $\overline{AB}^2 + \overline{BC}^2 = \overline{AC}^2$. But $AB = BC$. Therefore, $2\,\overline{AB}^2 = \overline{AC}^2$; or $\overline{AB}^2 = \frac{\overline{AC}^2}{2}$, or $AB = \sqrt{\frac{\overline{AC}^2}{2}}$. Hence,

RULE.—*Divide the square of the diameter of the circle by 2, and extract the square root.*

The side of a regular hexagon inscribed in a circle is equal to the radius of the circle.

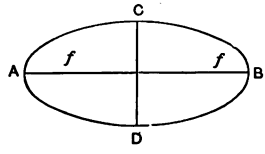
EXAMPLES.

459. 1. Find the side of a square that can be cut from a circle 10 in. in diameter.

2. The diameter of a round piece of timber is 18 in. What is the side of a square piece that can be cut out of it?

THE ELLIPSE.

460. An **Ellipse** is a plane figure bounded by a curved line, the sum of the distances from every point of which to two fixed points is equal to the line drawn through these points and terminated by the curve. The two fixed points are called the *foci*.



The *transverse* axis is the line passing through the foci and terminating in the curve; as AB. The *conjugate* axis is a line perpendicular to the transverse axis passing through the center and terminated by the curve.

461. To find the Area of an Ellipse.

RULE—*Multiply half of the two axes together, and that product by 3.1416.*

EXAMPLES.

1. Find the area of an ellipse whose axes are 12 ft. and 9 ft.

2. Find the area of an elliptical plot of ground 20 ft. long and 15 ft. wide.

462. To find the Cost of Painting, Plastering, and Kalsomining.

Painting, plastering and kalsomining are commonly estimated by the square yard.

Allowance is sometimes made for openings in the walls, as doors, windows, etc. Half the areas of the doors and windows is often deducted, but there is no established custom.

Laths are 1½ in. wide and 4 ft. long. They are placed about ½ in. apart for plastering, and 16 laths are reckoned to a sq. yd.

EXAMPLES.

1. What will it cost to plaster the walls and ceiling of a room 20 ft. long, 15 ft. wide, and 9 ft. high, at 25¢ a sq. yd., allowing 70 sq. ft. for doors and windows?

Solution:

Area of side walls = $2 \times 20 \times 9 = 360$ sq. ft.
" " end " = $2 \times 15 \times 9 = 270$
" " ceiling = $20 \times 15 = 300$
Total area = 930 sq. ft.
Area to be deducted = 70
Area to be plastered = 860 sq. ft.
$860 \div 9$ = $95\frac{5}{9}$ sq. yd.
Cost = $95\frac{5}{9} \times \$.25 = \$23.89$.

2. What will it cost to plaster the walls and ceiling of a room 18 ft. long, 14 ft. wide, and 9 ft. high, having 2 doors, $6\frac{1}{2}$ ft. by 3 ft., and 4 windows, 6 ft. by 3 ft., at 20¢ a sq. yd., deducting half the area of the doors and windows?

3. How much will it cost to kalsomine the walls and ceiling of a room 15 ft. long, 14 ft. wide, and $8\frac{1}{2}$ ft. high, with 2 coats, at 6¢ a sq. yd. each?

4. What will it cost to paint a barn 40 ft. long, 30 ft. wide, 18 ft. to the eaves, the gables being 10 ft. high, with 2 coats, at 7¢ a sq. yd. each?

463. To find the Cost of Roofing, Paving, etc.

Paving is usually computed by the square foot or square yard.

Roofing is generally estimated by the *square*, which consists of 100 square feet.

Shingles are estimated by the *thousand*.

Shingles that average 4 in. in width and are laid 5 in. to the weather, 720 shingles to the square; $5\frac{1}{2}$ in. to the weather, 655; 6 in. to the weather, 600; 7 in. to the weather, 515; and 8 in. to the weather will require 450 shingles to the square. Making due allowance for waste and defects, it is customary, however, to count 1000 shingles to the square.

About 5 lbs. of nails are required for each 1000 of shingles.

EXAMPLES.

1. What will be the cost of slating a roof 36 ft. long, each side being 14 ft. from eaves to ridge, at $8.25 per square?

Solution: The area $= 36 \times 14 \times 2 = 1008$ sq. ft., or 10.08 squares. Cost $= 10.08 \times \$8.25 = \$83.16$.

2. How much will it cost to pave a sidewalk 70 ft. long and $7\frac{1}{2}$ ft. wide, at $\$2.12\frac{1}{2}$ per sq. yd?

3. A roof is covered with shingles 4 in. wide, put 7 in. to the *weather*. What is the cost at $9 a thousand if the roof is 50 ft. long and each side is 19 ft. wide?

4. A barn is covered with shingles 4 in. wide, put 6 in. to the weather. What is the cost at $10 a thousand if the roof is 70 ft. long, each side being 30 ft., and the first course along the eaves being doubled?

464. To find the Cost of Carpeting.

In carpeting, the width of the carpet, the allowance for matching the figures, and whether the strips are to run lengthwise or crosswise, must all be considered.

1. Carpets are usually from ¾ yd. to 1 yd. in width; but matting, oilcloth, etc., are often much wider.

2. In matching figures it is often necessary to cut off or turn under one of the ends. To fit a room, if an exact number of strips is too wide, a part of one breadth is turned under.

RULE.—*Multiply the number of yards in a strip, by the number of strips required, and this product by the price per yard.*

If more convenient, the feet in length may first be multiplied by the number of strips and then reduced to lineal yards.

EXAMPLES.

1. What will it cost to carpet a room 20 ft. long and 17 ft. wide, with carpet 1 yd. wide, running lengthwise at 60¢ a yard?

Solution: Since the room is 17 ft. wide, it will require 6 strips. 20 ft. × 6 = 120 ft., or 40 yds. Cost = 40 × $.60 = $24.

2. What will it cost to carpet a room 18 ft. by 15 ft. with carpet ¾ yd. wide, at 75¢ per lineal yard?

3. Find the cost of a carpet 30 in. wide, at 85¢ per lineal yard, for a room 15 ft. long and 14 ft. wide, if the strips run lengthwise. Find the cost, if the strips run across the room.

4. How many yards of carpet, ¾ yd. wide, will it take to carpet a room 22 ft., by 18 ft., the strips running lengthwise, allowing 6 in. waste in each strip for matching?

5. What will it cost to carpet a room 15 ft. 4 in. long by 13 ft. 9 in. wide, with carpet 30 in. wide, running lengthwise, at $1.25 per lineal yard, allowing 8 in. waste in each strip for matching?

465. To find the Cost of Papering.

Wall paper is sold by the roll, and in estimates a part of a roll is *considered a whole roll.*

A roll of paper is 8 yd. long and 18 in. wide.

1. Imported wall papers differ in width and the length of the roll.

2. Wall paper is often put up in *double* rolls, 16 yd. long. Double rolls are counted as 2 rolls each.

3. Borders and friezes are sold by the yard. They vary in width from 3 in. upward.

4. On account of waste, it is rarely possible to find the exact cost of papering a room.

RULE.—*Measure the entire distance around the room in yards, and double the number of yards for the number of strips.*

Then divide the number of strips required for the room by the number of strips that can be cut from each roll. The quotient will be the number of rolls.

EXAMPLES.

1. How many rolls of paper will cover the walls of a room 18 ft. by 15 ft., and 7 ft. 8 in. high?

$2\ (18 + 15) = 66$ ft.,
or 22 yd. $= 44$ strips.
$24 \div 7\frac{2}{3} = 3$ strips.
$44 \div 3 = 15$ rolls.

EXPLANATION. — The distance around the room is 66 ft., or 22 yd. The number of strips equal 2×22 or 44. The number of full strips from a roll of 8 yd., or 24 ft. are 3. Hence, the number of whole rolls must be $44 \div 3$ or 15 rolls.

2. What will the paper cost at 15¢ a roll, to paper the walls of a room 16 ft. by 14 ft.. and 8 ft. high, deducting 2 rolls for doors and windows?

3. How much will the paper cost for a room 21 ft. by 17 ft., 10 ft. high, at 18¢ a roll, making a reduction of 3 rolls for doors and windows?

4. What will the paper cost for a room 20 ft. long, 18 ft. wide, and 11 ft. high, at 20¢ a roll, deducting 4 rolls for doors and windows?

5. What will it cost to paper a room 25 ft. long, 20 ft. wide, and 11 ft. 6 in. high, with mop-board 12 in. wide, at 25¢ a roll, having also a border 18 in. wide, at 15¢ a yard, if the work can be done by 2 men in 1 day at $1.75 each per day?

6. How much will it cost to paper the walls and ceiling of a room 20 ft. long, 18 ft. wide, and 10 ft. 6 in. high, at 20¢ a roll, *having also* a border 15 in. wide, at 15¢ a yard, making a deduction *of 4 rolls for doors*, windows, etc., and paying $2.50 for the work?

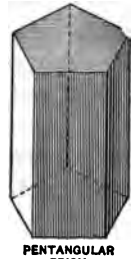
MEASURES OF SOLIDS

466. A **Solid** is anything that has length, breadth, and thickness or height. A solid is also called a *Body*.

The planes which bound a solid are called its *faces*, and their intersections, its *edges*.

TRIANGULAR PRISM.

SQUARE PRISM.



PENTANGULAR PRISM.

CYLINDER

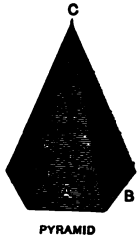
467. A **Prism** is a solid whose two ends are equal polygons and whose sides are parallelograms.

Prisms, from their bases, are named *triangular*, *quadrangular*, *pentagonal*, *hexagonal*, etc.

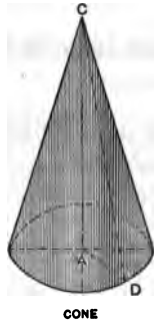
468. A **Rectangular Solid, Parallelopiped,** or **Square Prism,** is a prism whose six faces are all rectangles.

469. A **Cube** is a parallelopiped whose six sides are squares.

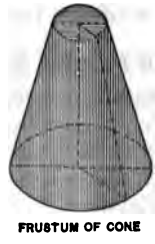
470. A **Cylinder** is a regular solid bounded by a uniformly curved surface with equal and parallel circles for its bases.



PYRAMID



CONE



FRUSTUM OF CONE

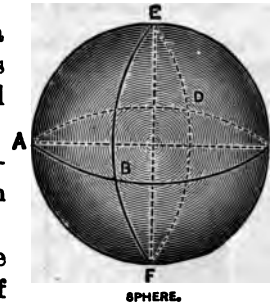
FRUSTUM OF PYRAMID

471. A **Pyramid** is a solid whose base is a polygon and whose faces are triangles meeting in a common point called the vertex.

472. A **Cone** is a solid whose base is a circle and whose curved surface tapers uniformly to a point called the vertex.

473. A Frustum of a pyramid or cone is the part of a pyramid or cone that remains after cutting off the top by a plane parallel to the base.

474. A Sphere is a solid bounded by a curved surface every point of which is equally distant from a point within called the center.



SPHERE.

The *Diameter* of a sphere is a line passing through its center and terminating in the surface.

The *Radius* is half the diameter, or the distance from the centre to the surface of a sphere.

The *Circumference* of a sphere is the greatest distance around it.

The *Altitude* of a solid is the perpendicular distance from its highest point to the plane of the base; as, AC (in cone or pyramid).

The *Slant height* of a pyramid is the altitude of any lateral face.

The *Slant height* of a cone is the distance from the vertex to the circumference of the base.

SURFACE OF SOLIDS.

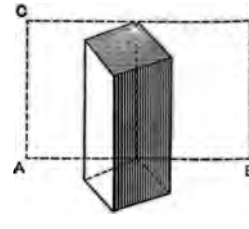
475. The Lateral Surface of a solid is all of the surface except that of its base or bases.

The lateral surface of a cylinder and a cone may properly be called the *convex* surface.

476. The Entire Surface of a solid includes the lateral surface and also the surface of the base or bases.

477. To find the Lateral Surface of a Prism or Cylinder.

If the lateral surface of a prism or cylinder were unfolded, it would form a rectangle, as *ABCD*, whose length is the perimeter of the base, and whose altitude is the height of the prism or cylinder. Hence,



RULE.—*Multiply the perimeter of the base by the altitude.*

EXAMPLES.

1. What is the lateral surface of a triangular prism whose sides are each 3 ft., and whose height is 5 ft.?

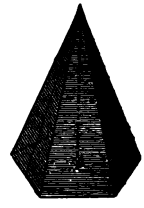
2. What is the convex surface of a cylinder whose diameter is 4 ft., and whose length is 7 ft.?

3. What is the entire surface of a square prism whose altitude is 15 ft. and the side of the base 5 ft.?

4. What is the entire surface of a cylinder whose length is 12 ft., and the radius $2\frac{1}{2}$ ft.?

478. To find the Lateral Surface of a Pyramid or Cone.

It is seen that the lateral surface of a pyramid is composed of triangles whose bases form the perimeter, and whose height is the slant height of the pyramid. This is also true when the pyramid has an infinite number of sides and coincides with a cone. Hence,



RULE.—*Multiply the perimeter of the base by half the slant height.*

EXAMPLES.

1. What is the lateral surface of a triangular pyramid, the sides of whose base are each 6 ft., and whose slant height is 20 ft.?

2. What will be the cost of painting a hexagonal church steeple at 25¢ per square yard, the sides of whose base are each 5 ft, and whose slant height is 40 ft.?

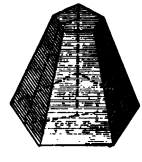
3. What is the lateral surface of a cone whose base is 7 ft. in diameter, and slant height 56 ft.?

4. What is the convex surface of a cone whose slant height is 48 ft., and whose base has a radius of 3 ft.?

5. Find the entire surface of a square pyramid the side of whose base is 8 ft., and slant height 25 ft.

479. To find the Lateral Surface of a Frustum of a Pyramid or Cone.

It is seen that the lateral surface of a frustum of a pyramid is composed of trapezoids whose parallel sides form the perimeter of the bases, and whose altitude is the slant height of the frustum. This is also true when the frustum has an infinite number of lateral faces, or trapezoids, and coincides with the frustum of a cone. Hence,



RULE.—*Multiply the sum of the perimeters of the two bases by half the slant height.*

EXAMPLES.

1. Find the lateral surface of the frustum of a square pyramid whose slant height is 20 ft., the side of the lower base 10 ft. and of the upper base 6 ft.

2. What is the convex surface of the frustum of a cone whose slant height is 14 ft., the diameter of the lower base 8 ft. and of the upper base 5 ft.?

3. How many square feet in the lateral surface of the frustum of a pentagonal pyramid whose slant height is 10 ft., and each side of whose lower base is 6 ft. and of the upper base 4 ft.?

4. What is the entire surface of the frustum of a cone whose slant height is 8 ft., the diameter of the lower base being 5 ft. and of the upper base 4 ft.?

480. To find the Convex Surface of a Sphere.

The following rules are derived from principles in geometry:

RULE.—*Multiply the circumference by the diameter; or, multiply the square of the radius by 4 times 3.1416.*

The convex surface of a sphere may also be found by multiplying the square of the diameter by 3.1416.

EXAMPLES.

1. What is the convex surface of a sphere whose diameter is 12 inches?

2. What is the convex surface of a sphere whose radius is 5 inches?

3. What is the convex surface of a cannon-ball whose circumference is 25 inches?

4. What would it cost to plate with silver a sphere 15 in. in diameter, at $2.25 a square foot?

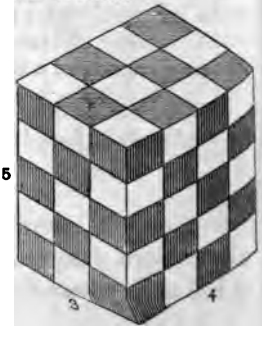
VOLUME OF SOLIDS.

481. The **Volume** of any body is the number of cubic units it contains.

The volume of a body is also called the *solid contents*, *contents*, or *capacity*.

482. To find the Volume of a Rectangular Solid.

It is seen that the volume of a rectangular solid is the number of small cubes it contains. Each layer contains the number in each row multiplied by the number of rows, and this multiplied by the number of layers, will give the number; but this equals the product of the length, breadth, and height, or thickness. Hence,



RULE. — *Multiply together the length, breadth, and thickness.*

Formulas for Rectangular Solids.

Length × Breadth × Height = Volume.
Volume ÷ (Length × Breadth) = Height.
Volume ÷ (Length × Height) = Breadth.
Volume ÷ (Breadth × Height) = Length.

The length, breadth, and thickness must be expressed in units of the same denomination.

EXAMPLES.

1. How many cubic feet in a block 6 ft. long, 5 ft. wide, and 3 ft. thick?

2. How many cubic feet of water in a rectangular reservoir 16 ft. long, 14½ ft. wide, 7½ ft. deep.

3. What will it cost to dig a cellar 38 ft. long, 30 ft. wide, and 5 ft. 6 in. deep, at $1.25 a cubic yard?

4. How many cubic feet of air in a room 16 ft. long, 14 ft. 9 in. wide, and 8 ft. 6 in. high?

5. A rectangular block of stone contains 182 cubic feet. It is 4 ft. wide and 3½ ft. thick. How long is the block?

483. To find the Volume of a Prism or Cylinder.

It is seen that if a prism or cylinder were 1 inch high it would contain as many cubic inches as there are square inches in the area of the base; and if 2 in. high, the volume would be 2 times as many; if 3 in. high, 3 times as many, etc. Hence,

RULE.—*Multiply the area of the base by the altitude.*

For area of the base see Art. 452.

EXAMPLES.

1. Find the solid contents of a prism whose base is 8 in. square and whose height is 15 inches?

2. Find the volume of a cylinder whose diameter is 10 in. and whose height is 2 ft.

3. How many cubic feet of earth must be removed to dig a cistern 8 ft. in diameter and 13 ft. deep?

484. To find the Volume of a Pyramid or Cone.

If we take two vessels, the one a prism and the other a pyramid, having equal bases and altitudes, the pyramid will hold just one-third as much as the prism. This is also true when the prism and pyramid have an infinite number of sides and coincide with the cylinder and cone. Hence,

RULE.—*Multiply the area of the base by one third of the alti-*

EXAMPLES.

1. What is the volume of a pyramid whose altitude is 16 in. and the area of the base 25 sq. in. ?

2. What is the volume of a cone whose altitude is 24 ft. and the diameter of the base 8 ft. ?

3. What is the volume of a rectangular pyramid whose altitude is 20 ft. and the sides of the base 4 ft. and 5 ft., respectively ?

4. Find the volume of a cone whose altitude is 50 ft. and the radius of the base 7 ft.

5. Find the volume of a pyramid whose base is a triangle, each side of which is 6 ft. and whose altitude is 15 ft.

485. To find the Volume of a Frustum of a Pyramid or Cone.

It can be shown by geometry that the frustum of a pyramid or cone is equal to the sum of three pyramids or cones whose common altitude is the altitude of the frustum, and whose bases are the lower base, the upper base, and a mean proportional between the bases of the frustum. Hence,

RULE.—*To the sum of the areas of the two ends add the square root of their product, and multiply this sum by one-third of the altitude.*

EXAMPLES.

1. What is the solid contents of the frustum of a square pyramid the sides of whose bases are 5 ft. and 8 ft., and whose altitude is 21 ft. ?

Solution: The volume $= (5^2 + 8^2 + \sqrt{5^2 \times 8^2})\ \frac{1}{3} = (25 + 64 + 40)\ 7 =$ 903 cu. ft.

2 Find the volume of the frustum of a square pyramid the side of whose bases are 12 ft. and 18 ft., and altitude 30 ft.

3. Find the number of cubic feet in a log 15 ft. long, the diameter of the larger end being 3 ft., and of the smaller end 2 ft.

486. To find the Volume of a Sphere.

A sphere may be regarded as composed of an infinite number of pyramids whose bases form the surface of the sphere and whose altitude is the radius of the sphere. Hence,

RULE.—Multiply the convex surface by one-third of the radius; or, multiply the cube of the diameter by one-sixth of 3.1416.

EXAMPLES.

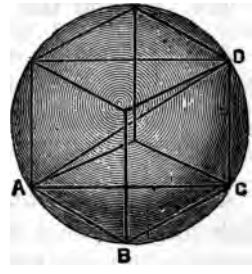
1. What is the volume of a sphere whose diameter is 8 inches?

2. The circumference of a sphere is 20 inches. Find its cubical ntents.

3. The outer diameter of a spherical shell is 10 in. and the inner ıneter is 5 in. What are the cubical contents of the shell?

4. If a cubic foot of cast iron weighs 450 lb., what is the weight a cannon-ball whose diameter is 12 inches?

487. To find the Side of a Cube inscribed in a given Sphere.



A cube is inscribed in a sphere when the vertices of its angles are in the surface of the sphere.

It is seen by Art. 438 that $\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 = \overline{AD}^2$. But $AB = BC = CD$. Therefore, $\overline{AB}^2 = \overline{AD}^2$; or, $\overline{AB}^2 = \frac{\overline{AD}^2}{3}$, or, $AB = \sqrt{\frac{\overline{AD}^2}{3}}$. Hence,

RULE.—*Divide the square of the diameter of the sphere by 3, and extract the square root.*

EXAMPLES.

1. What is the side of the largest cube that can be cut from a ere 9 in. in diameter?

2. Find the side of the largest cube that can be cut from a den ball 40 in. in circumference.

488. To find the Volume of an Irregular Body.

RULE.—1. *Place the body in a suitable vessel, pour in water until the body is just covered, and find the space occupied by both body and water.*

Remove the body from the vessel and find the space occupied by the water alone. The difference will be the volume of the body.

A vessel in the form of a cylinder is most convenient for finding the me of irregular bodies.

The volume may often be most conveniently found by taking the diam- of the vessel and the difference between the two altitudes of the water. 474 This will give the volume of the body at once.

A body lighter than water must be held down by pressure at the high- point.

EXAMPLES.

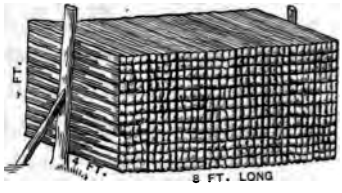
1. An irregularly shaped stone was placed in a cylindrical vessel 20 inches in diameter, and covered with water. The height of the water was then 15 inches; but when the stone was removed, the height of the water was only 8 inches. What was the volume of the stone?

2. A block of marble was placed in a 10-gallon vessel. It then required $4\frac{1}{2}$ gallons of water to fill the vessel. What was the volume of the block of marble?

489. To find the Quantity of Wood.

490. A Cord of wood is a pile 8 feet long, 4 feet wide, and 4 feet high. It contains 8 cord feet, or 128 cubic feet.

491. A Cord Foot is a cross section of this pile 1 foot long, and contains 16 cubic feet.



ONE CORD

EXAMPLES

492. 1. How many cords of wood in a pile 20 ft. long, 4 ft. wide, and 6 ft. high?

2. A certain pile of wood is 36 ft. long, 8 ft. wide, and 6 ft. high. What is its value at $3.50 per cord?

3. How many cords of wood can be placed in a shed 28 ft. long, 20 ft. wide, and 14 ft. high?

4. A man sold 10 loads of wood. Each load was 8 ft. long, 4 ft. wide, and 5 ft. high. What was the value of the wood at $4.25 a cord?

5. A tanner filled a shed 100 ft. long, 60 ft. wide, and 18 ft. high with bark which cost him $5.25 a cord. How much did the bark cost him?

BOARDS AND TIMBER.

493. Boards, Planks, Scantling, Joists, and other sawed timber are estimated by what is called *Board Measure*, the unit of which is the *Board Foot*.

494. A Board Foot is 1 ft. long, 1 ft. wide, and 1 in. thick. A *board an inch thick* is taken as the standard, and its contents in

board feet are the product of its length and breadth in feet, or the number of square feet it contains. The same measurement applies when the board is less than 1 inch thick.

Thus a board 16 ft. long, 14 in. wide, and 1 in. or less in thickness contains $16 \times 1\frac{1}{6} = 18\frac{2}{3}$ feet *board measure*, or $18\frac{2}{3}$ sq. ft.

When lumber is more than one inch thick the thickness is taken into account.

Thus, a board 16 ft. long, 14 in. wide, and $1\frac{1}{2}$ in. thick contains $16 \times 1\frac{1}{6} \times \frac{3}{2} = 28$ ft. *board measure.*

A cubic foot of lumber contains 12 board feet. Hence, board feet may be reduced to cubic feet by dividing by 12, and cubic feet to board feet by multiplying by 12.

495. To find the number of Board Feet in a Board.

RULE.—*Multiply the length in feet by the width in inches, and divide by 12.*

When a board tapers evenly, the mean or average width is used, which is half the sum of the two ends.

EXAMPLES.

Find the number of feet in the following boards:

1. 12 ft. long, 14 in. wide.
2. 14 ft. long, 15 in. wide.
3. 16 ft. long, 16 in. wide.
4. 16 ft. long, 14 in. wide.
5. 18 ft. long, 10 in. wide.
6. 14 ft. long, 18 in. wide.
7. How many feet in a board 16 ft. long, 15 in. wide, and $\frac{3}{4}$ in. thick?
8. How many feet in a board 18 ft. long, 16 in. wide at one end and tapering to 10 in. at the other?
9. Find the cost of 19 boards, each 14 ft. long, 15 in. wide, at $2.30 per C.
10. How much will it cost to lay 3 floors, each 20 ft. long and 16 ft. wide, at $25 per M, allowing $\frac{1}{5}$ for grooving?

496. To find the number of Board Feet in Timbers.

RULE.—*Multiply the length in feet by the width and thickness in inches and divide by 12.*

EXAMPLES.

1. How many board feet in a plank 14 feet long, 8 in. wide, and 2 in. thick?

2. How many board feet in a joist 20 ft. long, 9 in. wide, and 3 in. thick?

3. Find the number of board feet in a joist 24 ft. long, 7 in. wide, and 4 in. thick?

4. How many board feet in a piece of timber 18 ft. long, and 10 in. square?

5. What will be the cost of 10 girders, each 40 ft. long, 14 in. wide, and 12 in. thick, at $30 per M?

497. To find the Cubical Contents of Square Timber.

RULE.—*Multiply the area of one end in inches by the length in feet, and divide by 144.*

EXAMPLES.

1. What are the cubical contents of a stick of square timber 18 ft. long and 16 in. square?

Solution: $\frac{16 \times 16 \times 18}{144} = 32$ cu. ft.

2. What are the cubical contents of a stick of timber 20 ft. long and 14 in. square?

3. Find the cubical contents of a piece of timber, 12 by 14, and 24 feet in length.

4. Find the cubical contents of 3 pieces of timber, each 9 by 12, and 16 ft. long.

ROUND TIMBER.

498. The contents of "Saw Logs" are usually estimated by the quantity of square-edged inch boards they will produce.

There is no uniform nor accurate rule for determining the contents of a log *Board Measure.* Lumbermen generally use "Log Books" containing prepared tables showing the number of feet board measure which logs of various dimensions will produce; but they are not accurate. *Too much is allowed for waste.*

499. To find the number of Board Feet in a Log.

The following rule is often used; but like others is not strictly accurate, though perhaps one of the best.

RULE.—*Multiply the square of two-thirds of the smaller diameter, in inches, by the length in feet and divide by 12.*

* *1. On large logs* this allows too much for waste. Some dealers, on this *account, deduct only* 4 inches from the diameter, instead of one-third.

2. The following rule is said to be quite accurate for logs over 15 inches in diameter: *Multiply the square of two-thirds of the smaller diameter, in inches, by one-tenth of the length in feet.*

EXAMPLES.

1. What are the contents in board measure, of a log 18 ft. long, and 15 in. in diameter at the smaller end?

Solution: $\frac{2}{3}$ of $15 = 10$; $\frac{10 \times 10 \times 18}{12} = 150$ ft.

2. How many feet, board measure, in a log 20 ft. long, and 18 in. in diameter at the smaller end?

3. Find the number of feet, board measure, in a log 16 ft. long, and 14 in. in diameter at the smaller end.

4. Find the contents, in board measure, of a log 19 ft. long, and 21 in. in diameter at the smaller end.

5. Solve example 2, by both methods suggested in the notes and compare the results.

500. To find the number of Cubic Feet in a Log, when reduced to Square Timber.

The following rule is extensively used, and is practically correct.

RULE—*Multiply the square of two-thirds of the mean diameter, in inches, by the length in feet, and divide by 144.*

The mean diameter is half the sum of the two end diameters.

EXAMPLES.

1. How many cubic feet of square timber in a log 21 in. in diameter at the larger end, 15 in. at the smaller, and 25 ft. long?

Solution: $21 + 15 = 36$; $36 \div 2 = 18$, mean diameter.

$\frac{2}{3}$ of $18 = 12$; $\frac{12 \times 12 \times 25}{144} = 25$ cu. ft.

2. What are the cubical contents, in square timber, of a log 24 in. in diameter at the larger end, 18 in. at the smaller, and 20 ft. long?

3. Find the cubical contents, in square timber, of a stick of round timber 27 ft. long, having a diameter of 35 in. at one end, and 23 at the other.

4. A certain log, 40 ft. long, has a diameter of 52 in. at the larger end and 36 in. at the smaller end. How many cubic feet of square timber does it *contain*?

501. To find the Cubical Contents of Round Timber and Spars.

The following rule was formerly used when round timber was sold by the ton of 40 cubic feet. It is now seldom used.

RULE.—*Multiply the length in feet by the square of one-fourth the mean girt in feet.*

The mean girt of tapering timber is the circumference taken at the middle of the log.

The result as given by this rule is only about $\frac{11}{14}$ of the true contents as found by Art. 485.

ADDITIONAL RULES FOR MEASURING ROUND TIMBER.

Several additional rules for measuring logs are given below. These may be preferred by some to those already given.

502. To find the number of Feet Board Measure in a Log.

RULE.—*Multiply the diameter of the small end of the log in inches by half the same diameter in inches. Multiply this product by the length of the log in feet and divide by 12. The result will be the number of feet board measure.*

1. This rule does not allow for the thickness of the saw. Hence, to find how many feet square-edged inch boards can be sawed from a log, the result obtained by the rule must be divided by 1 plus the thickness of the saw.

2. If a log is to be sawed into plank 2 inches thick, divide the result obtained by the rule by 2 plus the thickness of the saw. The result will give the number of feet surface measure. But since the plank is 2 inches thick, the result must be multiplied by 2 for the number of feet, board measure.

EXAMPLES.

How many feet of lumber can be sawed from a log 20 inches in diameter at the smaller end, and 24 feet long, allowing $\frac{1}{4}$ inch for thickness of the saw?

Solution: $\frac{20 \times 10 \times 24}{12} = 400$; $400 \div 1\frac{1}{4} = 320$ ft.

503. Doyle's Log Rule.

Perhaps no better or more accurate rule for all sizes of logs can be given than the following:

RULE.—*From the diameter of the small end of the log in inches, subtract 4. The square of the remainder will be the number of board feet yielded by a log 16 feet long.*

The same result can be obtained by squaring $\frac{1}{2}$ of the remainder after subtracting 4 inches from the diameter of the small end and multiplying the result by the length in feet.

EXAMPLES.

How many feet of inch boards can be sawed from a log 20 feet long and 22 inches in diameter at the small end?

Solution:

$22 - 4 = 18$; $18^2 = 324$ ft. in a log 16 ft. long. A log 20 ft. long will contain $\frac{4}{16}$, or $\frac{1}{4}$, more feet. Hence, 324 ft. plus 81 ft. ($\frac{1}{4}$ of 324 ft.), or 405 ft. can be sawed from the log.

504. To find the number of Cubic Feet in the largest Square Piece of Timber that can be sawed from a Round Log.

RULE.—*Take the product of the diameter in inches, half the diameter in inches, and the length of the log in feet, and divide by 144. The result will be the number of cubic feet.*

EXAMPLES.

How many cubic feet of squared lumber can be sawed from a log 24 feet long and 30 inches in diameter?

Solution: $\frac{30 \times 15 \times 24}{144} = 75$ cu. ft.

505. To tell the number of Cubic Feet in Round Timber.

RULE.—*Multiply the square of the circumference at the middle of the log in feet by 8 times the length in feet, and point off two places in the product. The result will be the number of cubic feet.*

EXAMPLES.

$7^2 = 49$
22
98
98
1078
8
86.24

1. How many cubic feet in a log 7 feet in circumference and 22 feet long?

EXPLANATION.—Squaring the circumference, we have 49, which multiplied by 8 times the length, or by 22 and the product by 8, and pointing off two places, the result is found to be 86 24 cu. ft.

2. How many cubic feet in a log 15 feet long and having a circumference of 8 feet?

3. How many cubic feet in a log 13 feet in circumference and 38 feet long?

STONE-WORK.

506. Stone-work is usually estimated by the perch. In some places it is estimated by the cubic foot.

507. A **Perch** of stone-work formerly contained 24¾ cu. ft. It was 16½ ft. long, 1½ ft. wide, and 1 ft. high. The perch of 16½ cu. ft., which is 16½ ft. long, 1 ft. wide, and 1 ft. high, is now in general use.

508. Excavations and **Embankments** are estimated by the cubic yard. A cubic yard of earth is called a load.

In estimating the **Quantity of Material** for the walls of a building, deduction is made for the doors, windows, and corners. The deductions in the quantity of stone or brick, made for doors and windows, depend upon the size of the openings. The deduction for each corner is the thickness of the wall in feet multiplied by the height of the wall in feet.

In estimating the **Cost of Labor,** mechanics measure the length of the wall on the outside, taking the girt of a building; and no allowance is made for doors and windows except by special contract, when one-half the space for openings is usually deducted.

509. To find the number of Perches of Stone in a Wall.

The number of perches in a wall will, of course, depend upon whether a large perch of 24¾ cu. ft. or a small perch of 16½ cu. ft. is meant.

RULE.—*Divide the number of cubic feet in the wall by* 24¾ *for large perches, or, by* 16½ *if small perches are meant.*

EXAMPLES.

1. How many perches of stone, of 16½ cu. ft. each, in a wall 50 ft. long, 5 ft. high, and 2 ft. thick?

2. What will it cost to build a wall 36 ft. long, 6 ft. high, and 2½ ft. thick at $1.25 per perch of 16½ cu. ft.?

3. How much will it cost to dig a cellar 30 ft. long, 28 ft. wide, and 6 ft. deep, at 35¢ a load?

4. How many perches of stone, 24¾ cu. ft. each, will be required to build the walls of a cellar 30 ft. long and 28 ft. wide, the walls to be 8 ft. high and 1 ft. 6 in. thick, deducting 80 cu. ft. for openings?

5. What will be the cost of building the walls of the above *cellar, if the* mason charges $1.40 a perch (16½ cu. ft.) for his *labor, and no* deductions for openings?

BRICK-WORK.

510. Brick-work is usually estimated by the *thousand bricks.*

The dimensions of common bricks are 8 in. long, 4 in. wide, and 2 in. thick.

511. To find the Number of common Bricks for a Wall.

In estimating the number of bricks required to build a wall, $\frac{1}{4}$ of an inch for mortar is added to the length and thickness of each brick. The product of the length and thickness, thus increased, will give the exposed surface required for each brick. Dividing this product into 144, will give the number of bricks for each square foot of surface in a wall one brick in thickness. This will be found to be over 7 bricks, which is the number usually considered for common bricks. If the wall is two bricks thick, 14 bricks will be required, etc. Hence,

RULE.—*Multiply the number of square feet of surface by 7 when the wall is 1 brick thick; by 14 when the wall is 2 bricks thick; and by 21 when 3 bricks thick.*

When greater accuracy is desired, add $\frac{1}{4}$ of an inch to the length and thickness of a brick, and divide 144 by the product of the increased dimensions for the exact number of bricks to a square foot of wall surface.

EXAMPLES.

1. How many common bricks will be required to build the walls of a house 30 ft. long, 28 ft. wide, and 20 ft. high, the wall being 13 in. thick, allowing 200 sq. ft., for doors and windows?

Solution:

The outside surface $= (30 + 28) \times 2 \times 20 = 2320$ sq. ft.
The deductions $= 200 + (1\frac{3}{4} \times 4 \times 20) = 286\frac{2}{3}$ sq. ft.
No. of bricks $= (2320 - 286\frac{2}{3}) \times 21 = 42700.$

2. How many common bricks will be required to build the walls of a house 37 ft. long, 32 ft. wide, and 21 ft. high, the wall being 13 in. thick, deducting 220 sq. ft. for doors and windows?

3. What will the bricks cost for the walls of a house 40 ft. long, 36 ft. wide, and 24 ft. high, the walls being 13 in. thick, of common bricks, at $7.50 per M., deducting 260 sq. ft. for doors and windows?

4. What will the brick-work of the house in example 3, cost at $2.25 per M., 130 sq. ft. being allowed for openings?

5. A certain walk is 40 ft. long and 8 ft. wide. What will it cost to pave it with common bricks laid flat, if the bricks cost $8.35 per M., *and the* workman charges 30¢ a square yard?

512. To find the Capacity of Bins in Bushels.

513. The **Standard Bushel** of the United States is a cylindrical measure 18½ inches in diameter and 8 inches deep. The *Stricken Measure* (even full and stricken off by a rule) contains 2150.42 cubic inches. The *Heaped Measure* (the contents heaped in the form of a cone) contains 2747.71+ cubic inches.

Grains, seeds, and small fruits are sold by stricken measure. Corn in the ear, large fruits and vegetables, coal, lime, and other bulky articles are sold by heaped measure.

RULE.—*Divide the contents in cubic inches by 2150.42 to find the number of bushels stricken measure; or by 2747.71 to find the number of bushels heaped measure.*

EXAMPLES.

514. 1. How many bushels of grain will a box hold that is 4 ft. long, 2 ft. wide, and 3 ft. high?

2. A certain bin is 6 ft. long, 3 ft. wide, and 5 ft. high. How many bushels of grain will it hold?

3. How high must a bin be made to hold 100 bushels of grain, if it is to be 5 ft. 6 in. long, and 4 ft. wide?

4. A granary 12 ft. long, 9 ft. wide, and 8 ft. high is ¾ full of wheat. What is the value of the grain at 70¢ a bushel?

5. A corn crib is 20 ft. long, 5 ft. wide, and 10 ft. high. How many bushels of corn in the ear will it hold?

6. A certain corn crib flared at the sides is 18 ft. long, 10 ft. high, 8 ft. wide at the bottom and 12 ft. wide at the top. How many bushels of corn in the ear will it hold?

Suggestion.—The average width is half the sum of the top and bottom widths. (Art. 495).

515. To find the Capacity of Cisterns.

516. The **Capacity of Cisterns, Tanks,** etc., is usually estimated by the gallon or barrel.

517. The **Standard Liquid Gallon** of the United States contains 231 cubic inches.

In measuring the capacity of cisterns, tanks, etc., 31½ gallons are considered a *barrel* and 63 gallons a *hogshead*.

RULE.—*Divide the contents in cubic inches by 231, and the quotient will be the number of gallons.*

The gallon dry measure, contains 268.8 cubic inches. It is commonly alled a *half-peck*.

EXAMPLES.

518. 1. How many gallons of water will a tank 4 ft. long, 3 ft. ride, and 2 ft. high hold ?

2. A tank 6 ft. long, 3½ ft. wide, and 2 ft. high will hold how any gallons ?

3. How many barrels of water will fill a tank 10 ft. long, 4 ft. ride, and 5 ft. high ?

4. How many barrels of water will fill a cistern that is 5 ft. quare and 13 ft. deep ?

5. How many barrels of water will fill a circular cistern 8 ft. in iameter and 13 ft. deep ?

6. I wish to dig a circular cistern 8 ft. in diameter that will hold vhen filled 200 hogsheads of water. How deep must it be dug ?

7. A retangular cistern is 4 meters long, 2½ meters wide, and 2 neters deep. What is its capacity in hectoliters?

8. A tank is 2¼ meters long, 5 decimeters wide, and 60 centi-neters deep. How many liters of water will it contain?

9. A tank is 7.5 meters long, 3.8 meters wide, and 2.5 meters leep. How many hours will it take a pipe to fill the tank if 140 iters of water flow into the tank per minute?

GAUGING.

519. Gauging is the process of finding the capacity of casks nd other vessels.

1. Barrels and casks differ from cylinders in bulging more or ess in the middle. By first finding the mean diameter of a vessel, ts capacity can be found like that of a cylinder.

2. The mean diameter of a barrel or cask is found by adding to he head diameter ⅔, or, if the staves are only slightly curved, ⅝, f the difference between the head and bung diameters.

520. To find the Capacity of a Barrel or Cask in Gallons.

The contents of a cylinder $= D^2 \times .7854 \times$ length (Art. 483). Since a iquid gallon contains 231 cu. in., the contents of a vessel in gallons = mean $D^2 \times .7854 \times$ length) $\div$ 231. But, $.7854 \div 231 = .0034$. Hence,

RULE.—*Multiply the square of the mean diameter in inches by he length in inches, and this product by .0034.*

If a cask is only partly full, stand it on end, find the mean diameter of the art filled. Multiply its square by the height of the liquid, and that pro-luct by .0034.

EXAMPLES.

1. How many gallons in a barrel whose head diameter is 20 in., bung diameter 26 in., and length 34 in.?

2. How many gallons in a cask 20 in. long, and whose diameters are 12 and 17 inches, respectively?

3. What is the capacity in gallons of a barrel of slight curvature 30 in. long, the head diameter 19 in., and the bung diameter 21 in.?

RULES FOR APPROXIMATE MEASUREMENTS.

521. 1. To find the contents of a bin in stricken bushels. RULE.—*Multiply the cubic feet by* .8.

2. To find the contents of a bin in heaped bushels. RULE.—*Multiply the cubic feet by* .64.

3. To find the capacity of tanks, cisterns, etc., in gallons. RULE.—*Multiply the cubic feet by* $7\frac{1}{2}$.

4. To find the capacity of tanks, cisterns, etc., in barrels of 31½ gallons. RULE.—*Divide the cubic feet by* $4\frac{1}{4}$.

5. To find the number of perches of masonry in a wall, of 24¾ cu. ft. to the perch. RULE.—*Multiply the cubic feet by* .0404.

6. To find the number of perches of masonry in a wall, of 16½ cu. ft. to the perch. RULE.—*Multiply the cubic feet by* .0606.

7. To find the weight of ice in pounds. RULE.—*Multiply the cubic feet by* $58\frac{1}{8}$.

8. To find the weight of pure water in pounds. RULE.—*Multiply the cubic feet by* $62\frac{1}{2}$.

9. To find the weight of Lehigh white ash coal, egg size, in tons. RULE.—*Divide the cubic feet by* $34\frac{1}{2}$.

10. To find the weight of Schuylkill white ash stove coal in ton. RULE.—*Divide the cubic feet by* 35.

11. To find the weight of gray, pink, or red ash stove coal in tons. RULE.—*Divide the cubic feet by* 36.

12. To find the weight of hay in a load or shed loft, unpressed, in tons. RULE.—*Divide the cubic feet by* 540.

13. To find the weight of hay in a common hay barn, or a low stack in tons. RULE.—*Divide the cubic feet by* 405.

14. To find the weight of timothy hay in mow bases, compressed with grain, and in butts of large stacks, in tons. RULE.—*Divide the cubic feet by* 324.

EXAMPLES.

522. 1. A certain bin is $6\frac{1}{2}$ ft. long, $3\frac{1}{2}$ ft. wide, and 6 ft. high. About how many bushels of grain will it hold?

2. A potato bin is 15 ft. long, 4 ft. wide, and $4\frac{1}{2}$ ft. high. How many bushels will it contain?

3. A water tank is 8 feet long, $3\frac{1}{2}$ ft. wide, and $2\frac{1}{2}$ ft. high. How many gallons will it hold?

4. A cylindrical cistern is 6 ft. in diameter and 12 ft. deep. How many barrels of $31\frac{1}{2}$ gallons will it hold?

5. How many perches ($24\frac{3}{4}$ cu. ft.) of stone, laid dry, will build a wall 120 ft. long, 2 ft. wide, and 4 ft. high?

6. How many perches ($16\frac{1}{2}$ cu. ft.) of stone in a wall 100 ft. long, 18 in. thick, and 8 ft. high?

7. The inside dimensions of an ice-house are 30 ft. long, 20 ft. wide, and 14 ft. high. How many tons of ice will it hold, allowing 2500 cu. ft. for preserving material?

8. A tank is 8 ft. square at the base and 10 ft. high. What weight does the bottom support when the tank is full of water?

9. A wagon bed $11\frac{1}{2}$ ft. long, $3\frac{1}{2}$ ft. wide, and 3 ft. deep, is filled with Lehigh coal. Find the weight of the coal in tons.

10. A coal house 14 ft. long, and 5 ft. wide, is filled, to a depth of 4 ft. 6 in. with Schuylkill stove coal. How many tons does it contain?

11. A coal bin 10 ft. long, $7\frac{1}{2}$ ft. wide, and 8 ft. high is filled with red ash coal. How many tons are in it?

12. A shed loft 25 ft. long, 18 ft. wide, and averaging 7 ft high is filled with hay. What is the weight in tons?

13. A hay barn 40 ft. long, 30 ft. wide, and 15 ft. high, is two-thirds full of hay. How many tons of hay are in it?

14. A mow base 36 ft. long, 25 ft. wide, and 8 ft. high is full of compressed timothy hay. What is the value of the hay at $11.50 a ton?

15. Mr. Leidig sold a mow of hay 28 ft. long, 16 ft. wide, and 10 ft. high, at $10.25 a ton of 405 cu. ft. How much did he receive for it?

16. How many tons of compressed timothy hay in a mow 30 ft. long, 20 ft. wide, and 14 ft. high?

PERCENTAGE.

523. Percentage is the process of computing by hundre

524. The term **Per Cent.** comes from the Latin words *p tum*, meaning *by* or *on the hundred.*

Thus, 5 per cent. of any quantity means 5 of every hundred of th tity.

525. The **Sign** % is used in business to stand for the wo cent. The per cent. may also be indicated by a common f or a decimal.

Thus, $5\% = \frac{5}{100} = .05$.

526. The principal **Quantities** considered in percentage *Base*, the *Rate*, and the *Percentage.*

The *Amount* is the sum of the base and percentage. The *Differen* base less the percentage.

527. The **Base** is the number on which the percentage i puted.

528. The **Rate** is the number of hundredths of the base t

529. The **Percentage** is the result obtained by multiplyi base by the rate.

In the statement, 5% of 400 is 20, 400 is the *base*, 5 per cent. (.0 *rate*, and 20 is the *percentage.*

In all computations where a decimal *rate* is used, great care sh taken to express all decimal terms with exactness.

EXPRESSIONS OF THE RATE.

530. The Rate may be expressed as a *decimal* or as a *c fraction.*

Thus, $4\% = .04 = \frac{4}{100} = \frac{1}{25}$; $10\% = .10 = \frac{10}{100} = \frac{1}{10}$; $16\frac{2}{3}\% = .16\frac{2}{3} = \frac{50}{300} = \frac{1}{6}$; $\frac{3}{4}\% = .00\frac{3}{4} = \frac{\frac{3}{4}}{100} = \frac{3}{4} \times \frac{1}{100} = \frac{3}{400}$; or, $.00\frac{3}{4} = .0075 = \frac{75}{10000} = \frac{3}{400}$; $125\% = 1.25$.

531. Express decimally and as common fractions in their lowest terms.

1. 8%.	11. $6\frac{2}{3}$%.	21. 100%.	31. $\frac{1}{2}$%.
2. 9%.	12. $7\frac{1}{2}$%.	22. 115%.	32. $\frac{1}{4}$%.
3. 12%.	13. $12\frac{1}{2}$%.	23. 120%.	33. $\frac{3}{4}$%.
4. 25%.	14. $13\frac{1}{3}$%.	24. 145%.	34. $\frac{2}{5}$%.
5. 48%.	15. $22\frac{1}{2}$%.	25. 150%.	35. $\frac{9}{10}$%.
6. 49%.	16. $24\frac{1}{2}$%.	26. 185%.	36. $\frac{11}{20}$%.
7. 50%.	17. $33\frac{1}{3}$%.	27. $137\frac{1}{2}$%.	37. $\frac{18}{25}$%.
8. 62%.	18. $37\frac{1}{2}$%.	28. $262\frac{1}{2}$%.	38. $\frac{12}{25}$%.
9. 85%.	19. $62\frac{1}{2}$%.	29. $312\frac{1}{2}$%.	39. $\frac{38}{50}$%.
10. 90%.	20. $83\frac{1}{3}$%.	30. $416\frac{2}{3}$%.	40. $\frac{45}{75}$%.

532. To read an extended decimal as a *per cent.*, call the first two places per cent., and the added places, a fractional part of one per cent.

Thus, .0525 read 5 and one-fourth per cent.; .16375 read 16 and three-eighths per cent.; .25371, read 25 and three hundred seventy-one thousandths per cent. Strictly considered, a decimal can be called a rate per cent. only when expressed in *hundredths;* for per cent. means *by the hundred.* Thus, .07, .25, .03$\frac{1}{2}$, .10$\frac{41}{100}$, are each a rate per cent.

533. A common fraction may be expressed as a per cent. by reducing it to hundredths.

Thus, $\frac{1}{2} = \frac{50}{100} = .50 = 50\%$; $\frac{1}{10} = \frac{10}{100} = .10 = 10\%$.

1. Express $\frac{5}{6}$ as a per cent. :

6)5.
.83$\frac{1}{3}$, or 83$\frac{1}{3}$%.

EXPLANATION.—$\frac{5}{6}$ equals $\frac{1}{6}$ of 5. Hence dividing 5 by 6 to two decimal places, we have $\frac{5}{6} = .83\frac{1}{3}$, or $83\frac{1}{3}$%.

Express as a per cent. :

2. $\frac{2}{3}$.	5. $\frac{4}{5}$.	8. $\frac{8}{12}$.	11. $\frac{9}{10}$.	14. $\frac{1}{100}$.
3. $\frac{3}{4}$.	6. $\frac{7}{8}$.	9. $\frac{10}{14}$.	12. $\frac{6}{17}$.	15. $\frac{2}{100}$.
4. $\frac{1}{8}$.	7. $\frac{2}{11}$.	10. $\frac{6}{15}$.	13. $\frac{19}{25}$.	16. $\frac{5}{4}$.

534. When the rate per cent. is an **aliquot** or other small common fractional part of 100, it will often be more convenient to use this form than the decimal. The following table of aliquot per cents., with their fractional equivalents, is therefore worthy of careful *study and, perhaps,* of memorizing :

TABLE.

$1\frac{2}{3}\% = \frac{1}{60}$.	$16\frac{2}{3}\% = \frac{1}{6}$.	$60\% = \frac{3}{5}$.
$2\frac{1}{2}\% = \frac{1}{40}$.	$20\% = \frac{1}{5}$.	$62\frac{1}{2}\% = \frac{5}{8}$.
$3\frac{1}{3}\% = \frac{1}{30}$.	$25\% = \frac{1}{4}$.	$66\frac{2}{3}\% = \frac{2}{3}$.
$5\% = \frac{1}{20}$.	$30\% = \frac{3}{10}$.	$70\% = \frac{7}{10}$.
$6\frac{1}{4}\% = \frac{1}{16}$.	$33\frac{1}{3}\% = \frac{1}{3}$.	$75\% = \frac{3}{4}$.
$8\frac{1}{3}\% = \frac{1}{12}$.	$37\frac{1}{2}\% = \frac{3}{8}$.	$80\% = \frac{4}{5}$.
$10\% = \frac{1}{10}$.	$40\% = \frac{2}{5}$.	$87\frac{1}{2}\% = \frac{7}{8}$.
$12\frac{1}{2}\% = \frac{1}{8}$.	$50\% = \frac{1}{2}$.	$90\% = \frac{9}{10}$.

CASES AND FORMULAS.

535. The principal quantities considered in percentage are the Base, Rate, and Percentage. These quantities are so related that any two of them being given, the others may be found. The subject of percentage may therefore be treated under three cases:

1. Given the base and rate, to find the percentage.

2. Given the percentage and rate, to find the base.

3. Given the base and percentage, to find the rate.

The method of finding the unknown quantity may be represented by the following

FORMULAS.

I. $B \times r = P$

II. $B = \frac{P}{r}$

III. $r = \frac{P}{B}$

EXPLANATION.—Let B represent the base, r the rate, and P the percentage. It will be seen that formula I. is derived from the definition of percentage, Art. 523. Dividing both members of the equation, $B \times r = P$, by r, we obtain formula II.; and dividing the same equation by B, we obtain formula III.

1. When the *amount* (A) is given or required, the r in the appropriate formula becomes $1 + r$; and when the *difference* (D) is given or required r should be changed to $1 - r$. Thus, $B \times (1 + r) = A$, and $B \times (1 - r) = D$. In these formulas the 1 always represents 100%. Formula II. thus modified becomes, $B = \frac{A}{1 + r}$ and $B = \frac{D}{1 - r}$; and formula III. becomes $1 + r = \frac{A}{B}$, and $1 - r = \frac{D}{B}$. But these, as is seen, are only modifications of the three principal formulas.

2. The applications of percentage are so extensive and enter so largely in [illegible] calculations that the student should be satisfied with nothing [illegible] of its principles.

CASE I.

36. Given the Base and the Rate, to find the Percentage.

ORAL EXERCISES.

What is 20% of 40?

lution: 20% of 40 is $\frac{20}{100}$, or $\frac{1}{5}$, of 40, which is 8.

What is $12\frac{1}{2}$% of 72 bu.?

lutions: $12\frac{1}{2}$% of 72 bu. is $\frac{12\frac{1}{2}}{100}$, or $\frac{1}{8}$ of 72 bu., which is 9 bu.

ıe example may be solved more briefly thus: $12\frac{1}{2}$% equals $\frac{1}{8}$. $\frac{1}{8}$ of 72 s 9 bu.

'hat is

3. 10% of 50 yd.?
4. 25% of 40 lb.?
5. 30% of 60 oz.?
6. 40% of 25 ft.?
7. 50% of 80 men?
8. 60% of 55 rd.?
9. 75% of 100 cows?
10. 80% of 70 gal.?
11. 90% of 50 qt.?
12. 35% of 40 bu.?
13. 45% of 60 mi.?
14. 55% of $20?
15. 70% of 80 days?
16. 15% of 60 tons?
17. $12\frac{1}{2}$% of $96?
18. $16\frac{2}{3}$% of 54 bu.?
19. 65% of 40 yd.?
20. $33\frac{1}{3}$% of 120 lb.?
21. 40% of 75 acres?
22. $37\frac{1}{2}$% of 64 pears?
23. $62\frac{1}{2}$% of 32 sheep?
24. $87\frac{1}{2}$% of 56 tons?
25. $8\frac{1}{3}$% of $120?
26. $6\frac{1}{4}$% of 80 lb.?
27. 5% of $180?
28. $3\frac{1}{3}$% of $210?
29. $2\frac{1}{2}$% of $160?
30. $1\frac{2}{3}$% of $480?

EXAMPLES.

. What is 8% of $524?

524
.08
.92

EXPLANATION.—8% of $524 is .08 times $524, which is $41.92.

. What is $62\frac{1}{2}$% of $9.68?

$9.68 = $6.05

EXPLANATION.—$62\frac{1}{2}$% = $\frac{5}{8}$, and $\frac{5}{8}$ of $9.68 is $6.05.

ULE.—*Multiply the base by the rate.*

FORMULA.—$B \times r = P$.

The *amount* equals the base *plus* the percentage; the *difference* equals the base *minus* the percentage.

Find:

3. 16% of $14.50.
4. 28% of $73.40.
5. 32% of $650.
6. 7½% of $320.
7. 43% of $36.30.
8. 71% of $160.70.
9. 106% of $120.50.
10. 4⅝% of $318.20.

11. A man's income is $1800 a year. He pays 13% of it fo house rent. How much rent does he pay?

12. Mr. M. had 480 acres of land and sold 26⅔% of it. Ho many acres did he sell?

13. A man owned 460 acres of land. He sold to one neighbo 25% of his land and 20% to another. How many acres did h sell?

14. A man had 1260 cattle and bought 30% more. How man had he then?

Suggestion.—$B \times (1 + r) = A$. Hence, $1260 \times 1.30 =$ Amount. Or, s note.

15. A merchant having $4000 invested in goods, bought 15 more. What was the entire amount invested?

16. A store was sold for $3500, of which 24% was paid in ca h. How much remained unpaid?

Suggestion.—$B \times (1 - r) = D$. Hence, $3500 \times .76 =$ Difference. Or, see note.

17. A farmer having 540 bushels of wheat, sold 33⅓% of it. How many bushels remained?

18. From a bill of goods amounting to $29.50, 5% was deducted for cash. What was the sum paid?

19. A dealer having 2400 bushels of wheat, sold 12½% of it to one man and 62½% to another. How many bushels remained?

20. A man deposited $12500 in bank, and afterwards drew out ¼% of it to pay for a horse. How much did the horse cost him?

21. A house was bought for $1800. The buyer paid 20% in cash, 25% of the remainder was paid a month later, and at another time 33⅓% of what still remained was paid. How much did he still owe?

CASE II.

537. Given the Percentage and the Rate, to find the Base.

ORAL EXERCISES.

1. 30 is 20% of what number?

Solution.—If 30 is 20%, or $\frac{1}{5}$, of some number, $\frac{5}{5}$, or that number, is 5 times 30, or 150.

2. 18 is 40% of what number?

Solution.—If 18 is 40%, or $\frac{2}{5}$, of some number, $\frac{1}{5}$ of that number is $\frac{1}{2}$ of 18, or 9, and $\frac{5}{5}$, or that number, is 5 times 9, or 45.

Find the number of which:

3. 50 is 25%.	9. 7 is 1%.	15. $\frac{3}{5}$ is 10%.
4. 37 is 10%.	10. 12 is $16\frac{2}{3}$%.	16. $\frac{1}{8}$ is $6\frac{1}{4}$%.
5. 14 is 20%.	11. 27 is $37\frac{1}{2}$%.	17. $2\frac{1}{2}$ is 25%.
6. 15 is 30%.	12. 35 is $12\frac{1}{2}$%.	18. $4\frac{2}{5}$ is 20%.
7. 24 is 60%.	13. 60 is 50%.	19. $\frac{14}{25}$ is 70%.
8. 13 is 5%.	14. 40 is $66\frac{2}{3}$%.	20. $12\frac{1}{2}$ is $12\frac{1}{2}$%.

EXAMPLES.

1. $32.68 is 43% of what sum?

$$\begin{array}{r|l} .43 & \$32.68 \,(\$76 \\ & \underline{301} \\ & \quad 258 \\ & \quad \underline{258} \end{array}$$

EXPLANATION.—If $32.68 is 43% of the sum, then .43 times that sum is $32.68. Hence, the sum is $32.68 ÷ .43, or $76.

2. $220 is $68\frac{3}{4}$% of what sum?

$68\frac{3}{4}\% = \frac{68\frac{3}{4}}{100} = \frac{275}{400} = \frac{11}{16}$.

$\$220 \div \frac{11}{16} = \$220 \times \frac{16}{11} = \$320$.

EXPLANATION.—$63\frac{3}{4}$% equals $\frac{11}{16}$. If $\frac{11}{16}$ of the sum is $220, that sum is $\frac{16}{11}$ of $220, or $320.

RULE.—*Divide the percentage by the rate.*

FORMULA.—$B = \frac{P}{r}$.

Divide the *amount* by 1 *plus* the rate; the *difference* by 1 *minus* the rate.

Find the sum of which:

3. $3.45 is 23%.	7. $10.50 is $\frac{3}{4}$%.
4. $4.76 is 17%.	8. $12.30 is 1%.
5. $5.40 is $6\frac{1}{2}$%.	9. $21.20 is 2.5%.
6. $7.35 is 49%.	10. $3990 is 114%.

11. Mr. A. spent 12½% of his money for a horse which cost him $140. What was his money?

12. A man bought a house for $2800, which was 35% of his capital. What was his capital?

13. A teacher spent 38% of his salary and saved $744. What was his salary?

Suggestion.—$B = \frac{D}{1-r}$. See Note 1, Art. 535, or above note

14. Mr. James spent 40% of his fortune and had $3500 left. What was his fortune?

15. A bankrupt whose assets were $8750, paid 62½% of his liabilities. What were his liabilities?

16. A man invested 12½% of his money in horses and cattle, 62½% in a farm, and had $4500 left. How much money had he before his investments?

17. A farm was sold for $5832, which was 8% more than it cost. What was the cost?

Suggestion.—$B = \frac{A}{1+r}$. See Note 1, Art. 535, or above note.

18. A merchant paid $3465 for a store, which was 15% more than he paid for his house. How much did he pay for his house?

19. A cargo of wheat was so damaged that it was sold for $7700, which was 23% less than its original value. What was its value before it was damaged?

20. A merchant spent 25% of his income for clerk hire, 35% for his family, 8¾% for insurance, rent, etc., 10% for benevolence, and saved $1700. What was his income?

21. A man drew 66⅔% of his money from the bank, and paid 87½% of it for a farm costing $5600. How much money had he remaining in bank?

CASE III.

538. Given the Base and the Percentage, to find the Rate.

ORAL EXERCISES.

1. 20 is what per cent. of 80?

Solution.—20 is ¼ of 80. Hence, 20 is ¼ of 100%, or 25% of 80.

2. What per cent of 40 yd. are 16 yd.?

Solution.—16 yards are $\frac{16}{40}$, or ⅖, of 40 yards. Hence, 16 yards are ⅖ of 100%, or 40% of 40 yards.

What per cent. of

3. 25 is 5?	9. 50 is 20?	15. $40 is $24?
4. 30 is 3?	10. 60 is 15?	16. 48 gal. is 16 gal.?
5. 40 is 8?	11. 72 is 18?	17. 56 lb. is 42 lb.?
6. 24 is 6?	12. 80 is 60?	18. 72 bu. is 56 bu.?
7. 27 is 9?	13. 84 is 49?	19. 120 ft. is 90 ft.?
8. 36 is 6?	14. 96 is 64?	20. 150 rd. is 270 rd.?

EXAMPLES.

1. What per cent. of 235 bu. is 150 bu.?

235) 150.00 (.63$\frac{8}{47}$=63$\frac{8}{47}$%
1410
900
705
195 = 39/47

EXPLANATION.—235 bu. multiplied by some rate equals 150 bu. Hence, that rate is 150 divided by 235, which is .63$\frac{8}{47}$, or 63$\frac{8}{47}$%.

2. What per cent. of 75$\frac{1}{8}$ acres are 18$\frac{5}{6}$ acres?

$\frac{18\frac{5}{6}}{75\frac{1}{8}} = \frac{113}{452} = \frac{1}{4} = 25\%$

EXPLANATION.—Multiplying both terms of the fractional form by 6, the least common multiple of the fractions, we secure a common fraction or whole numbers for the base and the percentage.

RULE.—*Divide the percentage by the base.*

Formula.—$r = \frac{P}{B}$.

What per cent. of

3. $960 are $240?	7. $24.08 are $13.76?
4. 375 rd. are 250 rd.?	8. 343\frac{3}{4}$ are 172\frac{1}{2}$?
5. 780 gal. are 560 gal.?	9. 360 A.? are 137$\frac{1}{2}$A.?
6. 832 bu. are 260 bu.?	10. 81$\frac{9}{10}$ ft. are 163$\frac{4}{5}$ ft.?

11. A certain man's income was $1800 a year and his expenses were $1200. What per cent. of his income were his expenses?

12. If I receive only $58.50 from a debtor who owes me $90, what per cent. do I lose on the debt?

13. Mr. Jones and Mr. Smith engaged in partnership. Mr. Jones invested $2460 and Mr. Smith $2740. What per cent. of the capital of the firm did each invest?

14. If a miller takes 3$\frac{1}{5}$ quarts of every bushel he grinds for toll, what per cent. does he take for toll?

15. A man having deposited in bank $3606, afterward drew out $450.75. What per cent. of the sum deposited did he withdraw?

16. A teacher's salary was $3200 a year and his expenses were $2720. What per cent. of his salary was he able to save?

17. Mr. G. raised 1600 bushels of wheat. He sold at one time 480 bushels, and 270 bushels at another. What per cent. was still unsold?

18. A man began business with a capital of $4575. At the end of the first year his capital was $6405. What was the per cent. of increase?

Suggestion.—$1 + r = \frac{A}{B}$. Art. 535, note 1. Or, get the percentage and apply the general formula.

19. A grocer having 1600 pounds of sugar, sold $\frac{1}{4}$ of it in a week and $\frac{1}{3}$ of what remained the week following. What per cent. of the whole quantity remained?

20. A farm that cost $7500 was rented for $500. What per cent. did the owner realize on his investment?

21. A bankrupt had liabilities to the amount of $10800, and his assets were only $6804. What per cent. of his indebtedness could he pay?

REVIEW OF THE PRINCIPLES OF PERCENTAGE.

539. The **Principles of Percentage** are so extensively involved in commercial calculations, that a thorough mastery on the part of the student is necessary. To this end, the principles are here embodied in both rules and formulas.

540. Rules.

1. Given the base and the rate, to find the *Percentage*. RULE.—*Multiply the base by the rate.*

2. Given the percentage and the rate, to find the *Base*. RULE.—*Divide the percentage by the rate.*

3. Given the base and the percentage, to find the *Rate*. RULE.—*Divide the percentage by the base.*

4. Given the base and the rate, to find the *Amount*. RULE.—1. *Multiply the base by the rate, and add the base.* Or, 2. *Multiply the base by 1 plus the rate.*

5. Given the base and the rate, to find the *Difference*. RULE.—1. *Multiply the base by the rate, and subtract from the base.* Or, 2. *Multiply the base by 1 minus the rate*

6. Given the amount and the rate, to find the *Base.* RULE.—*Divide the amount by 1 plus the rate.*

7. Given the difference and the rate, to find the *Base.* RULE.—*Divide the difference by 1 minus the rate.*

8. Given the amount and the base, to find the *Amount Per Cent.* RULE.—*Divide the amount by the base.*

9. Given the difference and the base, to find the *Difference Per Cent.* RULE.—*Divide the difference by the base.*

541. Formulas.

1. $B \times r = P.$
2. $B = \frac{P}{r}.$
3. $r = \frac{P}{B}$
4. $A = B \times (1 + r).$
5. $D = B \times (1 - r).$
6. $B = \frac{A}{1 + r}$
7. $B = \frac{D}{1 - r}.$
8. $1 + r = \frac{A}{B}$
9. $1 - r = \frac{D}{B}.$

1. It will be seen that each formula is a concise expression of the rule corresponding in number.

2. If the student has thoroughly mastered the definitions under percentage and the first three formulas so as to comprehend the relations which these quantities sustain with one another, he will have no difficulty in applying the principles.

REVIEW.

542. 1. A man bought a house for \$1900, which is 25% of what was paid for a farm. How much was paid for both?

2. Mr. M. had \$4500 in bank, and drew out 35% of it. How much had he remaining in bank?

3. In an army of 15000 men, 4000 were killed and 5000 were wounded. What per cent. of the army were unhurt?

4. A merchant having \$8750 to invest, bought an interest in a foundry for which he paid \$4000. He invested 40% of the remainder in grain, and with what was left he bought a lot. What did he pay for the lot?

5. After a clerk's salary had been increased 12%, he received \$1680. How much did he receive before the increase?

6. In order to pay a note of \$450, I drew from bank 37½% of my deposit. How much had I left in bank?

7. A man owning $\frac{3}{8}$ of a vessel, sold 35% of his share for $5600. At that rate, what was the value of the vessel?

8. A horse was sold for $212.80, which was 12% more than he cost. What was the cost?

9. A, B, and C were partners. A furnished 33% of the capital, B 37½%, and C $2450. What was the capital of the firm?

10. A druggist having 37 lb. 8 oz. of a mixture, sold 7 lb. 8 oz. of it. What per cent. did he sell?

11. A man having an income of $5000, spends 20% of it for house rent, 60% of the remainder for general expenses, and $500 for benevolences. What per cent. of his income does he save?

12. A merchant deposited 80% of $\frac{5}{9}$ of his money in bank, and then had $2345 on hand. What was the amount of his money?

13. D. C. Lix & Co. charged me $12.25 for collecting an account of $350. What rate was charged?

14. A section of Western land was bought at $10.50 an acre, and sold at an advance of 30%. How much was received for the section?

15. Mr. H. paid 42½% of his debts, and still owed $4725. What was the sum paid?

16. A gentleman paid $190 for a horse, which was 24% less than he paid for a carriage. How much did he pay for both?

17. A farmer raised 216 bushels of wheat on a field of 12 acres. If he had sown 1 bu. 3 pk. to the acre, what was the per cent. of yield?

18. In settling an estate, an executor found that 12½% of it was invested in telephone stock, 16½% in bank stock, 25% in a manufacturing industry, $10000 in real estate, and $1830 in personal property. There was also $410 in cash. What was the total value of the estate?

19. Mr. Burr having $1800 in bank, drew out at one time 15% of it, and soon after 30% of the remainder. Having realized 35% on what he had withdrawn from bank, he deposited it. How much did he then have in bank?

20. Three railroad companies carried 12500 pounds of "through freight" a distance of 500 miles. The total charge was 60¢ per 100 pounds. The distance over the first line was 125 miles, and over the second, 175 miles. What per cent. of the total charge, *and how much money*, should each company have received?

APPLICATIONS OF PERCENTAGE.

543. The **Applications of Percentage** to business transactions are extensive. This is due to the convenience of computing by hundredths, and to the fact that our money system reckons a hundred cents to the dollar.

These applications are of two classes: those in which the element of time is not considered, and those involving time. To the former belong Profit and Loss, Trade Discount, Commission, etc. To the latter class belong Simple Interest, True Discount, Bank Discount, etc.

In the applications of percentage the student will find little difficulty if he has thoroughly mastered the three cases of percentage. Care should be taken to see clearly the *base* upon which the percentage is reckoned.

PROFIT AND LOSS.

544. Profit and Loss treats of the method of reckoning gains or losses in business transactions.

545. The **Gross Cost** of anything is its first cost, increased by all expenses incident to its purchase and holding to date of sale.

546. The **Net Selling Price** of anything is its gross selling price, decreased by all charges incident to its sale.

When the net selling price is *greater* than the gross cost, the excess is a *Profit* or *Gain;* and when less than gross cost, the deficit is a *Loss.*

547. The **Quantities** should be considered as follows:

1. The **Cost,** as the *base.*
2. The **Per Cent.** of gain or loss, as the *rate.*
3. The **Gain** or **Loss,** as the *percentage.*
4. The **Selling Price,** if at a gain, as the *amount;* if at a loss, as the *difference.*

548. PRINCIPLE.—*The gain or loss is reckoned on the cost.*

ORAL EXERCISES.

549. James bought a book for 40 cents and sold it at a profit of 20%. What was the gain?

Solution.—A profit of 20% is $\frac{20}{100}$, or $\frac{1}{5}$, of the cost, $\frac{1}{5}$ of 40 cents is 8 cents

2. A man bought a watch for $24, and sold it at an advance of 25% What was the gain?

3. A colt was bought for $60, and sold at 10% less than cost. What was the loss?

4. How much is lost in selling a $160 organ for 12½% less than cost?

5. Mr. L. bought a coach for $320, and sold it at a gain of 30%. What was his profit?

6. A merchant bought an article for $3.60, and sold it at a profit of 20%. What was the selling price?

7. By selling a coat for $2 more than it cost 20% was gained. What was the cost of the coat?

Solution.—At 20% gain, $\frac{20}{100}$, or $\frac{1}{5}$, of the cost is the gain, which is $2. $\frac{1}{5}$, or the cost, is 5 times $2, or $10.

8. A grocer lost 10% by selling butter at 2¢ per lb. less than cost. What was the cost?

9. A profit of 8¢ per lb. was made by selling tea at 16⅔% advance. What did it cost?

10. I cleared $12 by selling a watch at a gain of 24%. What was the cost?

11. A second hand buggy was sold for $35 less than cost. If the loss was 28%, what was the cost?

12. Mr. Dunmire sold his watch for $70, which was a gain of 40%. What was the cost?

Solution.—At 40% gain, $\frac{40}{100}$, or $\frac{2}{5}$, of the cost is the gain, which added to $\frac{5}{5}$, the cost, is $\frac{7}{5}$ of the cost, which equals the selling price, or $70. $\frac{1}{5}$ of the cost is $\frac{1}{7}$ of $70, or $10, and $\frac{5}{5}$, or the cost, is 5 times $10, or $50.

13. A cow was sold for $65, which was a gain of 25%. What was the cost?

14. A suit of clothes was sold for $21, at a gain of 16⅔%. What was the cost?

15. What was the cost of a wagon sold for $26, at a loss of 35%?

16. Mr. B. sold his team for $166, and lost 17%. What was the cost?

17. A house was sold for $2600, which was at a gain of 160%. Find the cost.

18. A boat was sold for $132, which was at a loss of 45%. How *should it have* been sold to gain 45%?

19. Mr. K. received 15% less than cost by selling his horse for $153. What was the loss?

Solution.—At 15% loss, $\frac{15}{100}$, or $\frac{3}{20}$, of the cost is the loss. $\frac{20}{20}$, the cost, minus $\frac{3}{20}$, of the cost, or $\frac{17}{20}$ of the cost equals the selling price, or $153. $\frac{1}{20}$ of the cost is $\frac{1}{17}$ of $153, or $9, and $\frac{3}{20}$, or the loss, is 3 times $9, or $27.

20. A wagon was sold for $70, at 10% less than cost. What was the loss?

21. A sewing machine was sold for $59, at a profit of 18%. What was the gain?

22. Mr. Hill received $165 for a binder by selling it at an advance of 32%. What was his profit?

23. A man sold two lots for $120 each. He gained 20% on the one, and lost 20% on the other. What was his gain or loss?

24. A cow that cost $30, was sold at a profit of $6. What was the gain per cent.?

Solution.—A gain of $6 on $30, is a gain of $\frac{6}{30}$, or $\frac{1}{5}$, of the cost. Hence, the gain is $\frac{1}{5}$ of 100%, or 20%.

25. An organ was bought for $60, and sold at a profit of $15. What was the gain per cent.?

26. A book which cost $4, was sold at a profit of 80¢. Find the gain per cent.

27. A coat was bought for $24 and sold for $30. What was the gain per cent.?

28. A coat was bought for $30 and sold for $24. What was the loss per cent.?

29. A dealer sold a horse for $125, and cleared $\frac{1}{5}$ of this money. Had he sold the horse for $30 less, what would he have lost per cent.?

EXAMPLES.

550. 1. Goods were bought for $4165, and sold at a profit of 26%. What was the gain?

$$\begin{array}{r} \$4165 \\ .26 \\ \hline 24990 \\ 8330 \\ \hline \$1082.90 \end{array}$$

EXPLANATION.—The gain was 26% of the cost, or .26 times $4165, which is $1082.90.

2. What is the gain on goods which cost $19.25, and are sold at 18% profit?

3. A farm was bought for $4675, and sold at a gain of 7½%. What was the profit?

4. A merchant paid $387.40 for a lot of choice goods which he sold at a profit of 37%. How much did he receive for the goods.

5. Mr. A. bought a lot of goods for $2760. Owing to damage by fire, he was obliged to sell at a loss of 18¾%. What was the loss?

6. A man owned real estate for which he paid $7400; but at a forced sale was obliged to sell at a loss of 23%. How much did the farm bring?

7. A dealer buys coal for $3.60 a ton, and sells it at 25% advance. What does he charge a ton?

8. A grocer bought 360 lb. of coffee at 16¢ a pound, and sold it at a gain of 12½%. How much did he sell it for per pound?

9. A dealer bought 4760 bu. of grain at 65¢ per bu., and sold it at a gain of 4½%. How much did he get for all of the grain?

10. A merchant gained $22.40 by selling a lot of goods at a profit of 16%. What was the cost of the goods?

.16) $22.40 ($140.
16
64
64
0

EXPLANATON.—Since $22.40 is 16% of the cost, then .16 times the cost is $22.40. Hence, the cost is $22.40 ÷ .16 or $140.

Or, *Solution:*

16% of the cost = $22.40.
∴ 1% of the cost = $\frac{1}{16}$ of $22.40 = $1.40.
100% of the cost = 100 times $1.40 = $140.

The former method preferred, because shorter.

11. What was the cost of goods which were sold at a gain of $79.80, if the rate of profit was 14%?

12. Find the cost of goods which were sold at a loss of $27.50, if the rate of loss was 40%.

13. A man lost 18½% of his money dealing in stocks and 5½% by neglecting his business. His total loss was $1800. How much money had he left?

14. Mr. Mine sold some cloth at 30% advance, and had a profit *of $1.05 on each* yard. What did it cost per yard?

15. A merchant had some goods marked 40¢. He dropped 10% from the marked price, and still had a profit of 20%. What did they cost him?

16. A manufacturer's capital produced an income of $3150 a year at a gain of 17½%. What would have been his income at 20%?

17. Mr. G. bought some goods at $1.20 a yard, and sold them at $1.56 a yard. What was the gain per cent.?

$1.56 − $1.20 = $.36, gain,
$.36 ÷ $1.20 = .30, or 30%.

EXPLANATION.—The gain, $.36, is the percentage and the cost, $1.20, is the base.

18. A house was bought for $2170 and sold for $125.20 more than it cost. What was the gain per cent.?

19. A lot which cost $230, was sold for $31.29 more than it cost. What was the gain per cent.?

20. A man sold a store for $3600, which cost him $4000. What was the loss per cent.?

21. Mr. L. sold a store which cost him $5280 at a gain of ⅛ of the cost. What was the gain per cent.?

22. Mr. B., who had sold the store mentioned in ex. 21, to Mr. L., lost ⅛ of the selling price. What was the per cent. of loss to Mr. B.?

23. I have goods which cost 75¢ a yard. How must they be marked to allow 12½ per cent. for bad debts and still gain 40% on the cost?

24. If apples cost 50¢ a bushel, at what price must they be sold to gain 20%, after making an allowance for 16⅔% decay, and 25% bad debts in selling?

25. Mr. M. lost 15% by selling his house for $450 less than it cost. How much would he have received for it if he had gained 15%?

26. A man sold a farm at $86.25 per acre, which was 15% more than he paid for it. How much did he pay per acre?

27. A lady sold a house for $4575, which was 3½% less than she paid for it. What was the cost?

28. A furrier sold a set of sable furs at 16% less than cost and lost $25.60. How much did he get for them?

29. Mr. Hale paid $2.80 for a certain article. How much must he receive for it to gain 20%?

30. If Mr. Hale gained 20% by selling a hat for $4.50, what was the cost?

31. What per cent. would a man gain by selling a city lot for $2250, which cost $2000?

32. A real estate man bought land for $2480, and sold it at a gain of 18¾%. The buyer sold it at a loss of 20%. What did the latter receive for it?

33. Mr. T. sold his house for $2640. If his loss equals ⅛ of the selling price, what was the per cent. of loss?

34. I bought two farms for $15714, paying 25% more for one than for the other. I sold the cheaper farm at 10% profit, and the higher priced at 8¼%. How much was my total gain?

35. A merchant bought 5 lots of Royal prints averaging 60 yd. in a lot, marked 10¢, at a discount of 10, 12½, 15, 20, and 25% respectively, and sold them all at 10% below marked price. What was his net profit?

36. A drover bought 60 cattle at an average cost of $27. Five of the number were killed by accident. How must he sell the remainder per head to gain 15% on the investment, the expenses being $117?

37. A lot of silks were bought by a merchant for 60¢ per yard. How must the goods be marked that he may throw off 20% from the marking price and still make a profit of 20%?

38. Mr. E. was offered $2420 for his house, but refused to sell, as he would lose thereby 8⅓%. Later he sold the house for $2970. What per cent. did he gain?

39. I sold my team for $450, and cleared 20% of this sum. What per cent. would I have gained by selling it for $480?

40. A man bought two farms for $6000 each. In selling them he gained 25% on the one farm and lost 25% on the other. Did he gain or lose by the transaction, and how much?

41. A man sold two farms for $6000 each. On the one he gained 25%, and on the other he lost 25%. Did he gain or lose by the transaction, and how much?

42. Mr. Brown gained 12½% by selling a house for $850 more than it cost him. How much did Mr. Brown pay for the house?

MARKING GOODS.

551. To **Mark Goods** is to represent the cost price and the selling price upon a list or upon tags attached to the articles on sale.

552. In marking the cost of goods, merchants usually take a word, phrase, or a sentence consisting of ten different letters, and let each letter in succession represent one of the Arabic figures. The cost, marked thus, can be known only to those who know the key, or private mark. Characters other than letters are sometimes used. Some merchants conceal the selling price in the same way.

553. The **Key** must contain ten different letters or characters. To prevent repetition an extra letter or character called a repeater may be used. Fractions are represented as with numbers.

The cost is usually written above a horizontal line, and the selling price below. Suppose our key is "Quick rates," then the letter values are $\frac{q\text{-}u\text{-}i\text{-}c\text{-}k\text{-}r\text{-}a\text{-}t\text{-}e\text{-}s}{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6\text{-}7\text{-}8\text{-}9\text{-}0}$. If the cost of an article were \$2.15, and the selling price \$2.60, they would be marked $\frac{u.qk}{u.rs}$. Articles which cost \$3.37½ could be marked $i.ia\frac{q}{u}$, or, if x be used as a repeater, $i.xa\frac{q}{u}$.

EXAMPLES.

554. 1. A merchant's key is "Much profit." He has cloth which cost him \$3.75 per yard. How must he mark it to gain 20%?

2. My key for marking goods is "Enough said." How much must I receive for hats marked $n.ad$ to gain 25%?

3. Mark the cost and selling price from the key "Enough said," of an article which cost \$1.40, and sold at a gain of 20%?

4. Mark the selling price of goods which cost \$2.40 to gain 30%, the key being "Pay me quick."

5. From the key word "Charleston" find the cost mark for each tie bought for \$4.20 per doz.

6. What will be the retail selling price mark of the ties to gain 20%?

7. Caps bought at \$8.40 per dozen, are sold at retail for 25% above cost. Mark the cost and selling price from the key "Cash profit."

8. Mark the selling price of goods that cost \$4.20, and sold at a gain of *33⅓%, if the key is* "Outlandish."

9. A lot of capes were bought for $36, $42, and $54 per dozen. What is the cost mark for one cape of each kind, and also the selling price mark, at a gain of 30%, the key being, "James Brown"?

10. If I sell an article at the rate of $18 per dozen I gain 25%. How should I mark the cost of each article from the key "Handsomely"?

555. How to Mark Goods Bought by the Dozen to gain a certain per cent.

To gain

10	%	,	divide	the	cost	per	dozen	by	10	and	deduct	$\frac{1}{12}$ of itself.
$12\frac{1}{2}$	"		"	"	"	"	"		10	"	"	$\frac{1}{16}$ " "
$16\frac{2}{3}$	"		"	"	"	"	"		10	"	"	$\frac{1}{36}$ " "
$18\frac{3}{4}$	"		"	"	"	"	"		10	"	"	$\frac{1}{96}$ " "
20	"		"	"	"	"	"		10.			
25	"		"	"	"	"	"		8	and	deduct	$\frac{1}{8}$ of itself.
26	"		"	"	"	"	"		10	"	add	$\frac{1}{20}$ " "
28	"		"	"	"	"	"		10	"	"	$\frac{1}{15}$ " "
30	"		"	"	"	"	"		10	"	"	$\frac{1}{12}$ " "
32	"		"	"	"	"	"		10	"	"	$\frac{1}{10}$ " "
$33\frac{1}{3}$	"		"	"	"	"	"		9.			
35	"		"	"	"	"	"		10	and	add	$\frac{1}{8}$ of itself.
40	"		"	"	"	"	"		10	"	"	$\frac{1}{8}$ " "
50	"		"	"	"	"	"		8.			
60	"		"	"	"	"	"		10	and	add	$\frac{1}{3}$ of itself.
80	"		"	"	"	"	"		10	"	"	$\frac{1}{2}$ " "
100	"		"	"	"	"	"		6.			

EXAMPLES.

556. 1. A merchant bought hats at $25 per dozen. How should he sell them to gain 20 per cent.?

$25 ÷ 10 = $2.50 EXPLANATION.—By referring to the table it is seen that to gain 20%, the cost per dozen should be divided by 10. $25 divided by 10 equals $2.50.

2. Caps were bought at $4.80 per dozen. For how much a piece must they be sold to gain 25%?

8) $4.80
6).60
.10
.50

EXPLANATION.—By referring to the table it is seen that to gain 25%, the cost per dozen should be divided by 8, and deduct $\frac{1}{8}$ *of itself.* $4.80 divided by 8 is $.60; and $\frac{1}{8}$ of $.60 is $.10. $.60 *minus* $.10 equal $.50.

3. A merchant bought shoes at $16.50 per dozen pairs. At hat price must he sell them per pair to gain 28%?

4. Boots were bought by a dealer for $27.60 per dozen pairs. For how much must they be sold per pair to gain 30%?

5. Lamps were bought at wholesale for $6 per dozen. What must be the selling price of each to make 26%?

6. A hatter imported fur hats at $75 per dozen. At what price must he retail them to make 32%?

7. A dealer in queensware bought dishes for $13.50 per dozen sets. What must be his selling price per set to realize a gain of 3⅓%?

8. Mr. Ford bought 3 doz. sets of silver spoons for $36 per dozen sets. How must he sell them per set to make 40%?

TRADE DISCOUNT.

557. A Trade Discount is an allowance made by manufacturers and merchants upon their catalogue or list prices.

Since the prices of articles vary with the supply and demand, invention of cheaper methods of manufacture, etc., instead of publishing a new catalogue or list price with each fluctuation which would often involve great expense, manufacturers or jobbers simply change the rate of discount with each variation of price.

Merchandise is frequently sold "on time" subject to certain rates of discount if paid at an earlier date.

558. A Discount Series, that is several successive discounts, is often allowed.

Thus, a discount of 15%, 10%, and 5%, means a discount of 15% from the list price, 10% from the remainder, and 5% from what still remains.

559. Trade Discount is computed in accordance with the principles of percentage, the list price being the base; and the discount and net selling price the percentages.

560. Given the list price and discount series, to find the selling price.

EXAMPLES.

1. What is the net amount of a bill of $600, subject to a discount of 20% and 10%?

8	$600	EXPLANATION.—A discount of 20% will leave 80% or .8; and a second discount of 10% will leave 90%. or .9 of .8, or .72.
9	.72	
72	$432.00	*.72 of $600* is .72 times $600, or $432.

2. What single discount is equivalent to a series of 25%, 20%, and 5%?

.75	1.	EXPLANATION.—A discount of 25% will leave 75%,
.8	.57	or .75; and a second discount of 20% will leave 80%, or
.600	.43	.8 of .75, or .6; and a third discount of 5% will leave
.95		95%, or .95 of .6, which is .57 net proceeds. Hence, the
.570		discount equivalent to the given series is 1 minus .57,
		or .43, or 43%.

RULE.—1. *First find the net proceeds of 1 by multiplying together the proceeds of each discount. Then multiply the list price by the net proceeds of 1.*

2. *To find a single equivalent discount, subtract the net proceeds of 1 from 1.*

1. This method is much shorter than to compute the net price for each sale through a series of discounts.

2. The order in which the discounts of a series is considered does not affect the result. Thus, 25%, 20%, and 5% off is the same as 20%, 25%, and 5% off, or 5%, 25%, and 20% off.

3. When more convenient use common fractional equivalents.

3. What is the net price of a bill of goods invoiced at $350, and sold at 20% and 15% off?

4. Find the selling price of an invoice of goods amounting to $540, at a discount of 20%, 10%, and 5%.

5. The gross amount of a bill of goods is $780. What is the net amount, the rates of discount being 15%, 10%, and 5%?

6. What is the net value of a bill of iron amounting to $650, at a discount of 20%, 10%, and 5%?

7. What single discount is equivalent to 20%, 15%, 10%, 10%?

8. Having bought a bill of goods at 30% discount from the list price of $500, I sell it at 20% and 10% from the same list price. Do I gain or lose, and how much?

9. An invoice of hardware amounting to $4500 was sold June 24, 1904, at a discount of 25%, 20%, and 12½%, with a further discount of 10% if paid within 30 days. How much will pay the bill on July 15, 1904?

10. A merchant sold a bill of goods amounting to $2500, with a discount of 20%, 20%, and 10%. How much did he deduct from the amount of the bill?

11. Goods were sold at a discount of 20%, 16⅔%, 12½%, and 10%. What single discount would have been equally satisfactory to the purchaser?

12. A's list price of a certain quality of hats is $15 per dozen, 20% and 5% off. B's list price of the same quality and size of hats is $16 per dozen, 25% and 10% off. How much can be saved by ordering 10 dozen of B?

13. A merchant bought a bill of goods upon which the total discount was $329.80, the discount series being 20%, 15%, and 10%. What was the selling price of the goods?

561. Given the cost, the per cent. of profit or loss desired, and the discount series, to find the marked price.

EXAMPLES.

1. What must be the marked price of steel tanks costing $17.28 each, that 25% may be gained, after allowing a discount of 20% and 10%?

$17.28 .8
4.32 .9
.72) $21.60 .72
$30

EXPLANATION.—To gain 25%, or ¼ of the cost, the price received must be $17.28, the cost, plus $4.32, the gain, or $21.60.

Since a discount of 20% and 10% is allowed, only .72 of the marked price is received, which is $21.60. Hence the marked price is $21.60 divided by .72, or $30.

RULE.—*Add to the cost the gain required or subtract the loss, and divide the result by the net proceeds of 1 after allowing for all discounts.*

2. At what price must goods be marked to net $1.20 after allowing a discount of 20%?

3. What must be the marked price of goods to net $15.30, after allowing a discount of 15% and 10%?

4. How much must be the marked price of a stove which cost $4 to gain $3, after allowing a discount of 20% and 15%?

5. What must be the asking price of a coat which cost $8, that 25% may be gained, after allowing the purchaser a discount of 10%?

6. What must be the marked price of goods costing $1.50 per yard to gain 20% after dropping 10% from the marked price?

7. What must be the list price of an article costing $30 to gain 10%, after allowing a discount of 25% and 20%?

8. What must be the list price of goods costing $325 to gain 10%, *after allowing* 15%, 10%, and 5%?

9. Wishing to dispose of a stock of goods which cost $1200, I so marked it as to allow discounts of 25%, 20%, and 10%, and yet sell it at a loss of only 10%. What was the marked price?

10. Goods which cost 60¢ per yard are marked 80¢. What per cent. of discount must be allowed on marked price to net cost?

Solution.—Discount ($.80 − $.60 = $.20) ÷ marked price ($.80) = 25%.

11. What per cent. of discount must be allowed on marked price to net cost, if the goods cost $7 and are marked $10?

12. What per cent. of discount must be allowed on marked price to net 25% profit on a watch which cost $12 and is marked $18?

COMMISSION.

562. Commission is a sum charged by a person for transacting business for another.

Commission is usually estimated at a per cent. of the gross proceeds of a sale or of the prime cost of a purchase; but sometimes it is computed at a certain price per bushel, barrel, bale, etc.

563. An **Agent** is a person who transacts business for another.

An agent who buys or sells merchandise for another is sometimes called a *Factor*, *Commission Merchant*, or *Broker*. A commission merchant usually receives the goods to be sold and acts in his own name; a broker does not take possession of the goods bought or sold, but simply effects contracts to buy or sell in the name of his employer.

564. The **Principal** is the person for whom the business is transacted.

565. A **Consignment** is a shipment of goods by one party to another to be sold on account of the shipper or on joint account of the shipper and the receiver of the goods.

The shipper of the goods is called the *Consignor*, and the one to whom the goods are shipped is called the *Consignee*.

566. The **Gross Proceeds** of a sale or collection is the total amount received by the agent before any deductions are made.

567. The **Net Proceeds** is what remains after deducting the commission and all other charges from the gross proceeds.

568. An **Account Sales** is a statement in detail rendered by an agent to his principal, showing the sales of the consignment, all of the charges or expenses attending the same, and the net proceeds.

The charges may be for commission, guaranty, freight, cartage, inspection, insurance, advertising, storage, or any other outlay incident to the sale.

569. An **Account Purchase** is a statement in detail rendered by an agent to his principal, showing the quantity, grade and price of goods bought on his account, the charges or expenses incident to the same, and the gross amount of the purchase.

570. The **Quantities** should be considered as follows:

1. The **Prime Cost** or **Gross Selling Price,** as the *base.*
2. The **Per Cent.** of commission as the *rate.*
3. The **Commission** for either buying or selling, or for guaranty of quality or credit, as the *percentage.*
4. The **Remittance,** including investment and commission, as the *amount.*
5. The **Selling Price,** *less* the commission, as the *difference.*

EXAMPLES.

571. 1. An agent sold goods to the amount of $465. What was his commission at 3½%?

Suggestion.—$B \times r = P$. Hence, $\$465 \times .03\frac{1}{2} = \$16.28$, Ans.

2. An agent sold property to the value of $3240. What was his commission at 2½%?

3. What is the commission at 2¾% for selling goods amounting to $2300?

4. What is the commission at 2%, if the gross proceeds of a sale are $362.80?

5. A commission merchant sold 275 bbl. of apples at $2.40 per bbl., at a commission of 3¼%. What was his commission?

6. A real estate agent sold a farm of 150 acres at $75 per acre, for which he received 2% commission. What was his commission, and how much was due his employer?

7. A lawyer employed to collect a debt of $1250 compromised for 95%. He charged 4¼% commission. How much was his commission?

8. An agent sold for his employer goods to the amount of $1864.78, at a commission of 5%, and purchased for him raw material amounting to $3795, at a commission of 1¾%. What was the total commission received by the agent?

9. An agent's commission at 5% for selling a quantity of goods was $75. What was the amount of the sales?

Suggestion.—$B = \frac{P}{r}$. Hence, $\$75 \div .05 = \$1500$, Ans.

10. An agent's commission for purchasing goods at 2% was $46.50. What was the amount of goods purchased?

11. What is the amount sold if the commission at 5% is $104.50

12. A grain dealer paid his agent $71.28 commission for buying corn. The rate of commission was $1\frac{1}{2}$%, and the price paid was 54¢ per bushel. How many bushels did he buy?

13. A commission merchant sold goods for a firm at $2\frac{1}{2}$% and charged $1\frac{1}{4}$% for guaranteeing the payment of the money. What was the amount of goods sold, if his whole commission was $197.25?

14. I sold some goods on commission at 5%, through an agent who charged me 3%. My commission, after paying the agent, was $194. What was the agent's commission, and the sum paid to my employers?

15. A dealer sent his agent $1545 to be invested in wheat, after deducting his commission of 3%. What was the sum invested?

Suggestion.—$B = \frac{A}{1+r}$. Hence, $1545 ÷ 1.03 = $1500, Ans.

16. An agent received a remittance of $1352 to be invested in rye, after retaining his commission of 4%. What was the amount of the purchase?

17. I sent my agent $2214.50 to be invested in cotton, after deducting his commission, 3%. What sum was paid for the cotton?

18. Mr. Roe received $931.50 to invest in oats at 30 cents per bushel, which included his commission at $3\frac{1}{2}$%. How many bushels did he buy?

19. A real estate agent received $9816 to invest in choice land, which included his commission of $2\frac{1}{4}$%. He purchased 160 acres. What was the price paid per acre?

20. An agent remitted to his employer $1349, after deducting his commission of 5%. What were the gross sales?

Suggestion.—$B = \frac{D}{1-r}$. Hence, $1349 ÷ .95 = $1420, Ans.

21. My agent remitted me $2444.80 as the net proceeds of a sale of grain. His commission was $4\frac{1}{2}$%. What was the amount of the sale?

22. A farmer received $239.40 as the net proceeds of the sale of *140 bbl. of* apples. How much did they sell for per barrel, the *commission* being 5%?

23. The net proceeds of a shipment were $926.50. The commission was 4%, and other charges amounted to $12.50. What was the gross amount of the sales?

24. An agent received $38.25 commission for sales amounting to $850. What was the rate of commission?

Suggestion.—$r = \frac{P}{B}$. Hence, $38.25 ÷ $850 = .045, or 4½%, Ans.

25. A factor sold property for $4500, and retained as his commission $112.50. Find the rate of commission.

26. An agent bought grain on which he paid $25.60 storage, and he charged $70.75 commission. The entire bill was $2926.35. What was the rate of commission?

27. The gross proceeds of a sale were $105, and the net proceeds $102.90. What was the rate of commission?

28. The prime cost of a purchase was $872, and the gross cost $909.06. What was the rate of commission?

29. An agent's commission was $46.80 for selling 360 yards of cloth at $3.25 per yard. What was his rate of commission?

30. The net proceeds of a sale were $465, the rate of commission 5%, and the other charges $7.50. What were the gross proceeds?

31. The gross cost of a purchase was $367.50, the rate of commission 2½%, and the other charges $3.50. What was the prime cost?

32. What is the brokerage for selling 12800 bu. of wheat at 75¢ per bushel, if the rate of brokerage is ⅛%?

33. Find the net proceeds of the following:

Account Sales.

PHILADELPHIA, PA., June 21, 1900.

Sold for account of D. E. CLARK & CO.,

By EDWARD BURNS.

1900									
June	2	600 bu. Wheat	.80	480					
	5	800 bu. Corn	.45	360					
	7	376 bu. Oats	.32	120	32	*	*	*	* *
		Charges:							
June	1	Freight,		74	20				
		Cartage,		12	35				
	7	Commission 5%,		* *	* *	*	*	*	* *
		Net Proceeds,				*	*	*	* *

Put the following in the form of Account Sales:

34. H. W. Cannon & Co., New York, N. Y., sold for account of O. S. Carter, the following: 1900, Jan. 4, 36 bbl. Royal Flour @ $5.25; 55 bbl. Family Flour @ $4.90; 75 bbl. Minn. Flour @ $5.75; Jan. 6, 100 bbl. Royal Flour @ $5.25. The charges are, Jan. 2, Freight, $58.72; Cartage, $15; Commission, Jan. 6, 5%. What were the net proceeds?

35. James Stillman & Co., Chicago, Ill., sold for account of J. S. Chase & Co., St. Joseph, Mo., the following: 1904, Mar. 6, 700 bu. wheat @ $.75; 580 bu. rye @ $.62; Mar. 9, 1300 bu. wheat @ $.76; 2000 bu. corn @ $.45. The charges are, Mar. 1, Freight, $63.75; Cartage, $17.60; Mar. 9, Commission 5%. Find the net proceeds.

36. Find the gross cost of the following:

Account Purchase.

BOSTON, MASS., June 25, 1904.

Bought for account of G. S. WHITSON & Co.,

By F. H. BREEZE, JR.

160	pr. Boots	2.60				
100	pr. Shoes	1.40				
75	pr. Slippers	.90			***	**
	Charges:					
	Cartage,		1	15		
	Commission 3%,				**	**
	Gross Cost,				***	**

Put the following in the form of Account Purchase:

37. L. F. Gardner, Philadelphia, Pa., bought for Jones, White & Co., the following: 1903, Apr. 25, 3 pc. Black Silk, 124 yd., @ $1.35; 5 pc. Summer Silk, 186 yd., @ $.68; 4 pc. Alpaca, 125 yd., @ $.35; 8 pc. Calico, 436 yd., @ $.06½. The charges for Cartage were $2.75, and Commission, 2½%. What was the entire cost?

38. John T. Slater, Chicago, Ill., bought for account of Geo. Bermont & Co: 1904, May 4, 250 bbl. Pork, @ $13.50; 200 bbl. *Beef,* @ $14.25. The Cartage was $11.75; Commission 2½%. *What was the* gross cost?

INTEREST.

572. Interest is money paid for the use of money.

573. The **Principal** is the money for the use of which interest is charged.

574. The **Rate** of interest is the per cent. of the principal charged for its use for one year.

When the rate of interest is specified in a note or other written contract without the period of time being mentioned with it, the rate is understood to be by the year.

575. The **Time** is the period during which the principal is at interest.

576. The **Amount** is the sum of the principal and interest.

The interest on \$500 for one year at 6% is \$30. Here \$500 is the principal, \$30 is the interest, and \$500 plus \$30, or \$530, is the amount.

577. Simple Interest is interest on the principal only, for the full time of the loan or credit.

When the kind of interest is not specified in a contract, simple interest is understood.

578. Common Interest is the simple interest computed on a bases of 30 days for a month, and 12 months or 360 days for a year.

This is the method generally used by business men in finding the interest on notes and debts that bear interest for more than a year.

579. Bankers' Interest is the simple interest computed by counting the exact number of days as the time, but allowing only 360 days for a year.

This method is used by bankers, and also by other business men in finding the interest for short periods of time.

580. Accurate, or Exact, Interest is the simple interest computed for the exact time in days, and on the basis of 365 days to a year. This method is used by the United States Government.

Accurate interest for any time expressed in days is the common interest diminished by $\frac{1}{365}$, or $\frac{1}{73}$ of itself. Common interest is accurate interest increased by $\frac{1}{360}$, or $\frac{1}{72}$ of itself.

581. Compound Interest is interest on both the principal and its unpaid interest after the latter becomes due.

The difference between simple interest and compound interest may be readily seen by the following: If $500 were loaned on Jan. 1, 1904, for 2 years at 6% simple interest, the amount due on Jan. 1, 1906, would be $500, the principal, plus $60, the simple interest, or $560. At compound interest, the amount due at the end of the first year, Jan. 1, 1905, would be $500 plus $30 interest, or $530. This sum would be the new principal for the second year. The interest on $530 for 1 year at 6% is $31.80, which added to $530 gives $561.80, the amount due on Jan. 1, 1906. By a comparison of the results it is seen that the simple interest for 2 years is $60, and the compound interest for the same time is $61.80. The difference of interest, $1.80, is thus seen to be the interest on the first year's interest, $30, which has been compounded with the principal and interest for 1 year.

582. Legal Interest is the interest or rate established by law for cases in which the rate is not specified.

This rate can be collected on all debts remaining unpaid after maturity unless another rate was agreed upon.

583. The **Contract Rate** is the highest rate allowed by the law of the state to be binding in a contract.

In certain states by special contract higher rates of interest are permitted than the legal rate.

584. Usury is the charging of a higher rate of interest than allowed by law.

The penalty for usury varies in the different states. In some states no penalty is attached; in others, a forfeiture of contract; or of excess of interest; of all interest; of all interest and costs; of three times the excess of interest; of interest and principal; of interest, principal, costs, etc., depending on the statute of the state.

585. Interest is allowed on notes, debts, and other contracts when it is agreed upon by the parties. When a note is written "with interest" it draws interest from date; if interest is not *mentioned, only* after it becomes due. Book accounts draw interest *after the* expiration of the term of credit.

The **Interest Laws and Statutes of Limitations** of the states and territories are shown in the following table:

AND TERRITORIES.	INTEREST LAWS.		STATUTES OF LIMITATIONS.		
	Legal Rate.	Rate Allowed by Contract.	Judgments, Years.	Notes, Years.	Open Accts. Years.
	Per Ct.	Per Ct.			
.....	6	8	20	6*	3
.....	7	Any rate.	5	5	3
s.....	6	10	10	5	3
a.....	7	Any rate.	5	4	2
.....	8	Any rate.	6	6	6
cut.....	6	6	No law.	6	6
.....	6	6	10	6†	3
of Columbia.....	6	10	12	3	3
.....	8	10	20	5	4
.....	7	8	7	6	4
.....	7	12	6	5	4
.....	5	7	7	10	5
.....	6	8	20	10	6
.....	6	8	20	10	5
.....	6	10	5	5	3
y.....	6	6	15	15	2
a.....	5	8	10	5	3*
.....	6	Any rate.	20	6†	6
l.....	6	6	12	3	3
usetts.....	6	Any rate.	20	6	6
.....	6	8	6*	6	6
ia.....	7	10	10	6	6
pi.....	6	10	7	6	3
.....	6	8	10	10	5
.....	10	Any rate.	10	8	3
a.....	7	10	5	5	4
.....	7	Any rate.	6	6	4
npshire.....	6	6	20	6	6
sey.....	6	6	20	6	6
xico.....	6	12	7	6	4
k.....	6	6	20	6	6
rolina.....	6	6	10	3*	3
akota.....	6	12	10	6	6
.....	6	8	5	15	6
a.....	7	12	5	5	3
.....	8	10	10	6	6
ania.....	6	6	5	6†	6
land.....	6	Any rate.	20	6	6
rolina.....	7	8	20	6	6
akota.....	7	12	10	6	6
e.....	6	Any rate.	10	6	6
.....	6	10	10	4	2
.....	8	Any rate.	8	6	4
.....	6	6	8	6	6
.....	6	6	10	5*	2
ton.....	7	12	6	6	3
ginia.....	6	6	10	10	5
n.....	6	10	20	6	6
g.....	8	12	5	5	3

*er seal, 10 years. †Under seal, 20 years.

587. Interest is an application of percentage in which the element of *time* is introduced. The quantities considered are the Principal (Base), the Rate, the Time, the Interest (Percentage), and the Amount.

In the following problems, interest will mean common interest unless otherwise stated.

588. Given the principal, the rate per cent., and the time to find the interest or the amount.

COMMON METHOD.

589. 1. What is the interest of \$480 for 3 yr. 4 mo. 15 da., at 7%?

\$480.
.07
\$33.60
$3\frac{5}{8}$
1260
10080
\$113.40

Solution.—At 7%, the interest for 1 yr. is .07 of \$480, or \$33.60; and for 3 yr. 4 mo. 15 da., or $3\frac{5}{8}$ yr. (Art. 285), it is $3\frac{5}{8}$ times \$33.60, or \$113.40.

RULE.—*To find the interest, multiply the principal by the rate per cent. expressed decimally, and that product by the time expressed in years.*

To find the amount, add the principal and the interest.

Formula.—$B \times r = P$, or interest for 1 yr.

EXAMPLES.

Find the interest of

2. \$340 for 2 yr. at 5%.
3. \$575 for 3 yr. at 7%.
4. \$420 for 5 yr. at 6%.
5. \$760 for 4 yr. at 8%.
6. \$976 for 7 yr. at 7%.
7. \$684 for 3 yr. 4 mo. at 7%.
8. \$890 for 5 yr. 7 mo. at 4%.
9. \$12.60 for 7 yr. 5 mo. at 9%.
10. \$87.50 for 6 yr. 11 mo. at 7%.
11. \$78.75 for 8 yr. 10 mo. at 8%.
12. Find the interest of \$980 for 2 yr. 4 mo. 15 da. at 6%.
13. Find the interest of \$675.40 for 3 yr. 6 mo. 20 da. at 5%.
14. Find the interest of \$786.25 for 2 yr. 7 mo. 18 da. at 7%.
15. Find the amount of \$480 for 4 yr. 3 mo. 15 da. at 5%.
16. A farmer borrowed \$300 for 2 yr. 4 mo. at 5%. What amount will be due the lender at the end of that time?

What is the interest of $960 from May 15, 1890, to Oct. 15, at 6%?

jestion.—Find the time by Compound Subtraction (Art. 353).

What is the interest of $1200 from Apr. 1, 1900 to July 11 at 7%?

What is the amount of $1350 from Sept. 12, 1900, to Dec. 903, at 5½%?

On June 5, 1903, James Field gave his note for $460 with ɔst at 6%. He paid the note Sept. 17, 1905. What amount ɔ pay?

BUSINESS METHOD.

). A modification of the *Common Method* of finding the inter- any sum of money at any rate per cent. is the **Business od, or Method of Aliquot Parts.**

s method is used by many business men, and is preferred by many to the *Common Method.* The method will be explained, and if there icient time, the examples under Art. 589 should be solved by the ess Method.

What is the interest on $480 for 3 yr. 4 mo. 15 da. at 7%?

:80
07
60 = Int. for 1 yr.
3
80 = " " 3 yr.
20 = " " 4 mo.
40 = " " 15 da.
40, Ans.

EXPLANATION.—We find the interest as by the common method for the years. 4 mo. are ⅓ of a year, hence the interest is ⅓ of $33.60, or $11.20. 15 days are ½ of a mo. or ½ of 4 mo., hence the interest for 15 da. is ½ of $11.20 or $1.40. Adding, we have the total interest for the required time, or $113.40.

What is the interest of $275.50 for 4 yr. 5 mo. 21 da. at 5%?

5.50
.05
7750 = Int. for 1 yr.
4
1000 = " " 4 yr.
5916 = " " 4 mo.
1479 = " " 1 mo.
5739 = " " 15 da.
2295 = " " 6 da.
3429
+, or $.2295.
.64

EXPLANATION.—Find the interest for the whole number of years as before. 5 mo. are not an aliquot part of a year, hence we take 4 mo. and 1 mo. The interest for 4 mo., or ⅓ yr., is $4.5916. 1 mo. is ¼ of 4 mo., hence the interest is ¼ of $4.59+ or $1.1479. 21 da. are not an aliquot part of a month, hence we take 15 da. and 6 da. 15 da. are ½ of a mo., hence the interest for 15 da. is ½ of $1.14+, or $.5739. 6 da. are ⅒ of a mo., hence the interest for 6 da. is ⅒ of Adding, we have the total interest for 4 yr. 5 mo. 21 da.

SIX PER CENT. METHOD.

591. This method is perhaps more generally used by accountants than any other because of the ease with which the interest on $1 can be computed.

TABLE.

The interest on $1 for 1 yr. = $.06.
The interest on $1 for 2 mo. = .01.
The interest on $1 for 1 mo. = .005, or ½¢.
The interest on $1 for 6 da. = .001.
The interest on $1 for 1 da. = .000⅙, or ⅙ of a mill.

1. What is the interest of $350 for 5 yr. 6 mo. 24 da. at 6%?

$$\begin{aligned} 5 \times \$.06 &= \$.30 \\ 6 \times .00\tfrac{1}{2} &= .03 \\ 24 \times .000\tfrac{1}{6} &= .004 \\ & \$.334 \\ 350 \times \$.334 &= \$116.90 \end{aligned}$$

EXPLANATION.—By reference to the table it is seen that the interest of $1 for 5 yr. 6 mo. 24 da. is $.334; and the interest of $350 is 350 times $.334, or $116.90.

RULE.—*Multiply the rate by the number of years, take ½ of the months as cents, and ⅙ of the number of days as mills. The sum will be the interest of $1 for the given time at 6%. Multiply this sum by the principal, and the product will be the interest at 6%.*

The interest for any other rate may be found from the interest at 6% as follows: At 1%, divide by 6; at 1½%, divide by 4; at 2%, divide by 3; at 3%, divide by 2; at 4%, subtract ⅓; at 4½%, subtract ½; at 5%, subtract ⅔; at 5½%, subtract $\frac{1}{12}$; at 6½%, add $\frac{1}{12}$; at 7%, add ⅙; at 7½%, add ¼; at 8%, add ⅓; at 9%, add ½; at 10%, divide by 6, and multiply by 10 by placing the point one place to the right. At any per cent., divide by 6 and multiply by the number expressing the rate.

EXAMPLES.

Find the interest of

2. $500 for 2 yr. 4 mo. 12 da. at 6%.
3. $450 for 1 yr. 6 mo. 18 da. at 6%.
4. $375 for 3 yr. 2 mo. 24 da. at 6%.
5. $135.50 for 4 yr. 8 mo. 6 da. at 6%.
6. $243.25 for 2 yr. 5 mo. 8 da. at 6%.
7. $372.40 for 5 yr. 4 mo. 20 da. at 6%.
8. $433.75 for 3 yr. 6 mo. 24 da. at 6%.
9. $580.30 for 4 yr. 7 mo. 15 da. at 6%.
10. $276.80 for 5 yr. 10 mo. 27 da. at 6%.
11. $442.25 for 3 yr. 4 mo. 18 da. at 5%.
12. $170.60 for 2 yr. 11 mo. 21 da. at 7%.
13. $340.25 for 4 yr. 9 mo. 12 da. at 5½%.

14. $275.40 for 1 yr. 7 mo. 16 da. at 4%.

15. $421.50 for 2 yr. 10 mo. 25 da. at 8%.

16. Find the amount of $640 for 4 yr. 9 mo. at 6%.

17. Find the amount of $437.50 for 3 yr. 4 mo. at 5%.

18. Find the amount of a note of $360 for 5 yr. 7 mo. 18 da. at 4%.

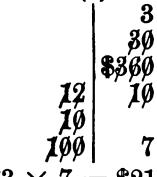
19. Find the amount of a note of $735 for 3 yr. 4 mo. 18 da. at 5½%.

20. A note for $250 at 7%, was given June 26, 1898, and paid April 2, 1900. What was the amount paid?

CANCELLATION METHOD.

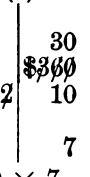
592. 1. Find the interest of $360 for 10 mo. at 7%.

(1)



$3 \times 7 = \$21$

(2)



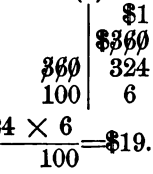
$\$30 \times 10 \times 7 = \$21.00$

EXPLANATION.—The interest on any sum is always the product of the principal, time (in years), and rate. Here the time is only 10 mo., or $\frac{10}{12}$ of a year, hence we make 12 a divisor. The rate is 7%, or $\frac{7}{100}$, hence 100 is a divisor. By cancellation we find the result to be $21 interest. By inspection of the work, we see that the same result may be obtained by placing the principal, time in months, and rate on the right of the line, 12 on the left, then cancel and move the decimal point two places to the left.

2. Find the interest of $360 for 10 mo. 24 da. at 6%.

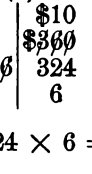
10 mo. 24 da. = 324 da.

(1)



$\frac{\$1 \times 324 \times 6}{100} = \$19.44$

(2)



$\$10 \times 324 \times 6 = \$19.440$

EXPLANATION.—In this problem the time is $\frac{324}{360}$ of a year, hence 360 (days) is a divisor. By cancellation, we find the result to be $19.44 interest. Inspecting the work, we see that the same result may be obtained by placing the principal, time in days, and rate on the right of the line, 36 on the left, then cancel and move the decimal point three places to the left.

RULE.—*Upon the right of a vertical line write the principal, time in months or days, and the rate. If the time be in months, place 12 upon the left of the line. Then cancel, multiply the remaining factors together, and move the decimal point two places to the left.*

If the time be in days, place 36 upon the left of the line, cancel, multiply, and move the decimal point three places to the left.

The time must be expressed either in months or days. If accurate interest is required use 365 instead of 36, and move the decimal point only two places to the left.

EXAMPLES.

3. Find the interest of $240 for 4 mo. at 6%.
4. Find the interest of $510 for 7 mo. at 6%.
5. Find the interest of $160.75 for 9 mo. at 4%.
6. Find the interest of $346.64 for 5 mo. at 5%.
7. Find the interest of $450 for 2 mo. 12 da. at 8%.
8. Find the interest of $217.49 for 3 mo. 10 da. at 6%.
9. Find the amount of a note whose face is $300, for 2 mo. 15 da. at 6%.
10. Find the amount of a note whose face is $224.38, for 2 mo. 25 da. at 7%.

BANKERS' 60-DAY METHOD.

593. This is called the **Bankers' Method**, or the **60-Day Method**, because the process is based on the interest of any principal for 60 days at 6%. (See Bankers' Interest, Art. 597.)

Banking business being nearly all transacted on the basis of 30, 60, or 90 days, the work can be very much shortened, and sometimes the interest obtained without any calculation.

594. Interest at 6% for 1 year, is equivalent to 1% for 2 months, or 60 days. 1% of any principal is .01, or $\frac{1}{100}$ of it. Hence, the interest of any sum for 2 months, or 60 days, may be readily found by placing the decimal point two places to the left. Since 6 days is $\frac{1}{10}$ of 60 days, the interest of any sum for 6 days at 6% is $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$ of it, which may be found by placing the decimal point three places to the left.

1. A perpendicular line may be used to advantage in separating the dollars and cents in solving examples by this method.

2. No shorter method than this can be given for brief periods of time, and the student should thoroughly master it. Longer periods will also be given for practice.

595. Aliquot Parts of 60.—$1 = \frac{1}{60}$; $2 = \frac{1}{30}$; $3 = \frac{1}{20}$; $4 = \frac{1}{15}$; $= \frac{1}{12}$; $6 = \frac{1}{10}$; $10 = \frac{1}{6}$; $12 = \frac{1}{5}$; $15 = \frac{1}{4}$; $20 = \frac{1}{3}$; $30 = \frac{1}{2}$.

1. If the number of days given be not an exact number of times 60 or an quot part of 60, separate the number so that the component parts of the mber will be multiples or aliquot parts of 60. Thus, $7 = 6 + 1$; $11 =$ $+ 5$; $17 = 12 + 5$, or $15 + 2$; $29 = 20 + 6 + 3$; $90 = 60 + 30$; $120 = 60 +$; $130 = 60 + 60 + 10$; $175 = 60 + 60 + 30 + 20 + 5$.

2. To divide by 10, place the figures of the basis one place to the right.

3. To divide by 20, 30, or 60, divide by the first figure and write the quo- nt figures one place to the right.

1. Find the interest of \$1680 for 85 da., at 6%.

6	80	= int. for	60	da.	
5	60	= "	" 20	"	
1	40	= "	" 5	"	
3	80	= "	" 85	"	

EXPLANATION.—The interest of \$1680 for 60 da. at 6%, is seen to be \$16.80, by the process already explained. Since 25 da. is not an aliquot part of 60 da., we will separate it into 20 da. and 5 da., each of which is an aliquot rt of 60 da. If the interest for 60 da. is \$16.80, for 20 da. ($\frac{1}{3}$ of 60) it will $\frac{1}{3}$ of \$16.80, or \$5.60. The interest for 5 da. ($\frac{1}{12}$ of 60 or $\frac{1}{4}$ of 20) will be $\frac{1}{12}$ \$16.80 (or $\frac{1}{4}$ of \$5.60), or \$1.40. Hence the interest for 85 da. is \$16.80 + .60 + \$1.40, or \$23.80.

2. Find the interest of \$2450 for 2 yr. 5 mo. 18 da. at 6%

24	50	= int. for 2 mo., or 60 da.	
	14½	= ½ of 29 mo. (2 yr. 5 mo.)	
12	25	= int. for 1 mo., or 30 da.	
98	00		
45	0		
6	125	= int. for 15 da.	
1	225	= " " 3 "	
362	60	= " " 2 yr. 5 mo. 18 da.	

EXPLANATION.—The interest of \$2450 for 2 mo. is \$24.50. For 2 yr. 5 mo., (29 mo.) which is 14½ time 2 mo., the interest will be 14½ times \$24.50. The interest for 15 da. ($\frac{1}{4}$ of 30) is $\frac{1}{4}$ of \$12.25, and for 3 da. ($\frac{1}{20}$ of 15) is $\frac{1}{20}$ of \$6.125. Adding, we have \$362.60, the entire inter- t for the required time.

RULE.—*Draw a perpendicular line two places to the left of the cimal point. The result will be the interest for 2 months, or 60 ys, at 6%, the dollars being on the left and the cents on the right c this line. Multiply this result by one-half of the total number c months and add to the product that proportion of the interest c 60 days, which the given number of days, or its complements, is c 60.*

The process may often be shortened by taking as the basis the interest r 6 days instead of 60. When the interest for 6 days is desired, draw the rpendicular *line three* places to the left of the decimal point.

EXAMPLES.

Find the interest of the following at 6%:

3. $234 for 6 days.
4. $185 for 13 days.
5. $159 for 45 days.
6. $420 for 60 days.
7. $360 for 80 days.
8. $724 for 75 days.
9. $500 for 90 days.
10. $67.50 for 97 days.
11. $95.75 for 36 days.
12. $2000 for 57 days.

What is the interest of

13. $1260 for 6 mo. 20 da. at 7%?
14. $1440 for 10 mo. 15 da. at 5%?
15. $2360 for 11 mo. 24 da. at 8%?
16. $1890 for 1 yr. 2 mo. 10 da. at 4½%?
17. $78.50 for 2 yr. 3 mo. 25 da. at 3%?
18. $61.80 for 9 mo. 27 da. at 7½%?
19. $915 for 7 mo. 23 da. at 10%?
20. $1165 for 2 yr. 7 mo. 27 da. at 5½%?
21. $340 from Apr. 2, 1900, to July 2, 1900, at 6%?
22. $560 from June 16, 1900, to Aug. 14, 1900, at 6%?

ACCURATE INTEREST.

596. Accurate, or Exact, Interest is the simple interest computed for the exact time in days, and on the basis of 365 days to a year.

Accurate interest is used by the United States Government and by some bankers and merchants.

1. What is the accurate interest of $650 from May 14, 1900, to Oct. 5, 1900, at 5%?

May 17 days.	144 days $= \frac{144}{365}$ yr.	EXPLANATION.—From May 14, to Oct. 5, there are 144 days. The interest of $650 for 1 yr. at 5% is $32.50, and for 144 days, it is $\frac{144}{365}$ of $32.50, or $12.82.
June 30 days.		
July 31 days.	$650 × .05 = $32.50	
Aug. 31 days.		
Sept. 30 days.	$32.50 × $\frac{144}{365}$ = $12.82	
Oct. 5 days.		
144 days.		

RULE.—*Multiply the principal by the rate per cent. expressed decimally. The result will be the interest for one year. Then multiply the interest for one year by the exact number of days and divide by 365.*

The accurate interest for any number of days may also be found from the *common interest* by subtracting $\frac{1}{73}$ of itself. If preferred the cancellation *method may* be used.

EXAMPLES.

What is the accurate interest on

2. \$340 for 35 days at 4%?
3. \$285 for 127 days at 5%?
4. \$196 for 85 days at 6%?
5. \$225 for 140 days at 6%?
6. \$480 for 210 days at 4½%?
7. \$640 for 113 days at 7%?
8. \$530 from June 24, 1900, to Sept. 10, 1900, 6%?
9. \$1200 from July 10, 1903, to Apr. 1, 1904, at 7%?
10. \$735 from May 4, 1900, to July 5, 1900, at 5½%?
11. Find the interest on a \$1000 U. S. bond from June 1 to ept 5, at 4%.
12. What is the interest on four \$1000 U. S. bonds from Jan. , 1904, to April 15, 1904, at 4½%?
13. What is the interest on £200 6*s.* 3*d.* for 60 days at 7%?

Suggestion.—Reduce the shillings and pence to the decimal of a pound, ıen find the interest as in U. S. money, after which reduce the decimal of pound to lower denominations. See Art. 284, p. 125.

14. Find the interest on £360 9*s* 6*d.* from Mar. 8 to June 10, ₺ 5%.
15. Find the interest on £500 10*s* 9*d* from July 1 to Oct. 31, ₺ 4%.

COMMON, BANKERS', AND ACCURATE INTEREST COMPARED.

597. In Common interest we reckon 30 days to a month and ıd the time by compound subtraction. In Bankers' interest we ıd the exact number of days, but calculate the interest on the ısis of 360 days to a year. In Accurate interest we find the exact ımber of days and calculate the interest on the basis of 365 days a year.

1. Find the common, the bankers' and the exact interest on a ɔte of \$500, dated Jan. 3, 1900, and due Sept. 24, 1900, at 6%?

COMMON INTEREST.

1900 — 9 — 24	\$5 00 = int. for	60 da.
1900 — 1 — 3	15 00 = " "	180 "
8 21	1 25 = " "	15 "
	50 = " "	6 "
8 mo. *21 da.* = *261* da.	\$21 75 = " "	261 da.

BANKERS' INTEREST.

Jan.	28 da.	$5	00	= int. for	60 da.
Feb.	28 "	15	00	= " "	180 "
Mar.	31 "	1	$66\frac{2}{3}$	= " "	20 "
Apr.	30 "		$33\frac{1}{3}$	= " "	4 "
May	31 "	$22	00	= " "	264 da.
June	30 "				
July	31 "				
Aug.	31 "				
Sept.	24 "				
	264 da.				

ACCURATE INTEREST.

Jan.	28 da.
Feb.	28 "
Mar.	31 "
Apr.	30 "
May	31 "
June	30 "
July	31 "
Aug.	31 "
Sept.	24 "
	264 da.

264 da. $= \frac{264}{365}$ yr.

$\$500 \times \frac{6}{100} \times \frac{264}{365} = 21.698,$

or $21.70, Ans.

EXAMPLES.

2. What is the difference between the common interest and the accurate interest of $800 from May 5 to Aug. 25, at 5%?

3. What is the difference between bankers' interest and accurate interest on $1200 from June 20, to Sept. 20, at 6%?

4. What is the common interest on a note of $236.50, dated Feb. 2, 1899, and due Dec. 1, 1899, at 5%?

5. Find the bankers' interest on $1000 from Mar. 3, 1896, to June 18, 1896, at 6%.

6. Find the accurate interest on $609.45 from April 20, to July 27, at 6%.

7. Which is the greater, and how much, common interest or bankers' interest on a note of $648 from Feb. 14, 1899, to June 30, 1899, at 6%?

8. Mr. Leidig borrowed $2500 at common interest and loaned it at bankers' interest. How much did he gain from April 1 to Aug. 1, interest at 6%?

9. Mr. Wood borrowed $4000 at 5% common interest, and loaned *it at 7% bankers'* interest from Aug. 15, 1899, to May 1, 1900. *How much did* he gain?

PROBLEMS IN SIMPLE INTEREST.

598. Given the principal, the time, and the interest or amount, to find the rate.

1. What is the rate per cent. when the interest on \$360 for 1 yr. 4 mo. is \$28.80?

\$3|60 = int. 1 yr. 1%
1|20 = " 4 mo.
4|80 = int. 1 yr. 4 mo. 1%
\$28.80 ÷ \$4.80 = 6 = 6%

Solution.—The interest of \$360 for 1 yr. 4 mo. at 1% is \$4.80, and to give \$28 50 it will require as many times 1% as \$4.80 are contained times in \$28.80, which are 6 times or 6%.

RULE.—*Find the interest on the principal for the given time at 1%. and divide the given interest by it.*

If the amount be given, subtract the principal from it to find the interest.

EXAMPLES.

2. What is the rate per cent. when the interest on \$175 for 2 yr. is \$14?

3. Find the rate per cent. when the interest on \$450 for 1 yr. 6 mo. is \$40.50.

4. Find the rate per cent. when \$680 produce \$107.10 interest in 2 yr. 3 mo.

5. Find the rate of interest on a note of \$240 which produced \$18.70 interest in 1 yr. 5 mo.

6. Mr. Slyder lent me \$650 Jan. 2, 1900. On June 26, 1900, I paid the amount due, which was \$668.85. What rate of interest did I pay?

599. Given the principal, the rate, and the interest or amount, to find the time.

1. In what time will \$220 produce \$13.75 interest at 5%?

\$220 × .05 = \$11
\$13.75 ÷ \$11 = 1¼ = 1¼ yr., or 1 yr. 3 mo.

Solution.—The interest of \$220 for 1 yr. at 5% is \$11, and to give \$13.75 interest it will require as many times 1 yr. as \$11 are contained times in \$13.75, which are 1¼ times or 1¼ yr., or 1 yr. 3 mo.

RULE.—*Find the interest on the principal at the given rate for one year, and divide the given interest by it.*

If the amount be given, subtract the principal from it to find the interest.

EXAMPLES.

2. In what time will $305 produce $54.90 interest at 6%?

3. In what time will $475 produce $95 interest at 5%?

4. In what time will $562.25 produce $78.71 interest at 4%?

5. In what time will $780 produce $218.40 interest at 6%?

6. In what time will $1260 produce $357 interest at 8%?

7. How long must $1350 be at interest at 5% to amount to $1545.75?

600. Given the rate, the time, and the interest or amount, to find the Principal.

1. What principal will produce $19.55 interest in 1 yr. 5 mo. at 6%?

.06 = int. on $1 for 1 yr.
.025 = " " $1 " 5 mo.
.085 = " " $1 " 1 yr. 5 mo.
$19.55 ÷ .085 = $230.

Solution.—The interest on $1 for 1 yr. and 5 mo. at 6%, is $.085, and to produce $19.50 interest it will require as many dollars as .085 are contained times in $19.55, or $230.

RULE.—*Find the interest on $1 for the given rate and time, and divide the given interest by it.*

If the amount be given, divide it by the amount of $1 for the given rate and time.

EXAMPLES.

2. What principal will produce $39.24 interest in 2 yr. at 6%?

3. What principal will produce $49.50 interest in 2 yr. 3 mo. at 5%?

4. What principal will produce $65.40 interest in 3 yr. 4 mo. 15 da. at 6%?

5. I paid $33.45 interest on a note for a period of 1 yr. 10 mo. 9 da. at 6%. What was the face of the note?

6. What principal will amount to $930.23 from Oct. 1, 1898, to May 19, 1900, at 5%?

7. How much may I offer for a house which rents for $460 a year to realize 7% interest on my investment if the taxes are $40 a year?

ANNUAL INTEREST.

601. Annual, or **Periodic, Interest** is simple interest on the principal payable annually or at other regular periods of time.

602. If the Annual Interest is not paid when due, it draws interest from the time it becomes due until paid.

603. Periodic Interest is legalized in some States, but in many of the States it can not be legally enforced. It is, however, sometimes secured by a series of notes. In such cases not only is the principal secured by note (unless by mortgage or otherwise) but a note is also received for each interest payment maturing on the date upon which such payment is due. By this plan, Periodic interest can be enforced even where not sanctioned by law.

604. In States where periodic interest is legal, the contract should contain the words "interest payable annually," "annual interest," "interest payable semi-annually," or "interest payable quarterly," etc.

605. 1. Find the amount of \$1200 for 3 yr. 6 mo. 12 da. at 6%, interest payable annually and no payments being made until final settlement.

Int. of \$1200 for 3 yr. 6 mo. 12 da. =		\$254.40
\$1200 × .06 = \$72, 1 year's interest.		
1st yr.'s int. rem'd unpaid	2 yr. 6 mo. 12 da.	
2d " " " "	1 " 6 " 12 da.	
3d " " " "	6 " 12 da.	
	4 yr. 7 mo. 6 da.	
Int. of \$72 for 4 yr. 7 mo. 6 da. =		19.87
Total interest due,		\$274.27
Prin.,		\$1200.
Amt. due,		\$1474.27

EXPLANATION.—The simple interest on the principal for the full time is \$254.40. The interest for the 1st year (\$72) remained unpaid for 2 yr. 6 mo. 12 da. after it was due; the 2d year's interest (\$72) remained unpaid for 1 yr. 6 mo. 12 da. after it was due; and the 3d year's interest remained unpaid for 6 mo. 12 da. after it was due. This is equivalent to 1 year's interest (\$72) on interest for 4 yr. 7 mo. 6 da., or \$19.87. This added to the interest on the principal for the full time gives \$274.27, the total interest due. Since the principal is \$1200, the amount due is \$1474.27.

RULE.—*Find the simple interest on the principal for the full time, and add to it the interest on one period's interest for the aggregate of time for which the payments of interest were deferred. The result will be the entire interest due.*

If the amount be desired, add the principal to the entire interest.

EXAMPLES.

2. Find the amount due on a note of $500 for 2 yr. 6 mo. at 6%, interest payable annually, no payments having been made.

3. What is the amount due on a note of $350 for 3 yr. 4 mo. at 6%, interest payable annually, no payments having been made?

4. Find the amount due on a note of $600 for 4 yr. 2 mo. at 5%, interest payable annually, no payments having been made.

5. What is the amount due on a note of $475 for 3 yr. 8 mo. 10 da. at 6%, interest payable annually, no payments having been made?

6. Find the amount due on a note of $265.76 for 4 yr. 3 mo. 15 da. at 4½%, interest payable annually, no interest having been paid.

7. Find the amount due on a note of $800 for 1 yr. 8 mo. at 6%, interest payable semi-annually, and no payments having been made.

Suggestion.—At the expiration of each 6 mo., $24 is due, but not paid until the maturity of the note.

8. Find the amount of interest due at the end of 4 yr. 8 mo. on a note of $750 at 5%, interest payable annually, but remaining unpaid.

9. What is the entire interest due on a note of $400, interest payable quarterly from Jan. 1, 1898, to July 1, 1900, at 6%, no interest having been paid?

10. On a note of $960, dated June 24, 1894, written with interest payable annually at 6%, only the first two payments were made when due. What was the amount of the note Sept. 30, 1900?

COMPOUND INTEREST.

606. Compound Interest is interest upon the principal for the first period of time, which principal is compounded by having the unpaid interest added to it after the interest becomes due, thus forming a new principal for each subsequent period.

1. In other words, the principal increases by the addition of interest accruing, if unpaid, at the termination of each interest period, the amount thus found becoming the new principal on which to reckon interest for the following period, and so on throughout the entire time the contract may run.

2. Interest is usually compounded at the ends of certain periods, as for each year, half year, or quarterly, according to agreement.

3. Even though stipulated for in the contract, interest upon interest due *cannot be enforced* by law (*i. e.*, collected) in most of the States.

607. What is the compound interest of $500 for 3 years at 6%?

$500	
.06	
$30.	= Int. 1st yr.
500	
$530	= Amt. 1st yr.
.06	
$31.80	= Int. 2d yr.
530.	
$561.80	= Amt. 2d yr.
.06	
$33.7080	= Int. 3d yr.
561.80	
$595.51	= Amt. 3d yr.
500.	
$95.51	= Compound Int.

EXPLANATION.—We find the simple interest on the principal for 1 year, and adding it to the principal the amount is $530, which is the principal for the second year. Multiplying the new principal by the rate gives the interest for the second year, which added to the new principal gives the amount due at the end of the second year, $561.80. Proceeding in the same manner, we find the amount due at the end of three years to be $595.51. Subtracting the first principal, the remainder, $95.51, is the compound interest.

RULE.—*Find the amount of the principal for the first period of time for which interest is to be reckoned and make this the principal for the second. Find the amount of this principal for the second period; and thus continue to the end of the given time. The last amount will be the required amount.*

To find the compound interest subtract the given principal from the last amount.

1. When the interest is due semi-annually, quarterly, etc., find the interest for such time, and proceed as above directed.

2. When the time contains a fractional portion of a period as months and days, first find the amount due for the full periods, and then the simple interest upon this for the partial period.

EXAMPLES.

2. What is the compound interest of $350 for 2 years at 6%?

3. What is the compound interest of $500 for 3 years at 5%?

4. What is the compound interest of $850 for 2 yr. 6 mo. at 6%, payable semi-annually?

5. What is the amount of $400 for 7 yr. at 6% compound interest?

$1.5036303	
400	
$601.4521200	

EXPLANATION.—By referring to the table on page 252 it is seen that the amount of $1 for 7 yr. at 6% is $1.50363+. The amount of $400 is 400 times $1.5036303, or $601.45.

6. What is the amount of \$840 for 12 yr. at 5% compound interest?

7. What is the amount of \$375 for 15 yr. at 7% compound interest?

8. What is the amount of \$900 for 20 yr. at 3½% compound interest?

9. What is the interest of \$500 for 8 yr. at 5%, if compounded annually?

10. What is the compound interest of \$4000 for 25 yr. at 6%?

\$4.2918707
1.
\$3.2918707
4000
\$13167.4828000

EXPLANATION.—Subtracting \$1 from the compound amount of \$1 for 25 yr. at 6%, we have the compound interest of \$1 for the given time. This multiplied by 4000 gives the required interest, or \$13167.48.

11. Find the compound interest of \$3000 for 12 yr. at 4½%.

12. Find the compound interest of \$1400 for 14 yr. if compounded semi-annually at 6%.

Suggestion.—14 yr. at 6% payable semi-annually is equivalent to 28 yr. at 3%.

13. How much interest will \$1000 produce in 10 yr. at 6%, if compounded semi-annually?

14. Find the amount of \$250 for 5 yr. 4 mo. at 6% if compounded semi-annually.

15. Find the compound interest of \$1640.25 for 5 yr. 3 mo. if compounded quarterly at 8%.

16. If \$100 were deposited in a savings bank and the interest compounded quarterly at 4% per annum for 21 years, what would be the amount at the expiration of the time?

Suggestion.—See note 1, page 252.

17. Mr. L. deposited \$500 in a savings bank to be left on interest for 10 years. If compounded quarterly at 4% per annum, what will be the amount at the expiration of the time?

608. *Computations* in compound interest may be greatly shortened by the use of the following

Compound Interest Table,

Showing the amount of $1 at compound interest, at various rates per cent. for any number of years, from 1 to 50 years, inclusive.

Yrs.	1 per ct.	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per. ct.	Yrs.
1	1.0100 000	1.0150 000	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000	1
2	1.0201 000	1.0302 250	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1 0816 0000	2
3	1.0303 010	1.0456 784	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1 1248 6400	3
4	1.0406 040	1.0613 636	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856	4
5	1.0510 101	1.0772 840	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290	5
6	1.0615 202	1.0934 433	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902	6
7	1.0721 354	1.1098 450	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178	7
8	1.0828 567	1.1264 926	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905	8
9	1.0936 853	1.1433 900	1.1950 9257	1·2488 6297	1.3047 7318	1.3628 9735	1.4233 1181	9
10	1.1046 221	1.1605 408	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428	10
11	1.1156 683	1.1779 489	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1 5394 5406	11
12	1.1268 250	1.1956 182	1.2682 4179	1.3448 8882	1.4257 6089	1.5110 6866	1.6010 3222	12
13	1.1380 933	1.2135 524	1.2936 0663	1.3785 1104	1 4685 3371	1.5639 5606	1.6650 7351	13
14	1.1494 742	1.2317 557	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645	14
15	1.1609 690	1.2502 321	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351	15
16	1.1725 786	1.2689 855	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8601	1.8729 8125	16
17	1.1843 044	1.2880 203	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050	17
18	1.1961 475	1.3073 406	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652	18
19	1.2081 090	1.3269 507	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918	19
20	1.2201 900	1.3468 550	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314	20
21	1.2323 919	1.3670 578	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 0807	21
22	1.2447 159	1.3875 637	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879	22
23	1.2571 630	1.4083 772	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555	23
24	1.2697.346	1.4295 028	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417	24
25	1.2824 320	1.4509 454	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633	25
26	1.2952 563	1.4727 095	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6979	26
27	1.3082 089	1.4948 002	1.7068 8648	1.9478 000!	2.2212 8901	2.5315 6711	2.8833 6858	27
28	1.3212 910	1 5172 222	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332	28
29	1.3345 039	1.5399 805	1.7758 4469	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145	29
30	1.3478 490	1.5630 802	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9370	3.2433 9751	30
31	1.3613 274	1.5865 264	1.8475 8882	2.1500 0677	2.5000 8035	2 9050 3148	3.3731 3341	31
32	1.3749 407	1.6103 243	1.8845 4059	2.2037 5694	2 5750 8276	3.0067 0759	3.5080 5875	32
33	1.3886 901	1.6344 792	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483.8110	33
34	1.4025 770	1.6589 964	1.9606 7603	2 3153 2213	2.7319 0530	3.2208 6033	3.7943 1634	34
35	1.4166 028	1.6838 813	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899	35
36	1.4307 688	1.7091 395	2.0398 8734	2.4325 3532	2.8982 7833	3.4502 6611	4.1039 3255	36
37	1.4450 765	1.7347 766	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.:680 8986	37
38	1.4595 272	1.7607 983	2.1222 9879	2.5556 8242	3.0747 8348	3.6960 1132	4 4388 1345	38
39	1 4741 225	1.7872 103	2.1647 4477	2.6195 7448	3.1670 2698	3.8253 7171	4.6163 6599	39
40	1.4888 637	1.8140 184	2.2080 3966	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063	40
41	1.5037 524	1.8412 287	2.2522 0046	2.7521 9043	3.3598 9893	4.0978 3381	4.9930 6145	41
42	1.5187 899	1.8688 471	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391	42
43	1.5339 778	1.8968 798	2.3431 8936	2.8915 2008	3.5645 1677	4.3897 0202	5.4004 9527	43
44	1.5493 176	1.9253 330	2.3900 5314	2.96 8 0808	3.6714 5227	4.5433 4160	5.6165 1508	44
45	1.5648 107	1.9542 130	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568	45
46	1.5804 589	1.9835 262	2.4866 1129	3.1138 5086	3.8950 4372	4 8669 4110	6.0748 2271	46
47	1.5962 634	2.0132 791	2.5363 4351	3.1916 9713	4.0118 9503	5.0372 8404	6 3178 1562	47
48	1.6122 261	2.0434 783	2.5870 7039	3 2714 8956	4.1322 5188	5.2135 8898	6.5705 2824	48
49	1.6283 483	2.0741 305	2.6388 1179	3.3532 7680	4.2562 1944	5.3960 6459	6.8333 4937	49
50	1.6446 318	2.1052 424	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335	50

1. When the interest is to be compounded semi-annually, find, from the Table, the amount or interest at *one-half* the rate per annum for *twice* the the number of years if quarterly, at *one-fourth* the rate per annum for *four times* the number of years. Thus, 5 yr. at 7% per annum *payable semi-annually* is equivalent to 10 yr. at 3½% *payable annually*, and 6 yr at 8% per annum *payable quarterly is* equivalent to 24 yr. at 2% payable annually.

Compound Interest Table,

Showing the amount of $1 at compound interest, at various rates per cent. for any number of years, from 1 year to 50 years, inclusive.

Yrs.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 pe ct.	Yrs.
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1 0800 000	1.0900 000	1.1000 000	1
2	1.0920 2500	1.1025 000	1.1236 000	1 1449 000	1.1664 000	1.1881 000	1.2100 000	2
3	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000	3
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000	4
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100	5
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5868 743	1.6771 001	1.7715 610	6
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171	7
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 303	1.9925 626	2.1435 888	8
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477	9
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425	10
11	1.6228 5305	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167	11
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284	12
13	1.7721 9610	1.8856 491	2.1329 283	2 4098 450	2.7196 237	3.0658 046	3.4522 712	13
14	1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983	14
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482	15
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730	16
17	2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703	17
18	2.2084 7877	2.4066 192	2.8543 392	3 3799 323	3.9960 195	4.7171 204	5.5599 173	18
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390	19
20	2.4117 1402	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000	20
21	2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499	21
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749	22
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8 9453 024	23
24	2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327	24
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059	25
26	3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11 9181 765	26
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942	27
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936	28
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930	29
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023	30
31	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19 1943 425	31
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768	32
33	4.2740 3018	5.0031 885	6.8405 899	9.3253 398	12.6760 496	17 1820 284	23 2251 544	33
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13 6901 336	18.7284 109	25.5476 699	34
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369	35
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805	36
37	5.0968 6049	6 0814 069	8 6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486	37
38	5.3262 1921	6.3854 773	9.1542 524	13 0792 714	18 6252 756	26.4366 805	37.4043 434	38
39	5.5658 9908	6.7047 512	9.7035 075	13 9948 204	20.1152 977	28.8159 817	41.1447 778	39
40	5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556	40
41	6.0781 0094	7 3919 882	10.9028 610	16.0226 699	23.4624 832	34.2362 679	48.7851 811	41
42	6.3516 1548	7.7615 876	11.5570 327	17.1442.568	25.3394 819	37.3175 320	54.7636 992	42
43	6.6374 3818	8.1496 669	12.2504 546	18.3443.548	37.3666 404	40.6761 098	60.2400 692	43
44	6.9361 2290	8,5571 503	12 9854 819	19.6284 596	29.5559 717	44.3369 597	66.2640 761	44
45	7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3272 861	72.8904 837	45
46	7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321	46
47	7.9152 6849	9 9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853	47
48	8.2714 5557	10.4012 697	16.3938 717	25.7289 065	40.2105 731	62.5852.370	97.0172 338	48
49	8.6436 7107	10.9213 331	17.3775 040	27.5299 300	43.4274 190	68.2179 083	106 7189 572	49
50	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74.3575 201	117.3908 529	50

1. The amount of $1 for more than 50 years is equal to the product of the amount of $1 for any two or more numbers of years in the Table whose sum is equal to the given time. Thus, the amount of $1 at compound interest *for* 84 yr. at 2%, is equal to the product of the amount of $1 at 2% for 50 yr. *and the amount* of $1 at 2% for 34 yr.; that is, $2.69158803 × 1.99067603, or *$5.27733213+.*

COMMERCIAL PAPER.

609. Commercial Paper includes all documents used as representatives of money, which may be transferred by delivery merely, or by indorsement and delivery; such as negotiable notes, drafts, checks, and other written or printed evidences of indebtedness.

610. A Note (Promissory Note) is a written promise to pay a certain sum of money on demand, or at a specified time, for value received, signed by the party making the promise.

611. A Draft is an order in writing signed by one party, directed to another, requesting the payment of a certain sum of money unconditionally to a designated third party, or to his order, or to bearer, either at sight or at a specified time.

A draft drawn on a party in another state or country is called an Exchange or Bill of Exchange.

612. A Check is a draft or written order drawn on a bank or banker for a certain sum of money specified therein, supposed to be on deposit, and payable immediately to a certain person named therein, or to his order, or to bearer, on presentation and demand.

613. The **Parties** to a NOTE are the drawer, commonly called the **Maker,** who signs the note thus making himself responsible for its payment, and the **Payee,** the party to whom, or to whose order, it is made payable. The parties to a DRAFT are the **Drawer,** who draws the draft or check; the **Drawee,** to whom the order is addressed, who afterwards becomes the **Acceptor** (or maker), and the **Payee.** The **Indorser** is the person who writes his name on the back of the paper for the purpose of transferring it, or as security for its payment; the **Indorsee** is the person to whom it is indorsed; and the **Holder** is the person who holds or owns it. The last two parties are generally the same person.

1. **The original parties** to commercial paper are those who bring it into existence, or those named in the instrument when first drawn. The maker and payee are the original parties to a note; the drawer, drawee, and payee, are the original parties to a draft or check.

2. **The subsequent parties** to a commercial paper are those to whom the paper is transferred or negotiated,—who indorse the paper.

3. **There may be** only two parties to a draft or check, as when the drawer takes the paper payable to himself. The payee, indorser, and holder may be the same *person.*

614. The **Face** of a note is the sum promised. It is written in words in the body of the note, and is usually placed in figures in the upper left hand corner of the paper and at the end of the line in which the payee's name appears.

If the note reads "with interest," it draws interest from date; otherwise only after maturity until paid. It may also draw interest from a particular time after date, if so specified in the note. When no rate is mentioned the legal rate of interest is understood. The words "value received" should be inserted in a note to acknowledge a valuable consideration, which acknowledgment would be sufficient, in case of litigation, to cast upon the defendant the burden of proof that there was no valuable consideration received.

615. A **Negotiable Instrument** is one in which the legal right to the money value of the contract can be transferred from one party to another by delivery merely or by indorsement and delivery. A promissory note is negotiable when it is made payable to the "bearer," or to the "order" of the payee.

1. A note payable "to bearer" is negotiable without indorsement. A note payable "to order" becomes negotiable by the payee writing his name upon the back. This is called indorsing the note.

2. A **non-negotiable note** is one that is payable to that person only who is named in the note. It is not strictly a commercial instrument.

3. The words, "*without defalcation*" are required in Pennsylvania to make a note negotiable; in New Jersey, "*without defalcation or discount,*" and in Missouri, the words "*negotiable and payable without defalcation or discount.*"

4. When no **place of payment** is specified, a promissory note is payable at the maker's place of business or, if none is known, at his residence. A note or draft must be presented at the place where it is made payable, and during business hours. The holder need not call more than once if the maker is not ready with legal tender to pay the note.

5. A negotiable instrument must have the following requisites: 1. It must contain certain direction or order or promise to pay in writing. 2. It must be payable in money alone, and not in goods or chattels. 3. The sum of money must be stated with certainty or at least capable of being easily ascertained. 4. There must be no condition or contingency attached to the payment. 5. It must state the time when due, or at least give the facts by which this can readily be determined. 6. A valuable consideration must support the contract. 7. The maker or acceptor must be capable of becoming legally accountable for its payment at maturity. These essentials are generally set forth in the following orderly parts: *Place where made, date, time to run, name of payee,* negotiable words, amount, *consideration, place of payment, name of maker* or *drawer,* name of drawee.

616. An **Indorsement** is a writing on the back of a note or other commercial paper.

An indorsement may be made for three purposes: to secure the payment, to transfer the ownership, or to make a memorandum of partial payment.

617. The **Indorsement** of commercial paper to transfer ownership may be an *indorsement in blank;* an *indorsement in full;* a *conditional indorsement;* a *qualified indorsement;* or a *restrictive indorsement.*

1. An *indorsement in blank*, or general indorsement, is simply the signature of the indorser, who is usually the payee, or other lawful holder, without naming any indorsee. This makes the paper payable to the bearer.

2. An *indorsement in full*, or special indorsement, is one in which the indorser states over his signature to whose order it shall be paid. Thus, in Note 2, Art. 623, R. B. Dunmire by simply writing his name across the back indorses it *in blank*. Should he wish it paid to the order of a particular person, as G. E. Fisher, he would indorse it *in full;* thus, "Pay G. E. Fisher or order. R. B. Dunmire." Before it could again be transferred it would require the indorsement of G. E. Fisher, who could indorse it *in blank* or *in full.*

3. For greater security notes, checks, drafts, etc., to be sent by mail should be indorsed in full.

4. A *conditional indorsement* is one subject to a specified condition which must be complied with to hold the indorser liable; as, "Pay to G. E. Fisher or order, if not otherwise directed before maturity. R. B. Dunmire."

5. A *qualified indorsement* is one in which the indorser relieves himself from all liability for payment by writing over his signature "without recourse" or words of similar import; as, "Pay G. E. Fisher or order without recourse. R. B. Dunmire." The purpose of a qualified indorsement is to enable the indorser to sell or transfer his title to a negotiable paper, and, at the same time, free himself of liability for payment in case of protest for non-payment by the maker or acceptor.

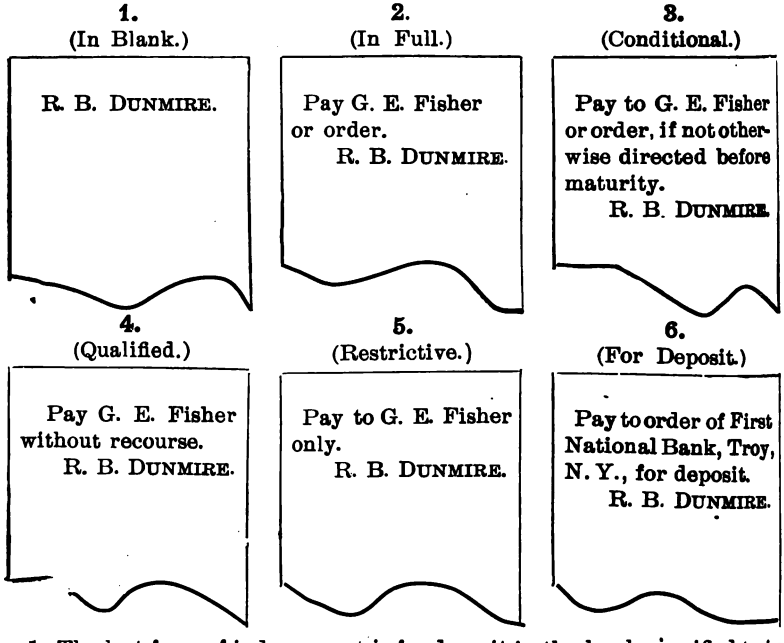
6. A *restrictive indorsement* is one which restricts the payment to the particular party specified; as, "Pay to G. E. Fisher only. R. B. Dunmire." With this indorsement, G. E. Fisher could not transfer the paper.

618. The Place for Indorsement is across the back of the paper, near the upper end, or left edge as held when reading it.

Place the paper before you, face up, as if to read. Now turn the face down without changing ends, and the left edge will be the top on which to place the first indorsement. This not only makes the writing of the indorsement right side up on turning from reading the face of the paper, but also affords all the blank space possible for further indorsement when needed.

FORMS OF INDORSEMENT.

619. The Forms of Indorsement, placed on the back and at the top of commercial paper, are shown in the following:



1. The last form of indorsement is for deposit in the bank specified to be there placed to the credit of the indorser's account. If a bank will not receive the paper with this form of indorsement (as is the case in New York), the words "for deposit" must be omitted, or it may be indorsed in blank.

2 If the drawee promises to pay a draft or bill of exchange at its maturity, he writes across the face above his name, "Accepted," with the date, when the paper is written a given number of days *after sight*, and his signature. The drawee then becomes the *acceptor*, and the draft or bill an *acceptance*. The paper then has the character of a promissory note.

620. Days of Grace are the three days allowed by law in some States for the payment of a note after the time specified in it has expired.

Days of grace have been abolished by statute in the following States: Arizona, California, Colorado, Connecticut, Delaware, Dist. of Columbia, Florida, Idaho, Illinois, Iowa, Maryland, Maine (except Sight Drafts), Mass. (except Sight Drafts), Montana, New Hampshire (except Sight Drafts), New Jersey, New York, North Dakota, Ohio, Oregon, Pennsylvania, Rhode Island (except Sight *Drafts), Tennessee*, Utah, Vermont, Virginia, Washington, West *Virginia, Wisconsin.*

1. The *Days of Grace* were originally three days allowed by the drawer to the drawee of the draft, more than the time stated in the draft. This was done as a favor and no interest charge was made for them. Finally days of grace were recognized by the courts as a right belonging to the debtor, but that he may be required to pay interest for them. In states where days of grace are allowed, banking houses charge interest for them. As has been seen above, many of the states have abolished days of grace by statute; and it is probable that before many years they will be abolished by statute in all of the states.

2. The "Negotiable Instrument Law" which has been ratified by many states abolishes grace and provides that when the day of maturity of an instrument falls on Sunday or a holiday the paper is due on the succeeding business day.

621. The **Maturity of Commercial Paper** is the day on which it becomes legally due for payment. It includes the days of grace, where allowed, unless the paper contain the words "without grace."

1. If the last day of grace falls on Sunday or a legal holiday, in nearly all of the States, except New York and Pennsylvania, the paper must be paid on the business day next preceding.

2. When the time of a note is expressed in months, calendar months are used to determine the day of maturity; when the time is expressed in days the exact number of days is considered. Thus, a note dated July 25, and payable two months after date would be nominally due September 25, and legally due September 28 (if grace is allowed); but if payable 60 days after date, it would be nominally due September 23, and legally due September 26. The date of maturity, where grace is allowed, is commonly indicated thus: Sept. 23/26.

3. A note dated Dec. 28, 29, 30 or 31, and payable two months after date, would be nominally due Feb. 28, and legally due Mar. 3 (with grace).

622. A **Protest** is a formal statement made by a Notary Public, over his official seal, at the request of the holder, that a note, check or draft has been properly presented for payment, and has been refused.

1. If the maker of a note refuses to pay it when due, each indorser can be held liable after a formal protest has been made, and proper notice given to him. In some States a written notice of demand and non-payment by the payee is sufficient, while in other States this notice must be written by a notary public.

2. Demand for the payment of a note should be made on the date of maturity, during business hours, at the place where payable. If no place of payment is specified in the note, it should be presented for payment at the residence or place of business of the maker, during reasonable hours. Upon the refusal of the maker to pay the note, it should be at once protested and notice sent to the indorsers on the day of maturity, or not later than the *next day; otherwise* they cannot be held responsible.

FORMS OF NOTES AND DRAFTS.

623. 1. Demand Note.

$475. ALBANY, N. Y., Jan. 13, 1899.

On demand I promise to pay H. C. Donat ~~~~ or bearer, Four Hundred Seventy-five Dollars. Value received.

J. T. HYATT.

This note is due and payable on demand to any lawful holder. It is *negotiable without indorsement.* If "or bearer" were omitted, it would be a non-negotiable note, since it would then be payable to H. C. Donat only. Although no interest is mentioned it draws interest from date at the legal rate of the State wherein it is made in accordance with the principle that all debts draw interest after maturity.

What will be the amount due on the above note if presented for payment April 1, 1902, finding time by compound subtraction?

2. **Time Note** (*with interest*).

$950. ORRSTOWN, PA., July 6, 1900.

Sixty days after date I promise to pay R. B. Dunmire or order, Nine Hundred Fifty ~~~~ Dollars, with interest, for value received, without defalcation.

JACOB MINICK.

How much was due on the above note at maturity?

3. **Time Note** (*without interest*).

$1000. LEWISBURG, PA., June 24, 1891.

Ninety days after date, for value received, I promise to pay to the order of Margaret O. Evans ~~~~ $1000 One Thousand ~~~~ Dollars, without defalcation.

PAUL TUSTIN.

This note could draw no interest except from date of maturity when, if not paid, the legal rate of the State could be collected.

What was due on the above note if paid on Dec. 30, 1893?

4. **Time Note.**

$260\frac{75}{100}$. CHICAGO, ILL., Sept. 5, 1898.

Three months after date I promise to pay to the order of A. L. Moore Two Hundred Sixty and $\frac{75}{100}$ Dollars, with interest at 7%. Value received.

CHARLES CAMPBELL.

Notes 2, 3, and 4 are *negotiable with indorsement.*

What was the date of maturity of the above note, and how much was then due?

5. **Principal and Surety Note.**

$135\frac{65}{100}$. COLUMBUS, O., Oct. 1, 1899.

Six months after date, for value received, I promise to pay to the order of C. G. Langford ~~~~~ *$135\frac{65}{100}$ One Hundred Thirty-five and $\frac{65}{100}$ Dollars, at the First National Bank.*

A. O. STEVENS, Principal.
H. W. D. KIRKENDALL, Surety.

This is a note in which another person becomes security for the payment of the note by the maker. Another form is to make the note payable to the order of the surety who indorses it on the back to the order of the creditor. It is claimed that the latter form is more binding on the surety.

What is the date of maturity of the above note? Where is it payable?

6. **Joint and Several Note.**

$6750. NEW YORK, N. Y., May 23, 1900.

Four months after date we jointly and severally promise to pay G. E. Fisher ~~~~~ *or order, Six Thousand Seven Hundred Fifty Dollars, with interest. Value received.*

F. H. STARKEY.
R. C. H. CATTERALL.

If this note were written "we promise," or "we jointly promise," etc., it would be called a *joint* note because it is signed by two persons who are jointly responsible for its payment; that is, each maker is individually liable for the payment of only one-half of the amount of the note. This is according to the Common Law. In New York both makers of a joint note are liable for the full amount. In a *joint and several* note either maker is individually liable for the payment of the full amount of the note if the other defaults. The makers of a joint note must be sued jointly, while those of a joint and several note may be sued separately, since either is responsible for the full amount.

How much was due on the above note at maturity?

7. **Sight Draft.**

$180. OMAHA, NEB., Apr. 1, 1899.

At sight pay to the order of G. E. King, One Hundred Eighty Dollars, value received, and charge to account of

A. B. HUGHES.

To P. D. MINICK,
Villisca, Ia.

In the above draft who is the drawer? Who the drawee? Who the payee?

8. **Time Draft.**

$3000. PHILADELPHIA, PA., Jan. 2, 1900.

At thirty days sight pay to the order of Bartow & Rockwood, Three Thousand Dollars, value received, and charge to the account of

GROFF & OWENS.

To HARRIS, PERRINE & Co.,
New York.

1. A draft written a certain number of days after sight, means after acceptance, but the acceptor should in accepting the draft write the date when it was first seen after the word "Accepted," which date as before stated, should be followed by the name of the acceptor. Drafts are sometimes drawn a certain number of "days after date." In such cases they mature the same as ordinary notes, hence the date of acceptance need not *be inserted.* The acceptor of any time draft may state in the acceptance *where he wishes* to make it payable.

PARTIAL PAYMENTS.

624. A **Partial Payment** is a payment in part of a note, mortgage, or other obligation.

625. An **Indorsement,** as here used, is an acknowledgment of a partial payment written upon the back of a note, mortgage or other written obligation, stating the date and the amount of the payment.

626. The Supreme Court of the United States and nearly all the States have adopted the following rule for partial payments called

THE UNITED STATES RULE.

Find the amount of the given principal to the time when the payment or the sum of the payments equals or exceeds the interest due. Subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal and proceed as before to the time of settlement.

1. This rule is based upon the principle that neither interest nor payment shall draw interest.

2. In applying the United States Rule to partial payments, find the time by compound subtraction.

627. 1. **Promissory Note.**

$400. POUGHKEEPSIE, N. Y., June 24, 1897.

Three years after date, I promise to pay D. C. Morrison or order, Four Hundred Dollars, with interest, for value received.

H. A. BITNER.

Indorsements on Back of Note.

MAR. 30, 1898. Received on the within note Sixty-eight and $\frac{4}{100}$ Dollars ($68.04).

NOV. 30, 1899. Received on the within note Twenty-five Dollars ($25).

FEB. 18, 1900. Received on the within note One Hundred Sixty and $\frac{55}{100}$ Dollars ($160.55).

How *much* remained due June 24, 1900?

PROCESS.

Principal or face of note	$400.
Interest to first payment	18. 40
Amount due Mar. 30, 1898	$418. 40
First payment to be deducted	68. 40
Balance due after first payment	$350.
Interest on balance to second payment is $35. The payment being less, is not deducted.	
Interest from first payment to third payment	39. 55
Amount due Feb. 18, 1900	$389. 55
Sum of second and third payments to be deducted	185. 55
Balance due after third payment	$204.
Interest from Feb. 18, to June 24, 1900	4.28
Balance due on settlement, June 24, 1900	$208.28

2. *$500.* HARRISBURG, PA., Apr. 1, 1895.

On demand, I promise to pay D. J. Faust, or order, Five Hundred Dollars, with interest, for value received, without defalcation.

CYRUS LEIDIG.

Indorsements: May 13, 1896, $80; Dec. 7, 1897, $20; July 1[illegible], 1898, $175. How much was due Sept. 25, 1899?

3. A note was given Jan. 1, 1893, for $1500. The following payments were indorsed upon it: Oct. 22, 1894, $135; June 2[illegible], 1895, $210; Aug. 14, 1896, $600. How much was due Dec. 3[illegible], 1896, with interest at 6%?

4. A note of $2500 was given Jan. 1, 1896. It received the following indorsements: Feb. 7, 1897, $400; Sept. 6, 1898, $50; May 23, 1899, $950; Nov. 5, 1899, $350. How much was due Jan. 1, 1900, with interest at 6%?

5. On Apr. 1, 1897, a note was given for $1000, at 5% interest. Payments were made as follows: Oct. 5, 1897, $75; May 1, 1898, $125; Sept. 13, 1899, $150. What was due Dec. 31, 1899?

6. I held a mortgage for $2000, dated June 1, 1896, on which the following payments had been made: Jan. 1, 1897, $300; Sept. 30, 1898, $500; Apr. 1, 1899, $400. The mortgage bore 6% interest. How much was required to pay the mortgage on Jan. 1, 1900?

7. On a mortgage for $2500 dated June 20, 1895, and bearing interest at 7%, the following payments were made: Apr. 1, 1896, $200; Jan. 2, 1897, $250; Sept. 2, 1897, $50; Apr. 16, 1898, $350. How much was due Oct. 28, 1899?

8. A note for $4000 dated Jan. 1, 1898, bearing 5½% interest was indorsed as follows: June 13, 1898, $150; Oct, 1, 1899, $200; Apr. 13, 1900, $500; Sept. 25, 1900, $300; Nov. 15, 1900, $650. How much was due Dec. 31, 1900?

623. The method generally employed by bankers and merchants for finding the balance due on a note payable within a year, is the following, called the

MERCHANTS' RULE.

Find the amount of the principal to the time of settlement, and also the amount of each payment to the time of settlement. Subtract the amount of the payments from the amount of the principal, and the remainder will be the balance due.

1. This rule is sometimes applied to notes for longer periods by finding the balance due at the end of the first year, and taking this as a new principal for the second year, and so on to the time of settlement. The times at which these balances are taken are called *Rests.*

2. In applying the Merchants' Rule use bankers' interest.

EXAMPLES.

1. A note for $1200 was given Jan. 12, 1898. It contained the following indorsements: Mar. 18, 1898, $300; June 5, 1898, $400; Sept. 21, 1898, $200. How much was due Dec. 12, 1898, at 6%?

2. On a note dated Apr. 1, 1899, for $1650, were the following indorsements: July 5, 1899, $360; Oct. 13, 1899, $540; Nov. 1, 1899, $135. How much was due Dec. 31, 1899, at 6%?

3. A note for $900 was dated Jan. 2, 1903, and contained the following indorsements: May 8, 1903, $100; July 15, 1903, $150; Aug. 12, 1903, $200. What was due Oct. 30, 1903, at 5%?

4. A note of $1750 with interest at 7%, dated Apr. 1, 1899, was indorsed as follows: June 1, 1899, $300; Aug. 1, 1899, $400; Oct. 1, 1899, $509; Dec. 1, 1899, $200. How much was due Feb. 1, 1900?

5. A note of $1400 with interest at 6%, dated Jan. 1, 1900, for 9 months, was indorsed as follows: Feb. 5, $120; Mar. 10, $180; May 1, $225; *June 16,* $175. How much was due at maturity?

TRUE DISCOUNT.

629. Discount is an allowance made for the payment of a debt, a note, or other obligation before it is legally due.

630. The **Present Worth** of a debt due at a future time is its value now. It is a sum which if placed on interest at the legal rate, will amount to the given debt when it becomes due.

631. True Discount is the difference between the amount of a debt when legally due and its present worth.

1. The *true discount* is the interest on the present worth for the time between the payment of the debt and the time it becomes due.

2. The present worth may be considered the principal, the discount the interest, and the debt the amount.

632. 1. What is the present worth and true discount of a debt of $345 due in 3 years, money being worth 5%?

$1.15 = Amount of $1 for 3 yr.
$345 ÷ $1.15 = 300. ∴ $300, present worth.
$345 − $300 = $45, true discount.

EXPLANATION.— The amount of $1 for 3 years at 5% is $1.15, hence the present worth of $345 is as many dollars as $1.15 is contained times in $345, or $300. The true discount is the difference between the debt ($345) and the present worth ($300), or $45.

RULE.—*To find the present worth, divide by the amount of $1 for the given time and rate.*

To find the true discount, subtract the present worth from the amount due.

To find the present worth of a debt drawing interest, first find the amount of the debt at maturity, and then find the present worth of this amount at the time the debt is discounted.

EXAMPLES.

2. What is the present worth of $560 due in 2 years, money being worth 6%?

3. What are the present worth and true discount of $840, due in 3 yr. 4 mo., when money is worth 6%?

4. Find the present worth and true discount of $360.50 due in 9 months, when money is worth 5%.

5. *Find* the present worth and true discount of $425.75 due in *1 yr. 2 mo.*, when money is worth 7%.

6. Find the present worth of $536.57 due in 1 yr. 3 mo. 18 da., ney being worth 6%.

7. A man owes $1200 due in 3 yr. 4 mo. What would pay the bt now, money being worth 6%?

8. I owe a debt of $150 due in 7 mo. 21 da. I wish to pay it w. How much will pay the debt, money being worth 5%?

9. I can buy a farm for $5000 cash or for $5500 due in 1 yr. 4). Which is the better investment and how much, money being rth 7%?

Suggestion.—Compare the cash value with the present worth of $5500.

10. Mr. Brown owes me $250, due in 6 mo. 21 da. He wishes pay me now. What sum should I receive, money being worth %?

11. I can buy a horse for $140 cash or for $150 due in 10 months. I pay cash do I gain or lose, money being worth 7%?

12. A farm was sold for $7000 on the following terms: $1000 in sh; one-third of the remainder in 1 yr.; and the balance in 2 ars. What was the equivalent cash value of the farm, money ing worth 6%?

13. Mr. M. owed a debt of $600 payable in 8 months with inter- t at 6%. At the end of 3 months he paid the debt at its then esent worth. How much was the debt at the time of payment?

14. A dealer bought $1200 worth of grain on 3 months' credit, d sold it on the same day for cash at a profit of 8½%. He paid e present worth of the grain, money being worth 6%. What is his gain on the grain?

BANK DISCOUNT.

633. Bank Discount is the simple interest on a loan, paid in vance, for the number of days the note has to run.

On notes without interest, bank discount is reckoned on their face; on es with interest it is usually reckoned on the amount due at maturity. some States notes mature 3 days later than the time specified. (See t. 620).

634. The **Proceeds** of a note is the sum received by the holder m the bank when the note is discounted. It is the sum dis- unted *less the discount.*

635. The **Term of Discount** is the number of days from the time of discounting to the time of maturity of the note.

1. In reckoning the time of discount exclude the day of date, but include the day of maturity, counting the actual number of days.

2. Some banks include both the day of discount and the day of maturity. This is notably true of banks in Philadelphia and Baltimore. The methods of bank discount depend upon State laws and in some cases even upon local customs. In the City of New York, some banks discount on the basis of 360, and others on the basis of 365 days to the year; though the greater number discount on the former basis.

636. Bank Discount is but an application of Percentage in which the Face of the note is the *base*, the Rate Per Cent. the *rate*, and the Bank Discount the *percentage*.

In the following problems omit days of grace unless otherwise stated, and count exact days for term of discount, excluding the day of discount but including the day of maturity. Use bankers' interest.

637. Given the face of the note, the rate, and the time, to find the discount and the proceeds.

1. A note dated June 24, 1899, at 90 days for $540, was discounted Aug. 1, 1899, at 6%. What was the proceeds?

June 24 + 90 da. = Sept. 22
Aug. 1 to Sept. 22 = 52 da.

2) $5	40	
2	70,	30 da.
1	80,	20 "
	18,	2 "
Dis. $4	68,	52 da.

Proceeds, $540 − $4.68 = $535.32

EXPLANATION.—The note matures 90 da. from June 24, or Sept. 22. Since the note was discounted Aug. 1, which was 52 da. before it was due, we find the interest of $540 for 52 da., which is $4.68, the bank discount. This subtracted from $540, the face of the note, leaves $535.32, the proceeds.

2. A note dated Apr. 1, 1898, at 4 months (grace) for $360, was discounted May 1, 1898, at 6%. What were the proceeds?

Apr. 1 + 4 mo. = Aug. 1/4.
May 1 to Aug. 4 = 95 da.

$3	60,	60 da.
1	80,	30 "
	30,	5 "
Dis. $5	70	95 da.

Proceeds, $360 − $5.70 = $354.30

EXPLANATION.—The note matures 4 mo. from Apr. 1, or Aug. 1/4. Since the note was discounted May 1, which was 95 da. before it was due, we find the interest of $360 for 95 da., which is $5.70, the bank discount. This, subtracted from $360, the face of *the note*, leaves $354.30, the proceeds.

RULE.—*To find the bank discount, compute the interest on the sum due at maturity for the term of discount.*

To find the proceeds, subtract the bank discount from the sum due at maturity.

EXAMPLES.

Find the bank discount on the following notes:

3. $280, dated June 1 for 60 days, discounted July 1 at 6%.

4. $510, dated Feb. 6 for 90 days, discounted Mar. 15 at 6%.

5. $435, dated May 20 for 2 mo., discounted May 31 at 6%.

6. $246.85, dated Jan. 16 for 3 mo., discounted Feb. 25 at 5%.

7. $615.38, dated Sept. 18 for 4 mo., discounted Nov. 1 at 7%.

Find the proceeds of the following notes:

8. $175, dated May 4 for 30 days, discounted on date of issue at 6%.

9. $745, dated July 5 for 3 mo. 12 da. (grace) and discounted Aug. 17 at 6%.

10. $575, dated Oct. 27 for 60 days, discounted Nov. 1 at 8%.

11. $930, dated Aug. 27 for 2 mo. 20 da. (grace), and discounted Sept. 10 at 7%.

12. $648, dated Mar. 13 for 3 mo. discounted Apr. 10 at 6%.

13. A note for $500, payable in 3 months was dated and discounted Apr. 1, 1898, at 6%. What was the date of maturity, and the proceeds?

14. A note for $750, payable in 4 months (grace) was dated and discounted Mar. 3, 1899. Find the proceeds when discounted at 6%.

15. A 90-day note for $800 was dated June 14, 1899. It was discounted July 20, 1899, at 6%. Find the discount and the proceeds.

16. Find the date of maturity and the proceeds of a note dated Sept. 12, 1898, payable in 90 days for $1200 with interest at 6%; discounted Oct. 10, 1898, at 6%.

17. Find the date of maturity and the proceeds of a note dated Apr. 2, 1900, payable in 6 months for $575, with interest at 5%; discounted June 25, 1900, at 6%.

18. What is the difference between the true discount and the bank discount of $960, due in 3 yr. 4 mo., money worth 6%, not considering *days of grace*?

Find the date of maturity, term of discount, and the proceeds of the following notes:

19. $700. BOSTON, MASS., July 21, 1903.

Three months after date I promise to pay J. H. Gise, or order, Seven Hundred Dollars, for value received, at the First National Bank.

HARRY HALL.

Discounted Aug. 4, 1903, at 6%.

20. $1000. GALVESTON, TEX., Jan. 6, 1904.

Sixty days after date, for value received, I promise to pay to the order of G. W. Minick, One Thousand Dollars, at the City National Bank.

F. S. WELLS.

Discounted Feb. 7, 1904, at 6%.

21. 275\frac{85}{100}$. BUFFALO, N. Y., Apr. 30, 1904.

Four months after date I promise to pay D. A. Kann, or order, Two Hundred Seventy-five and $\frac{85}{100}$ Dollars. Value received.

E. P. BALDWIN.

Discounted June 2, 1904, at 6%.

Days of grace have been abolished in Mass. (except Sight Drafts). Art. 620.

22. $456. DENVER, COL., Sept. 1, 1904.

Ninety days after date, we promise to pay J. A. Riker, or order, Four Hundred Fifty-six Dollars, for value received, at the Park National Bank.

W. H. WATT.
T. L. NASH.

Discounted Sept. 1, 1904, at 8%.

23. $1541. DETROIT, MICH., May 28, 1904.

Five months after date, either of us promises to pay John Crusey, or order, Fifteen Hundred Forty-one Dollars, for value received, with interest at 6%.

J. M. BELL.
C. C. CARR.

Discounted July 2, 1904, at 6%.

24. $850. SAN FRANCISCO, CAL., Aug 10, 1904.

Six months after date we, or either of us, promise to pay J. E. Johnson, or order, Eight Hundred Fifty Dollars, with interest at 6%. Value received.

J. M. LEE.
C. R. PAUL.

Discounted Nov. 1, 1904, at 8%.

25. $2000. PHILADELPHIA, PA., Apr. 2, 1904.

Three months after date, for value received, I promise to pay Geo. W. Polk, or order, Two Thousand Dollars, with interest at 6%, without defalcation.

CHAS. R. DICKINSON.

Discounted Apr. 30, 1904.

See note 2, Art. 635.

638. Given the time, the rate, and the proceeds or the discount, to find the face.

1. I wish to borrow $200 from a bank. For how much must I give my note at 60 days, discounting at 6%?

$1.00
.01
$ .99
$200 ÷ .99 = $202.02

EXPLANATION.—The discount on $1 for 60 days is .01. Hence, the proceeds of $1 is $1 minus $.01, or $.99. If the proceeds of $1 is $.99, to yield $200 proceeds, it will require a note whose face is as many dollars as $.99 is contained times in $200, which are $202.02.

RULE.—*To find the face of a note, divide the given proceeds of $1 for the given time and rate; or divide the discount by the discount of $1.*

EXAMPLES.

2. What is the face of a note at 90 days, the proceeds of which, discounted at 6%, are $500?

3. For what sum must a note be drawn at 60 days to net $2400, when discounted at 6%?

4. The discount at 6% on a 90-day note was $15. What was the face of the note?

5. Mr. Adams paid $24 discount for having a 6 months note discounted at 8%. What was the face of the note?

6. Mr. Hardup owing $450, gave a 60-day note which was discounted at 10%. What was the face of the note required to pay the *exact debt*?

STOCKS AND BONDS.

639. A **Company** is a number of persons associated together for the purpose of carrying on some business enterprise. It may or may not be incorporated.

640. A **Corporation** is an incorporated company authorized by law to transact business as a single individual.

A natural person may do anything not prohibited by law, while a corporation can do nothing but what is expressly permitted by its charter.

641. A **Franchise** is a right granted by the State to individuals or to corporations. Thus, the franchise of a railroad is the right to build and operate its road.

642. A **Charter** is a contract with the Government which defines the powers and limitations of a corporation.

One of the advantages of a charter is that it usually limits the liability of each stockholder to the amount he has contributed, while a member of an unincorporated company or firm is liable for all the debts of the partnership.

643. The **Capital Stock** of a company is the capital or funds (property) invested in the business.

This capital may be increased or diminished by a vote of the majority of the stockholders, representing a majority of the stock.

644. A **Share** is one of the equal parts into which the capital stock is divided. A share usually represents $100 of the capital stock, though it may be of any value agreed upon by the members of the corporation. The owner of stock is called a *Stockholder.*

645. A **Stock Certificate** is the paper issued by a company or corporation to its stockholders, signed by the proper officers of the company, certifying that they are the owners of a stated number of shares of its capital stock at a certain par valuation.

These certificates are transferable by assignment and may be bought and sold the same as other property.

646. The **Par Value** of a share is the value specified in the certificate and is usually $100.

647. The **Market Value** of a share is the price at which it can be sold.

When the market value of the shares is greater than their par value, as is the case when the business is very profitable, they are said to be *above par*, or *at a premium;* when the market value is less than the nominal value the shares are *below par*, or *at a discount* (business is unprofitable).

648. The **Gross Earnings** of a company are the total receipts of the business before the expenses have been paid.

649. The **Net Earnings** of a company are what is left of the gross earnings after all expenses have been paid.

The net earnings of a company are its clear profits, or net gains.

650. A **Dividend** is such portion of the profits of a corporation as is divided among its stockholders. It is not customary to pay a larger dividend than good interest on the capital invested.

Stocks sell *Dividend-on* between the time the dividend is declared and the day the books of the company close for transfer. After that date they sell *ex-dividend, i. e.*, the dividend does not go to the buyer.

651. A **Surplus Fund** is a fund created out of the profits remaining after expenses and dididends have been paid. This excess belongs to the stockholders.

652. An **Assessment** is a sum levied upon stockholders to make up losses, to meet expenses, or to provide for improvements.

653. Common Stock is the capital stock subscribed by the original organizers of a corporation.

654. Treasury Stock is a certain number of shares set aside to be sold from time to time to secure working capital.

655. Preferred Stock is that which has preference when dividends are declared.

1. The holder of preferred stock will receive interest theron out of the profits of the business before the holder of common stock receives any share of such profits.

2. Preferred stock is frequently issued in an effort to raise money when a company is financially embarrassed. To those who extend pecuniary aid at such a time, preferred stock is issued which entitles the holder to a dividend of a specified rate per cent. before any dividends can be declared on the common stock.

656. Guaranteed Stock expresses on its face the guaranty of a certain *dividend*.

657. Cumulative Stock is that form of security on which a corporation agrees to pay past dues and unpaid preferences before paying any dividends on the Common Stock.

Hypothecated stocks or bonds are those deposited as collateral security for borrowed money.

658. Watered Stock is stock issued by a company and distributed among its stockholders without subscribing additional capital or requiring payment therefor in any other form.

This is often done, when, by the terms of its charter, a corporation is forbidden to declare a dividend exceeding a certain rate per cent. of the par value of its stock, or when the stockholders wish to keep the public in ignorance of its large dividends. By distributing additional shares to the stockholders a low rate of dividend may be declared upon the increased number of shares, which will produce as much income as a high rate upon the original capital stock.

659. A Bond is a written or printed obligation, under the makers' seal, whereby the *obligor* (maker) binds himself, or himself and his heirs, executors or administrators, to pay a certain sum of money to another (called *obligee*) on demand or at a specified time. The current forms of bonds generally make interest at a fixed rate per cent. payable at stated periods.

1. Bonds are issued by the U. S. Government, States, Counties, Cities, business corporations, and individuals as security for money borrowed.

2. Government Bonds are issued by the general government. They are usually named from the interest they bear and the year when due; as U. S. 4's of 1907, means United States bonds, bearing 4% interest, payable in 1907.

3. Bonds of business corporations and individuals are commonly secured by mortgage on their real estate and other property.

4. A corporation may issue several series of bonds and secure their payment by mortgages on the same property, in which case each mortgage will have priority in its registered order in the County Clerk's Office.

(a) *First Mortgage Bonds*, being secured by the first mortgage placed on the property, constitute a lien prior to any later mortgage.

(b) *Consolidated Bonds* are issued when a number of debts, evidenced by first, second, third, etc., mortgage bonds, are consolidated in one large (or *blanket*) mortgage, thus putting the several series of bonds on an equal footing.

Thus, the English Consols (abbreviation of *Consolidateds*) are bonds of this *type,—the* British having many years ago consolidated their various debts *under one issue* in this way.

660. A **Coupon Bond** is one with small interest certificates attached which may be cut off and presented for payment as they become due.

These bonds are payable to bearer, and, hence, transferable by mere delivery.

661. A **Registered Bond** is one payable to the owner (or to his order) whose name is registered in the books of the corporation, etc., which issued it.

These bonds can be transferred only by personal assignment, and the bond must be presented both at the transfer office and at the register office when the transfer is made.

662. A **Debenture** is a form of corporation bond in general use.

663. A **Sinking Fund** is the portion of the earnings of a corporation set aside periodically to pay off bonds and other obligations as they mature.

664. A **Funded Debt** is one that has a permanent fund provided for its payment. The term is also frequently applied to an issue of new bonds, floated at a lower rate of interest to raise money to pay off bonds bearing a higher rate.

665. An **Unfunded,** or **Floating Debt,** is one of a temporary nature, payable out of the current income, and generally not secured by mortgage or other collateral.

666. The **Interest on Bonds** is usually payable quarterly or semi-annually.

1. The interest on U. S. bonds is reckoned by the method of exact interest; other bonds by the method of common interest.
2. The checks for interest on government bonds will be cashed by any bank, when properly indorsed.

667. A **Broker** is a person who brings the buyer and seller together. He buys and sells stocks, bonds, and other securities or property as an agent for others for a commission.

668. A **Stock Broker** is one who buys and sells stocks as an agent for other parties. He is usually called simply a **Broker.**

Brokers are really agents for others, but *on change* they stand in the mutual relationship of principals.

669. Brokerage is the charge made by brokers for buying and *selling stock and other* commodities.

670. The **Stock Exchange** is a place where brokers meet daily to buy and sell stocks, bonds, and such other securities as may be listed at the Exchange.

Stock Exchange members are of three classes:

(a) *Scalpers*, or *Room traders*, who buy and sell for themselves and not on commission and who close up all their trades each night.

(b) *Speculators*, *dealers* or *jobbers* who also deal for themselves alone but hold indefinitely for a rise or decline.

(c) *Brokers* or *Commission Merchants* proper, who deal for others on commission, and rarely trade for themselves. The number of brokers on the Paris Bourse is limited, and they are forbidden by law to deal on their own account.

671. The **Commission** for buying and selling at the New York Stock Exchange is $\frac{1}{8}\%$ on the par value of all stocks and bonds. This is $\$\frac{1}{8}$ or $\$.12\frac{1}{2}$ on a share whose par value is $100, or $1.25 on a $1000 bond. The rates of commission depend considerably upon the kind of stock bought and sold, and to some extent upon the local custom of the city brokers. Usually, however, it is $\frac{1}{8}\%$.

672. Listing Stock means having it entered on the list of securities regularly traded in on a certain Exchange. To have its securities listed gives a corporation the advantage of having at all times a quotation showing the market value of its stock.

673. Stock Quotations are statements giving the market price at which stock can be bought and sold.

1. The quotations are usually given at the market value of each $100 of the par value of stocks and bonds. Thus, stock quoted at $104\frac{1}{2}$ is selling at $4\frac{1}{2}\%$ premium, at 95 at 5% discount; for it means that a share of the former is selling for $104.50, and of the latter at $95 a share.

2. The *Ticker* reports market quotations by printing automatically abbreviated names of stocks with their prices on a narrow ribbon of paper.

3. The par value of some kinds of stock may be only $10 a share, and of others $25, or $50, though, as already stated, a share is usually quoted at the par value of $100.

674. A **Margin** is a sum of money or value in securities deposited with a broker to protect him against loss in buying or selling for a customer's account. It is generally a partial payment of 5% to 10% in amount of the par value of the stock, but may be more or less according to agreement.

1. Brokers allow their customers interest on money deposited as margin, *and charge them* interest for all sums expended in cash for their accounts.

2. Stocks are usually bought or sold "*Cash*" to be delivered on the day sold, "*Regular*" to be delivered the next day, "*seller three*" (s 3), or "*buyer three*" (b 3). Stock bought "seller three" means that the seller can deliver on the day sold, the next day, or the day after the next without interest. "Buyer three" means that the buyer may demand delivery of the stock at any time within three days without interest, but must take and pay for it by the third day. When stock is sold at the buyer's or seller's option for more than three days, the buyer pays 6% interest, unless "flat" is stipulated in the deal.

3. A person is said to be "*Short*" when he sells stocks which he does not own, expecting to buy in time to deliver, and at a lower price so as to make a profit. Stock is often borrowed in order to make a delivery, and afterwards bought to repay the loan.

4. An operator is said to be *Long* when he owns or has bought a certain stock expecting an advance.

5. A *Bull*, on the stock Exchange, is a member who expects the price of stocks to *advance*, and hence *buys* "for long account" with the hope of advancing prices for profit.

6. A *Bear* is one who expects the price of stocks to *fall* (decline in value), and hence *sells* "short" that he may profit by the *lower price* at which he expects to *buy*.

7. *To Hedge* is to buy one stock and sell another with the expectation that the one bought will advance and the one sold will decline.

8. A *Corner* is a practical monopoly enabling the operator to control the stock or thing cornered and force the price far above its actual value. Bears cannot buy the stock cornered to deliver in fulfillment of their contracts, except at exorbitant prices.

9. A *Syndicate* is a party of capitalists who unite their resources for the accomplishment of some financial object.

10. A *Pool* or *Ring* is a combination formed to control the price of stocks. Many transactions on the Stock Exchange amount to a wager in which the buyer assumes that the price will appreciate and the seller that it will decline.

11. *Stock Privileges* are options in the nature of formal contracts to buy and sell which are sold at a value agreed on and generally described as *puts* and *calls*.

12. A *Put* is a contract to accept upon one day's notice, except the day of expiration, a certain commodity at a stipulated price. A put is frequently bought as a protection to a speculator against a considerable decline in the value of stocks he holds.

13. A *Call* is the reverse of a put, giving its owner the right to demand or call for the delivery of the stock or commodity at a stipulated price on or before a fixed day. A call may serve a speculator who desires to purchase certain stocks at the call price on a future day, provided the stocks are not below *that price* in the open market on the day his call contract *expires*.

14. *Buyer's option* is applied to a contract in which the purchaser has the privilege of taking a certain stock at a price named within a specified time. This option is similar to a *call*.

15. *Seller's option* gives the privilege of delivering a certain stock within a specified time. This option is similar to a *put*.

16. *A Straddle* is the sale of a privilege to either buy or sell.

17. *To cover* is to buy in stock to fill a contract in which the operator is "short."

18. *A limited order* is one given to a broker in which the customer fixes a price beyond which he will not go.

19. *A stop order* is one given by a customer when buying on a margin to sell and close out the trade if the stock declines below a certain point. Such an order has the effect of limiting the risk on the transaction to a certain amount.

20. *Settlement day* is the last day of each month when all trades that were bought for "the account"—that is, not to be delivered or paid for until settlement day—must be settled.

675. Computations in Stocks and Bonds may be made in accordance with the principles of Percentage, the Par Value being the *base*, the Rate of Premium or Discount the *rate*, the Premium or Discount, the *percentage*, the Market Value above par the *amount*, and the Market Value below par the *difference*.

1. In stock and bond computations it is much shorter to consider all rates per cent. based on the par value as so much money per share or bond. Thus, a premium of 8% is \$8 per share; a dividend of 6¼% is \$6.25 per share; a brokerage of ⅛% is \$⅛, or \$.12½ per share. This is also the logical method of computing in stocks and bonds, for they are bought and sold by the share or bond.

2. When stocks or bonds are bought through a broker, the brokerage is added to the market value of the stock, or when sold through a broker, it is deducted from the selling price of the stock. Thus, stock bought at 96 with ⅛% brokerage, would cost the purchaser \$96⅛, or \$96.12½, per share. If sold at 96, brokerage ⅛%, the seller would receive \$95⅞, or \$95.87½ per share.

In the following problems, regard the par value of all stocks and bonds at \$100 each unless otherwise stated.

EXAMPLES.

676. 1. What is the dividend on \$4800 par value of stock, if the rate of dividend is 5%?

$48 \times \$5 = \$240$ EXPLANATION—Since the value of the stock is \$4800, there are 48 shares; and if the rate of dividend is 5%, each share is \$5, and on 48 shares it is 48 times \$5, or \$240.

2. What is the dividend on $5500 par value of stock, if the rate of dividend is 4%?

3. Find the dividend on $13500 par value of stock, if the rate of dividend is $4\frac{1}{2}\%$.

4. What is the assessment on $8700 par value of stock at $2\frac{3}{4}\%$?

5. What is the premium on $600 par value of stock, if the rate of premium is $3\frac{1}{4}\%$?

6. What is the discount on $14500 par value of stock, when the rate of discount is $3\frac{1}{8}\%$?

7. A company declares a dividend of $4\frac{3}{4}\%$. What is the dividend on 135 shares?

8. What is the market value of 40 shares of stock, dividend-on, if the rate of premium is $5\frac{1}{2}\%$?

Suggestion.—The market value of 1 share is $105.50.

9. What is the market value of 32 shares of stock, dividend-on, if the rate of premium is $6\frac{1}{4}\%$?

10. What is the market value of $3000 par value of stocks, if the rate of premium is $3\frac{3}{8}\%$?

11. What is the market value of 25 shares of stock, ex-dividend, if the rate of discount is $4\frac{1}{2}\%$?

12. What is the market value of 130 shares of stock, ex-dividend if the discount is $7\frac{3}{4}\%$?

13. I bought $5000 par value of stock at $5\frac{1}{2}\%$ discount. How much did I pay for the stock?

14. Find the market value of $2700 par value of stock, when the quotation is $101\frac{1}{4}\%$?

15. Find the market value of $4500 par value of stock, when the quotation is $97\frac{1}{8}$.

16. What is the market value of $21600 par value of stock, when the quotation is $105\frac{5}{8}$?

17. Find the cost of $4500 par value of stock at 5% premium, brokerage $\frac{1}{8}\%$.

$105.125
45
525625
420500
$4730.625

EXPLANATION.—Since the stock is bought at 5% premium and the brokerage is $\frac{1}{8}\%$, the cost of one share is $105.125. The cost of 45 shares is 45 times $105.125, or $4730.63.

18. What will be the cost of 60 shares of stock at 3% premium brokerage $\frac{1}{8}\%$?

19. What must be paid for 37 shares of stock at $3\frac{1}{4}\%$ premium, brokerage $\frac{1}{8}\%$?

20. How much must be paid for 50 shares of stock ($50) at 4% premium, brokerage $\frac{1}{8}\%$?

Suggestion.—50 shares of $50 stock is equivalent to 25 shares of $100 stock.

21. What will be the cost of 40 shares of stock ($50) at 6% premium, brokerage $\frac{1}{8}\%$?

22. What will be the cost of 70 shares of stock ($50) at $51\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

23. Find the cost of 110 shares of stock at 4% discount, brokerage $\frac{1}{8}\%$.

24. Find the cost of 130 shares of stock at $98\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

25. A man sold 142 shares of N. Y. C. & H. R. R. R. stock at 114, brokerage $\frac{1}{8}\%$. How much did he realize from the sale?

26. Find the net proceeds of the sale of 80 shares of stock at 99, brokerage $\frac{1}{8}\%$.

27. Mr. Baker bought 85 U. S. 4's at $121\frac{1}{4}$, brokerage $\frac{1}{8}\%$. How much more than the face did he pay for the bonds?

28. I bought 120 shares of railway stock at $98\frac{1}{4}$, and sold them at $103\frac{3}{4}$. What was my net gain, brokerage in each case being $\frac{1}{8}\%$?

29. Mr. Gish purchased through a New York broker 200 shares Del. & Hud. R. R. stock at $108\frac{1}{2}$, 150 St. Paul R. R. at $89\frac{3}{4}$, 100 Long Island R. R. at 95, and 50 Rock Island R. R. stock at $105\frac{1}{4}$. How much did they all cost him?

30. Mr. French sold through a New York broker 60 U. S. $4\frac{1}{2}$'s at 105, 80 W. U. Tel. 6's at 91, 50 Mich. 7's at 112, 70 Cen. R. R. N. J. at $92\frac{1}{2}$, and 150 Lehigh Navigation ($50) at 46. What were the net proceeds of the entire sale?

31. The dividend on 50 shares of stock was $175. What was the rate per cent of dividend?

32. The dividend on 76 shares of stock was $342. Find the rate per cent. of dividend.

33. The dividend on $8100 par value of stock was $303.75. What was the rate per cent. of dividend?

34. How much must be paid for $10000 worth, par value, of U. S. 4's at $112\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

35. How many shares of stock at 4% discount can be bought *for $5000,* brokerage $\frac{1}{8}\%$?

$96.125
$5000 ÷ $96.125 = 52+, or 52 shares, and $1.50 over.

EXPLANATION.—Since the stock was bought at 4% discount, the cost of 1 share was $96 plus $⅛, brokerage, or $96.125. Hence, for $5000 as many shares can be bought as $96.125 is contained times in $5000, which is 52 times, or 52 shares with a surplus of $1.50.

36. How many shares of stock can be bought for $3000, if the stock is selling at 5% premium, brokerage ⅛%?

37. Long Island R. R. stock is quoted at 94. How many shares can be purchased for $4000, brokerage ⅛%?

38. When Penn. R. R. Stock is quoted at 108, how many shares can be secured through a broker for $8000, brokerage ⅛%?

39. I wish to invest $10000 in C. R. I. & Pac. stock. The quotation is 107. How many shares can be obtained, brokerage ⅛%?

40. How many shares of Ch. Mil. & St. P. R. R. stock can be obtained for $18,000 at 110⅝, if the brokerage is ⅛%?

41. Am. S. & W. stock ($100) was quoted at 72. How many shares could be bought for $2500, brokerage ⅛%?

42. How many shares of Reading R. R. stock, at 60½, can be bought for $12610?

43. What income will be derived from investing $5887.50 in 5% stock purchased at 98, brokerage ⅛%?

$5887.50 ÷ $98.125 = 60
60 × $5 = $300.

EXPLANATION.—Since the stock was purchased at 98 and the brokerage was ⅛%, each share cost $98.125; and for $5887.50 as many shares can be purchased as $98.125 is contained times in $5887.50, which is 60 times, or 60 shares. The income at 5% is $5 on each share, and on 60 shares it is 60 times $5, which is $300.

44. What income will be derived from $8606.25 invested in U. S. 4's coup. 1907, at 114⅝, brokerage ⅛%?

45. What income will a man derive from $11462.50 invested in U. S. 4's reg. 1907, at 114½, brokerage ⅛%?

46. Find the income from $9281.25 invested in Col. Mid. 4's at 74, brokerage ¼%.

47. If $7398 are invested in Chi. B. & Q. 3½'s at 102⅝, brokerage ⅛%, what will be the income?

48. What will be the income from $8680 invested in Sou. Ry. 5's at 108⅝, brokerage ⅛%?

49. How much must be invested in 5% bonds, purchased at 102½, brokerage ⅛%, to secure an annual income of $600?

$\$600 \div \$5 = 120$
$120 \times \$102\frac{3}{8} = \$12285$

EXPLANATION.—Since the income from 1 bond is \$5, to secure an income of \$600 it will require as many bonds as \$5 is contained times in \$600, which are 120 times, or 120 bonds. The price of each bond is $\$102\frac{1}{2} + \$\frac{1}{8}$, or $\$102\frac{3}{8}$. Hence, the 120 bonds will cost 120 times $\$102\frac{3}{8}$, or \$12285.

50. What sum must be invested in 4% bonds, at $97\frac{1}{2}$, to secure a yearly income of \$500, brokerage $\frac{1}{4}\%$?

51. How much must I invest in 7% bonds, at $119\frac{1}{4}$, to secure a yearly income of \$910, brokerage $\frac{1}{8}\%$?

52. How much must I invest in U. S. 4's, at 115, brokerage $\frac{1}{4}\%$, to secure an annual income of \$1000?

53. When Un. Pac. 4's are selling at $105\frac{1}{4}$, how much must be invested to produce an annual income of \$1200, brokerage $\frac{1}{8}\%$?

54. When the Met. El. 6's are selling at 115 how much must be invested to produce a yearly income of \$2400, brokerage $\frac{1}{8}\%$?

55. What per cent. income is received on an investment in 5% bonds at 25% premium?

$\$5 \div \$125 = .04$ or 4%.

EXPLANATION.—Since 1 bond costs \$125, and the income from it is \$5, the income is $\frac{1}{25}$, or 4% of the investment.

Or $r = \frac{P}{B}$. Hence, $r = \frac{5}{125} = .04$, or 4%.

56. What is my rate per cent. of income from 6% bonds bought at 75?

57. What is the rate per cent. of income realized on 7% bonds bought at 110?

58. Find the rate per cent. of income realized on 6% bonds at $96\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

59. Find the rate per cent. of income realized on $4\frac{1}{2}\%$ bonds bought at $87\frac{3}{8}$, brokerage $\frac{1}{8}\%$.

60. What is the rate per cent. realized on the investment when 8% bonds are bought at 160?

61. At what price must I buy 4% bonds to realize 5% on my investment?

Suggestion.—$B = \frac{P}{r}$. The percentage is \$4 per bond.

62. At what price must 7% bonds be bought to realize 8% on the investment?

63. What is the price of 9% bonds when only 6% is realized on the investment?

64. At what quotation must 5% bonds be bought to realize an come of 4%, brokerage $\frac{1}{8}$%?

$\$5 \div .04 = \$125$
$25 - \$\frac{1}{8} = \$124\frac{7}{8}$

EXPLANATION.—Since $5, the income of 1 bond, is 4% of its cost, the cost is $125; but this includes the brokerage, which is $\$\frac{1}{8}$. Hence, the quotation $125 minus $\$\frac{1}{8}$, or $\$124\frac{7}{8}$.

65. What is the quotation when 5% on the investment can be alized from 3% bonds, if the brokerage is $\frac{1}{8}$%?

66. Find the quotation when 8% on the investment is realized om 7% bonds, if the brokerage is $\frac{1}{8}$%?

67. What is the quotation when only 4% can be realized on the vestment in $5\frac{1}{2}$% bonds, if the brokerage is $\frac{1}{8}$%?

68. What is the average rate of income from 4% bonds bought 110, maturing in 20 years?

$20 \times \$4 = \$80$
100
$\$180$
110
$\$70$
$\$70 \div 20 = \$3.50$
$.50 \div \$110 = .0318+$
or $3\frac{18}{100}\% = 3\frac{1}{5}\%-$

EXPLANATION.—The income of one 4% bond in 20 years is $80, which, with the face value at maturity, give a return of $180. Since each bond cost $110 the income in 20 years is $70, or an average annual income of $3.50 per bond. Since they were bought at $110 each, and the average income on $110 is $3.50, the average rate of income is $3\frac{18}{100}\%+$, or $3\frac{1}{5}\%$ nearly.

69. What is the average rate of income from 5% bonds bought 112, maturing in 20 years?

70. What per cent. on the investment do $4\frac{1}{2}$% bonds, maturing 20 years, and bought at 95, pay?

71. Find the average rate of income from 6% bonds bought at 0, maturing in 15 years.

72. What per cent on the investment do 5% bonds, maturing in years, pay if bought at par?

73. How much may be paid for 5% bonds, maturing in 20 years, produce 4% income?

4 $5
20 20
80 $100
00 100
80 $200
00 $\div \$180 = 111\frac{1}{9}\%$
$111\frac{1}{9}$.

EXPLANATION.—A 5% bond bought at par will bring a full return of $200 in 20 years, but to produce only 4% income the full return is only $180. Hence, to produce only 4%, the price paid per bond bears the same relation to the par value that $200 bears to $180, or $111\frac{1}{9}\%$ of par value, or $111\frac{1}{9}$.

While $111\frac{1}{9}$ is mathematically correct, yet ice quotations in stocks and bonds have fractions only in halves, fourths, d eighths, the result obtained must be modified to nearest eighth, which very nearly correct.

74. How much may be paid for 6% bonds maturing in 20 years to produce an income of 5%?

75. What may be paid for 5½% bonds maturing in 20 years to produce an income of 6%?

76. How much may be paid for 7% bonds maturing in 30 years to produce an income of 5%?

77. On June 1, I bought 500 Balt. & Ohio pf. stock at 70 (b 30), and paid for the same on June 25. How much did the stock cost, interest included? How much did I gain, if I sold the stock at 75 on June 25?

78. Which is the better investment, 6% bonds bought at 118, or 5% bonds bought at 90?

79. If stock bought at 10% premium will pay 5% on the investment, what per cent. will it pay if bought at 10% discount?

80. A dealer in stocks bought Chic. & Nor. pf. at 197. If it paid a dividend of 12%, what was the rate of income to the investor?

81. When U. S. 4's are quoted at 104, what sum must be invested to yield an income of $1000 a year?

82. Mr. G. owns 500 shares of Cumulative stock of a certain corporation. He received in one payment $1500 as past dues and a dividend of 5%. How much did Mr. G. receive?

83. A company paid 4½% on Hypothecated bonds amounting to $897400. What was the sum paid?

84. Mr. R. has $6000 invested in 7% first mortgage bonds. He also owns 75 shares ($50) of a certain mining stock which pays him 5% semi-annually. How much is Mr. R.'s income from these two investments?

85. Mr. S. has an annual income of $975 from an investment of $15000 in Consolidated bonds. What rate per cent. does he receive?

86. What is the annual income from $12000 invested in bonds bearing 1¾% interest quarterly?

87. What would be the cost of 400 Mil. L. S. & W. 6's at 128½, brokerage ⅛%?

88. When Mor. & Es. cons. gtd. 7's are quoted at 130⅜, what *rate per cent.* does the stock pay on the actual investment, if the *brokerage is* ⅛%?

89. If 4% bonds ($100) should sell at 61 what rate per cent. would they pay on the investment, brokerage $\frac{1}{8}$%?

677. Buying and selling on margin.

1. On Mar. 1, 1900, I. M. Flush instructed his broker to buy for him 100 shares of Man. El. stock and deposited with the broker $1000 margin. On Mar. 10, the stock was purchased at 90. On Mar. 22, he sold the stock at 88. What was the balance due Mr. Flush on Mar. 31, and what was his loss?

I. M. FLUSH.

1900					1900				
Mar.	10	Bot. 100 Man. El. at 90$\frac{1}{8}$,	9012	50	Mar.	1	Cash deposited,	1000	
	22	Int. on Bal.,	16	03		10	Int. on deposit,	1	50
	31	Balance,	761	63		22	Sold 100 Man. El. at 87$\frac{7}{8}$.	8787	50
						31	Int. on Bal.,	1	16
			9790	16				9790	16

Margin deposited, $1000
Balance due, 761.63
Loss, $238.37

EXPLANATION.—The form here given shows a customer's account as commonly kept by a broker. The brokerage is added to the purchase and deducted from the sale, and interest is calculated on the balances. There is a credit balance of $1000 from Mar. 1 to Mar. 10, on which the interest is $1.50; a debit balance of $8012.50 from Mar. 10 to Mar. 22, on which the interest is $16 03; and a credit balance of $775 from Mar. 22 to Mar. 31, on which the interest is $1.16.

On closing the account, it is found that there is a balance of $761.63 due Mr. Flush. Since he had deposited $1000, and has a credit balance of only $761.63, his loss is the difference, or $238.37.

EXAMPLES.

2. Mr. L. K. Wise directed his broker to purchase 200 Del. & Hudson stock, and deposited $2000 margin, Jan. 1, 1900. On Jan. 7, the stock was purchased at 108, and sold Jan. 25, at 110$\frac{1}{8}$. How much did Mr. Wise gain, and what was the balance due him by the broker on Feb. 1?

3. On Apr. 2, 1900, I deposited with my broker $2500 margin for purchasing on the same day 250 Balt. & Ohio stock at 73$\frac{3}{4}$. He sold the stock Apr. 20, at 76$\frac{1}{4}$, and remitted the balance due me. *How much did he* remit, and what was my net profit?

4. Mr. Greene deposited $5000 margin with his broker to purchase 500 Peo. Gas & C. stock. The margin was deposited on May 25, and the stock was purchased on June 6, at 98½. On June 15, the stock was sold at 101. Find the balance due Mr. Greene on July 1, and his gain.

5. On Sept. 1, 1900, I. L. Risk deposited with his broker $5000 margin and instructed him to buy 500 shares of a certain stock at once. The stock was bought on the same day at 95. Instead of raising, as was expected, the stock immediately began to depreciate. On Sept. 25, Mr. Risk was forced by circumstances to instruct his broker to sell all the stock at the quotations for the day, which was 85⅜. What was Mr. Risk's loss?

678. Buying and selling short.

1. Wm. H. Snoke sold "short" July 18, 1900, through his broker 200 shares Fed. Steel pf. stock at 66, and "covered" the sale July 30, at 62. Mr. Snoke placed $2000 as margin with the broker on July 18. What was his net profit?

WM. H. SNOKE.

1900 July	30	Bot. 200 Fed. Steel pf. at 62½,		12125		1900 July	18	Cash deposited,		2000	
		Balance,		2754				Sold 200 Fed. Steel pf. at 65⅝,		13175	
				15179			30	Int. on deposit,		4	
										15179	

Balance, $2754
Deposit, 2000
Gain, $ 754

The manner and form of keeping the customer's account in buying and selling "short" differs little from the method when buying and selling on margin. A margin is usually deposited on which interest is allowed. On "short" sales the stock is often borrowed for delivery and replaced when purchase is made. No interest is charged for this accommodation.

EXAMPLES.

2. On Oct. 1, 1900, E. O. Wells deposited $1000 margin with his broker and sold through him 100 shares Den. & R. G. pf. stock at 66½, "short." On Oct. 15, the "short" was "covered" at 63⅜. What was the profit, the stock having been borrowed for delivery?

3. Nov. 8, 1900, I deposited $3000 margin and sold "short" through my broker 300 shares M. & St. L. pf. at 92, and Nov. 26 "*covered*" *my* "short" by buying at 90. Allowing 6% interest, *what was the balance* due me, and what was my net profit?

EXCHANGE.

679. Exchange is the system of making payments between persons in distant places by means of *Drafts* or *Bills of Exchange.*

680. A **Draft,** or **Bill of Exchange,** is a written order drawn by one person (the drawer) upon another person (the drawee) directing him to pay a specified sum of money to the order of the drawer or to a third person (the payee).

A draft, or bill of exchange, is frequently called an "exchange."

681. Domestic, or **Inland, Exchange** is the exchange between places in the same state or country.

682. Foreign Exchange is the exchange between places in different states or countries.

683. The **Balance of Trade** between two places is the difference in the value of their commercial transactions. This difference consists in the net amount due from one to the other.

Thus, if the purchases made by London of New York amount to $30,000,000 and the purchases made by New York of London amount to $28,000,000, the *balance of trade* is $2,000,000 in *favor* of New York, or *against* London.

684. Exchange is *at par* when a draft or bill sells for its face. It is *above par*, or at a *premium*, when it sells for more than its face; and *below par*, or at a *discount*, when it sells for less than its face.

1. The rate of exchange between two places depends upon the course of trade. If the trade between New York and Chicago is equal, exchange is at par. If New York owes Chicago more than Chicago owes New York, the drafts are at a premium in New York; for the demand in New York for drafts on Chicago is greater than the demand in Chicago for drafts on New York.

2. The reason why the banks in New York, in the above case, would charge a premium is because they must be at the expense of sending money to the Chicago banks, or be charged with interest on their unpaid balance. On the other hand, the Chicago banks would sell at a discount in order to get the money owed them in New York immediately.

3. The rate of exchange is limited by the cost of shipping gold or currency by express. The premium or discount will not exceed this cost; otherwise the money would be sent by express.

4. A time draft, or bill of exchange, is also subject to a bank discount on its face value for the time it has to run before becoming due.

DOMESTIC EXCHANGE.

685. Domestic Exchange is the exchange between two places in the same state or country.

686. A Bank Draft is a written order drawn by one bank upon another bank directing it to pay a specified sum of money to a third party or to his order.

BANK DRAFT.

City National Bank. **No. 1347.**

$365\frac{00}{100}$ NEW YORK, July 20, 1900.

Pay to the order of Wm. J. Milne $365\frac{00}{100}$

Three Hundred Sixty-five ~~~~~ *Dollars.*

TO FIRST NATIONAL BANK, Chicago, Ill.

G. S. WHITSON, Cashier.

EXPLANATION.—Suppose Wm. J. Milne of New York wishes to remit $365 to A. B. Cook in Chicago, he buys of a New York Bank a draft on a Chicago bank, having the draft made payable to the order of himself (as above). He then indorses it *in full*, "Pay A. B. Cook or order. Wm. J. Milne," encloses the draft in a letter, and mails it to A. B. Cook. When Mr. Cook desires the draft cashed, he goes to the bank with which he does business, or to any other bank, indorses the draft *in blank*, presents it for payment, and receives the money.

Mr. Milne could have had the draft written "Pay to the order of A. B. Cook" instead of to himself when he would have mailed it without any indorsement; but the former method is preferred because Mr. Milne's name appears upon the draft and the paper itself is evidence of payment by him to Mr. Cook. In cases of litigation, the draft can be obtained from the bank which issued it, as a bank's own cancelled drafts are always returned.

687. Bank drafts are the principal means by which remittances are made from one part of the country to another. Nearly all brokers keep money deposited with some one bank, called a correspondent, at one or more of the great money centers, such as New York, Chicago, Boston, St. Louis, Philadelphia, New Orleans, and San Francisco. Upon these correspondents banks draw drafts to sell to their customers for remittances to creditors. The banks make a profit on the drafts by charging "exchange," "premium," or "brokerage."

The drafts are drawn on the correspondent nearest to the creditors, and *are received* either at the bank drawn on or at any other bank tributary *to the money* center at their face value.

688. A **Commercial Draft** is a written order drawn by one person or firm requesting a second person or firm to pay to the order of the drawer or to a third person or firm, at sight or on time, a specified sum of money. (See p. 260).

1. Bank drafts are drawn against cash balances on deposit; commercial drafts are drawn to collect a debt. Business men generally draw their drafts in favor of the bank where they keep their deposits. This is called drawing on a person or firm *through the bank*. But they are also frequently drawn to cancel a debt. Thus, if A in Denver owes B in Boston $200, and in Boston owes C in Denver $200, B by drawing on A in favor of C, can avoid the expense and inconvenience of transmitting money to and fro. C in Denver presents the draft for payment to A of Denver. A pays C $200, and the indebtedness of both A and B is cancelled.

2. Remittances may also be made by Post Office money order, express money order, and telegraphic money order. Coin and currency may be sent by express or by registered letter. The various orders are sold at schedule prices which are subject to change, and not at a certain rate per cent. To register a letter costs eight cents besides the regular postage.

689. Computations in exchange are based upon the principles of percentage, the Face of Draft being the *base*, the Rate of Exchange the *rate*, the Premium or Discount the *percentage*, and the Cost of Draft the *amount or difference*.

Time drafts are discounted in the same manner as promissory notes, though it is usually more convenient to first find the cost of $1 of draft as a basis.

690. 1. What will be the cost of a sight draft of $200 at $\frac{1}{8}\%$ premium?

$\$1 + \$.00\frac{1}{8} = \$1.00\frac{1}{8}$.
$\$1.00\frac{1}{8} \times 200 = \$200.25$

EXPLANATION.—Since exchange is at $\frac{1}{8}\%$ premium, every dollar of draft will cost 1.00\frac{1}{8}$, and a draft of $200 will cost 200 times 1.00\frac{1}{8}$, or $200.25.

2. What will be the cost of a bank draft of $160 at $\frac{1}{8}\%$ premium?

3. What will be the cost of a draft of $300 at $\frac{1}{4}\%$ premium?

4. What will be the cost of a draft for $495.20 at $\frac{1}{8}\%$ premium?

5. What will be the cost of a draft for $108 at $\frac{1}{4}\%$ discount?

6. Find the cost of a draft of $275 at $\frac{3}{8}\%$ discount.

7. Find the cost of a draft of $367.50 at $\frac{1}{2}\%$ discount.

8. Find the cost of a draft on Chicago for $425 at $\frac{1}{4}\%$ premium.

9. Find the cost of a draft on Philadelphia for $800 at $\frac{1}{8}\%$ discount.

10. Find the cost of a draft on Charleston for $560 at $\frac{3}{4}$% discount.

11. What will be the cost of a draft in Topeka for $300 on Baltimore payable 60 days after date at $\frac{1}{4}$% premium, and interest at 6%?

$\$1 + \$.00\frac{1}{4} = \$1.00\frac{1}{4}$
$\$1.0025 - \$.01 = \$.9925$
$\$.9925 \times 300 = \$297.75$

EXPLANATION.—Since the exchange is at $\frac{1}{4}$% premium, every dollar of the draft would cost 1.00\frac{1}{4}$, or $1.0025, if paid at sight. But the draft is not to be paid in Baltimore for 60 days, hence the banker in Topeka has the use of the money for 60 days, and allows the bank discount on the face of the draft for that time. The bank discount at 6% for 60 days is $.01 on $1, which subtracted from $1.0025 gives $.9925, the cost of $1 of the draft. Since the draft is for $300, it will cost 300 times $ 9925, or $297.75.

12. What will be the cost of a 60-day draft on Bangor for $1200 at $\frac{3}{4}$% premium, interest at 6%?

13. How much must be paid for a draft of $560 on Denver, payable in 2 months, exchange being at $\frac{1}{4}$% discount, interest at 8%?

14. What will be the cost of a 90-day draft on New York for $1500 at $\frac{1}{8}$% premium, interest at 6%?

15. Find the cost of a draft on St. Louis for $1350, payable in 60 days, exchange being at $\frac{3}{4}$% discount, and interest at 8%.

16. What will be the cost of a draft on Savannah for $437.65 payable 3 mo. after date, exchange being at $\frac{1}{4}$% premium, and interest at 7%?

17. What is the face of a draft which can be bought for $796 when the exchange is at $\frac{1}{2}$% discount?

$\$1 - \$.005 = \$.995$
$\$796 \div \$.995 = 800$ times, or $800.

EXPLANATION.—Since exchange is at $\frac{1}{2}$% discount, a draft of $1 will cost $.995, and $796 will buy a draft for as many times $1 as $.995 is contained times in $796, or $800.

18. What is the face of a draft which can be bought for $457.80, when the exchange is at $\frac{5}{8}$% discount?

19. Find the face of a draft which can be bought for $1000, when the exchange is at $\frac{1}{4}$% premium.

20. Find the face of a draft which may be bought on New Orleans for $1488.75, when the exchange is at $\frac{3}{4}$% discount.

21. How large a draft on Kansas City may be bought in Washington *for $2000,* when the exchange is $\frac{3}{8}$% discount?

2. The cost of a Chicago draft at $\frac{3}{4}\%$ premium, payable 30 days r date, was $1604. What was the face of the draft, money g worth 6%?

$1. + $.0075 = $1.0075.
075 — $.005 = $1.0025.
04 ÷ $1.0025 = 1600 times, 1600.

EXPLANATION.—Since the exchange was at $\frac{3}{4}\%$ premium, it would cost $1.0075 to buy a draft of $1, payable at sight. But since the draft was payable 30 days after , the banker allowed the bank discount on the face of the draft for time, or $.005 on every dollar. Hence, a draft of $1 cost $1.0025; and st $1604, the face of the draft was as many times $1 as $1.0025 is con- d times in $1604, or $1600.

3. The cost of a 60-day draft at $\frac{1}{2}\%$ premium was $683.50. at was the face of the draft, money being worth 6%?

4. I bought a draft on New York payable 2 months after date % discount, for $580. What was the face of the draft, money g worth 6%?

5. Mr. M. bought a 45-day draft at $1\frac{1}{4}\%$ discount. What was face of the draft, money being worth 5%, that cost $981.25?

6. An agent in Chicago bought for a Boston firm 5000 bushels wheat at $71\frac{1}{4}$ cents a bushel. He charged $1\frac{1}{2}\%$ commission for ing the wheat. The firm sent him a bank draft for the full unt. What was the cost of the draft, exchange being at $\frac{1}{8}\%$ ount?

7. I bought goods of a wholesale merchant in Philadelphia unting to $2367 at 5% off if paid within 30 days. At the end 0 days I bought a 15-day draft at $\frac{5}{8}\%$ discount, to pay for the ds. What was the face of the draft and what did I pay for it, ney being worth 6%?

8. A New York firm received a shipment of flour from St. l, amounting to $6000, and remitted the money by a 30-day ft at 7%, exchange being at $\frac{5}{8}\%$ discount. What did the firm for the draft?

9. Mr. B. received a draft for $375, which cost him $\frac{1}{4}\%$ to get ished. What should have been the face to realize $375?

0. A cotton broker of Boston bought a 45-day draft on Mobile 7200 at $\frac{3}{8}\%$ discount, paying $\frac{1}{10}\%$ for brokerage. How much he pay for the draft, money being worth 6%?

FOREIGN EXCHANGE.

691. Foreign Exchange refers to drafts, or bills of exchange, drawn in one state or country and payable in another.

1. Foreign bills of exchange are usually expressed in the money of the country on which they are drawn. Thus, drafts drawn on persons and banks in England, Ireland, and Scotland, are expressed in pounds, shillings, and pence; on those in France, Belgium, and Switzerland in francs; on those in Germany in marks; on those in Holland in guilders.

2. The draft, or bill of exchange, is the oldest negotiable paper. The early Italian and Jewish merchants devised credit instruments which later developed into bills of exchange. Even as far back as the fourteenth century, bills of exchange of much the same form as those of the present day, were used and for similar purposes.

692. The Par of Exchange is the established value of the money of one country expressed in the currency of another.

1. The *intrinsic par* of exchange is the real value of the coin of one country in the coin of another, based upon the relative weight and fineness of the metals of which they are composed. Thus, the pound sterling, or sovereign, of Great Britain contains 113 grains of pure gold, and the dollar of the United States 23.22 grains of pure gold. The pound contains practically 4.8665 times as many grains of pure gold as the gold dollar, hence its intrinsic value is 4.8665 times as much, or $4.8665.

2. The *commercial par* of exchange is the market value of the currency of one country as compared with that of another, determined by the requirements of trade.

693. A Set of Exchange is a bill of exchange drawn in sets of two or three, of the same date and tenor, each containing a condition that it shall continue payable only while neither of the others is paid.

1. It was formerly the custom to send each bill of a set of foreign exchange by a different steamer to guard against loss or delay; and when one of the set was paid, the others were worthless. Now, owing to the efficient mail service which is rapid and about as sure as on land, only one is usually sent, especially between America and Europe, the others being retained.

2. Bills of exchange are usually made payable at sight or at sixty days sight. The former are frequently called "short" exchange, and the latter "long" exchange. The latter is sold at a rate below that of "short" exchange sufficient to make the difference in interest of the classes of bills. Three days of grace are usually allowed on all foreign bills.

3. Foreign bills of exchange are drawn upon Amsterdam, Berlin, Hamburg, and other commercial centers, but drafts upon London and Paris are *much more common*, because they are paid in any part of Europe.

Set of Exchange.

1

Exchange for £150. New York, July 25, 1900.

Sixty days after sight of this First of Exchange (Second and Third of same date and tenor unpaid), pay to Cyrus Leidig, or order, One Hundred Fifty Pounds Sterling. Value received, and charge the same to the account of

To Brown Bros.,
London.
No. 1267.

P. D. Bermont & Sons.

2

Exchange for £150. New York, July 25, 1900.

Sixty days after sight of this Second of Exchange (First and Third of same date and tenor unpaid), pay to Cyrus Leidig, or order, One Hundred Fifty Pounds Sterling. Value received, and charge the same to the account of

To Brown Bros.,
London.
No. 1267.

P. D. Bermont & Sons.

3

Exchange for £150. New York, July 25, 1900.

Sixty days after sight of this Third of Exchange (First and Second of same date and tenor unpaid), pay to Cyrus Leidig, or order, One Hundred Fifty Pounds Sterling. Value received, and charge the same to the account of

To Brown Bros.,
London.
No. 1267.

P. D. Bermont & Sons.

694. A Documentary Bill of Exchange is a bill drawn by a shipper upon his consignee against merchandise shipped, accompanied by a bill of lading indorsed to order of the payee, and an insurance certificate covering the merchandise against which the bill is drawn.

695. A Letter of Credit is a letter issued by a banking house to a person who expects to travel abroad, and addressed to its foreign correspondents requesting them to furnish the traveler with money in sums to suit his convenience until the total amount specified in the letter has been obtained.

The advantages of a letter of credit are that it is payable at several places, at different times, and in variable sums. A bill of exchange is payable only at one time and in one payment.

696. A Cable Transfer is a remittance of money to a foreign country by telegraph (cablegram).

Cable transfers are made through bankers. The amount of business done in this way is constantly increasing.

697. The **Rate of Exchange** is the market value in one country of bills of exchange drawn on another.

1. The commercial par of exchange is about the same as the intrinsic value of the coin represented by its face plus the express charges and insurance of shipping coin or bullion from one country to another. When it is greater than this, gold can be exported at a profit; and when less than its intrinsic value and cost of shipment and insurance, gold can be imported at a profit.

2. When quotations of Foreign Exchange are given, exchange on England is quoted by giving the value of £1 in dollars and cents; exchange on France, Belgium, and Switzerland, by giving the value of $1 in francs and centimes (hundredths of a franc); exchange on Germany by giving the value of 4 marks (reichsmarks) in cents; exchange on Amsterdam by giving the value of 1 guilder (gülden) or florin in cents.

3. When bills of exchange are bought and sold through brokers, the brokerage is computed on the market value and added to the market value to find the total cost to the purchaser, or deducted from the market value to find the net proceeds due the seller.

698. 1. What will be the cost of a bill of exchange on Liverpool for £250 8 *s.* 6 *d.*, when exchange is $4.86¾?

Solution.—£250 8 *s.* 6 *d.* = £250.425. Since £1 costs $4.86¾, £250.425 will *cost 250.425 times* $4 86¾, or $1218.94.

2. A bill of exchange on London cost $2565.30. What was the face of the bill, exchange being $4.87?

Solution.—$2565.30 ÷ $4.87 = 526.7556, or £526.7556. £526.7556 = £526 15 *s.* 1 *d*, the face of the bill.

EXAMPLES.

3. What will be the cost of a bill of exchange on London for £150, exchange at $4.86¼?

4. What will be the cost of a sight exchange on London for £445 15 *s.* when exchange is quoted at $4.86½?

5. A merchant in New York bought a bill of exchange on London for which he paid $3600. What was the face of the bill, exchange at $4.85?

6. Find the cost of a bill of exchange on Liverpool for £185 12 *s.* 6 *d.*, when the quotation is $4.86⅛.

7. I purchased through a broker a bill of exchange on London of £540 at 4.85⅝. The brokerage was ⅛%. What was the total cost?

8. What will be the cost of a bill of exchange on Paris for 3000 francs, when exchange is at 5.20?

Suggestion.—See p. 292, note 2.

9. Find the cost of a bill of exchange on Paris for 500 francs, bought at 5.19½.

10. What will be the cost of a bill of exchange on Hamburg for 1200 marks when exchange is at 94½?

11. What will be the cost of a bill of exchange on Berlin for 10000 marks when exchange is quoted at 95¼?

12. I sold a bill of exchange on Berlin for 12000 marks, through a broker who charged me ⅛%. What were the net proceeds due me?

13. Find the cost of a bill of exchange on Amsterdam for 6000 guilders, at 40⅛.

14. What will be the cost of a draft on Bremen at 60 days' sight of 4560 marks when exchange is at 94¼?

Suggestion.—The exchange 94¼ is the quotation for 60 days. Quotations of Foreign Exchange are usually given for sight bills, and also for bills at 60 days. See Art. 693, note 2

15. A Philadelphia merchant bought a bill of exchange on Amsterdam at 60 days' sight for 16784 guilders, at 39⅞. How much did he pay for the bill?

16. The cost of a bill of exchange on Lyons, bought at 5.20, was $1263.75 which included a brokerage of $\frac{1}{8}\%$. What was the face of the bill?

17. A merchant of London bought 25000 bushels of Minnesota wheat at 70 cents a bushel, and remitted a draft in settlement What did he pay for the draft, exchange being $4.86\frac{5}{8}$?

18. When Penn. R. R. stock is quoted at 128, what should be the equivalent London quotation of the stock, exchange being 4.86?

128% of $5 = $6.40
$6.40 ÷ $4.86 = 1.3168+,
or $131\frac{3}{4}\%+$ or $131\frac{3}{4}$.

EXPLANATION.—American securities are quoted in London on a fixed basis of $5 to the £. Hence, to find the equivalent London quotation, multiply the American quotation by 5 and divide the product by the rate of exchange. (See explanation to problem 73, page 281.)

19. Find the equivalent London quotation of C. R. I. & Pac. R. R. stock quoted in New York at 108, exchange at $4.86\frac{1}{2}$.

20. Am. Tin Plate pf. stock was quoted in New York at $74\frac{1}{2}$. What was the equivalent quotation in London if the exchange was 4.85?

21. When Del. & Hud. R. R. stock is quoted in London at $112\frac{1}{2}$, what is the equivalent New York quotation, exchange at 4.86?

Suggestion.—Multiply the London quotation by the rate of exchange and divide the product by 5.

22. When Louis. & Nash. R. R. stock is quoted in London at 76, what is the equivalent New York quotation, if the rate of exchange is $4.86\frac{3}{4}$?

23. When Chic. & Nor. R. R. stock is quoted in New York at 197, what is the equivalent London quotation, the course of exchange being $4.85\frac{5}{8}$?

24. When Union Pac. pf. stock is quoted in London at 80, what is the equivalent New York quotation, exchange being quoted at $4.86\frac{7}{8}$?

25. An English manufacturer drew a bill of exchange for £360 15 *s.* 6 *d.* on a Chambersburg, Pa., merchant. The draft was presented to the drawee by a local bank, and paid by check. Find the face of the check, exchange being $4.85\frac{1}{2}$, and collection $\frac{1}{8}\%$.

26. A New York Jeweler bought a bill of exchange on Geneva for which he paid $5785. What was the face of the bill, the course of exchange being $5.16\frac{1}{8}$?

BANKS AND BANKING.

699. A **Bank** is an incorporated institution which deals in money and commercial paper.

700. The business of a bank consists chiefly in receiving deposits of money for the safe keeping and convenience of customers, lending money, discounting commercial paper, issuing notes for circulation, and in facilitating the sending of money from one place to another by means of checks, drafts, or bills of exchange.

Banks receive deposits without charge for safe keeping, and usually pay no interest on them. Loans, however, are made from deposits at a lawful rate and in sums sufficient to fully pay for keeping the accounts of depositors.

701. Banks are of two principal classes, National banks and State banks.

There are also banks conducted by private individuals just as any other business. These are called Private banks.

NATIONAL BANKS.

702. A **National Bank** is one which is organized under the National Banking Act of Congress.

The National Banking Act provides for the organization of banking associations to be formed of any number of persons not less than five.

No banking association can be organized with a capital of less than $100000 except in towns whose population does not exceed 6000, where, with the approval of the Secretary of the Treasury, they may be formed with a capital of $50000, and in towns of less than 3000 inhabitants with a capital of $25000.

In cities of over 50,000 inhabitants, the capital must not be less than $200000. This is to be divided into shares of stock of $100 each.

The stockholders of national banks are individually liable equally and ratably, and not for one another, for the debts of the bank to an amount equal to their stock in the bank at the par value thereof, in addition to the amount invested in such shares.

National banks are not allowed to lend money on real estate security; and real estate purchased or mortgaged to secure a previous debt must be disposed of within five years.

All national banks are subject to periodical visitation and examination by the National Bank Examiner who can, upon the detection of any dishonesty or failure to comply with the law, order a bank closed for full investigation.

703. National Bank Notes. A national bank may issue notes for circulation by depositing as security with the United States Treasurer registered bonds. These bonds are held as security for the circulating notes, which in case the bank should fail, the Government will redeem. A bank is allowed to issue circulating notes to the amount of the par value of the bonds deposited, but in no case can a bank have a circulation greater than its capital stock actually paid in.

1. A national bank is required to keep on deposit in the treasury of the United States, a sum equal to 5% of its circulation for the purpose of redeeming its notes.

2. No national bank may complete its organization or commence the banking business until it shall have deposited with the Treasurer of the U. S. interest bearing U. S. bonds to an amount not less than one-fourth of the capital.

704. Reserve Fund. The national banks in the cities of Boston, Albany, New York, Philadelphia, Baltimore, Washington, Pittsburg, Cleveland, Detroit, Chicago, Milwaukee, Louisville, St. Louis, New Orleans, and San Francisco, are required to keep a reserve fund equal to 25% of their deposits; and all other national banks are required to keep 15% as a *Reserve Fund.* Any excess above the requirements is called the *Surplus Reserve.*

705. Surplus Fund. National banks are also required by law to set aside, before the usual semi-annual dividends are declared, $\frac{1}{10}$ of their net earnings for the preceding half year as a *Surplus Fund* until this fund amounts to 20% of the capital.

706. Annual Tax. National banks pay, under section 13 of the Act of Congress approved March 14, 1900, a semi-annual tax of $\frac{1}{4}$ of 1% to the Government on the average amount of their circulating notes.

***707.* Advantages.** While a tax is paid upon the circulation, a *national bank* possesses the following advantages: It is exempt

from a tax on bonds deposited with the United States Treasurer, and also receives interest upon them. At the same time the bank lends and uses its circulating notes at a profit. In other words, the bank receives interest on the bonds deposited with the Government, and may issue the par value of the bonds in national bank notes which cost the bank but a trifle for the paper and engraving, and which it lends and uses as any other currency, thus making double interest on the same capital.

A national bank desiring to go into liquidation is required by law to deposit with the United States Treasurer six months before such liquidation, an amount of lawful money equal to its outstanding circulation. A sufficient amount of the money thus deposited must remain in the Treasury until the last outstanding note shall have been presented for payment. The Government derives the benefit of lost or destroyed notes.

708. 1. In a town of 4739 inhabitants it is desired to organize a national bank. How many persons are necessary for organization and what amount of capital is required?

2. A city having a population of 51209, wishes to have a new national bank. What must be the amount of capital necessary to organize the bank, and what will be the par value of a share of the stock?

3. A national bank having a paid up capital of $300,000 desires to issue circulating notes. What is the least amount of registered bonds that it can deposit as security for such notes? What amount of notes will it receive?

4. A bank deposited $375000 in bonds with the Government. How much is its redemption fund?

5. What is the annual tax paid by a bank having an average circulation of $287654?

6. A national bank in Philadelphia had deposits amounting to $675420. What was its reserve fund?

7. A certain national bank had a capital of $750000, but by poor management it lost $180000. The loss was made up by assessment of the stockholders. What was the rate of assessment?

8. Mr. B. owned 25 shares of stock in the bank referred to in the last problem. How much did he have to pay?

9. A national bank with a capital of $825000 earned $37500 net profits in 6 months. The surplus fund was 15% of the capital. What amount was carried to this fund, and how much will remain of the profits *after* declaring a semi-annual dividend of 3½%?

SAVINGS BANKS.

709. A **Savings Bank** is an institution which receives deposits of money for safe keeping, and allows interest to depositors.

A savings bank supplies each of its depositors with a bank book in which are entered the sums as deposited or withdrawn. At certain regular periods the interest at a fixed rate is computed on all sums that have been on interest for the required term. This interest, if not withdrawn, is placed to the credit of the depositor and draws interest the same as deposits.

710. An **Interest Term** is the period of time for which interest is regularly declared on deposits.

1. The interest term is usually 6 months, commencing Jan. 1 and July 1; but with some banks it is only three months, commencing Jan. 1, Apr. 1, July 1, and Oct. 1.

2. Some banks allow interest on money which has been on deposit during an entire quarter previous to interest day, while others allow interest on sums which have been deposited on or before the first of any month from the beginning of the month to the end of the interest term. No interest is allowed on sums withdrawn before interest day.

711. Savings banks are conducted in accordance with the laws of the state in which they are located. Some states place a limit to the size of deposits, fix the rate of interest which the banks must allow, and in other ways direct or control their management.

712. Interest is not usually allowed by savings banks on money deposited or withdrawn during an interest term. Hence, interest calculations are generally made according to the following:

RULE.—*Compute the interest at the end of the interest term on the smallest balance on deposit at any time during the term.*

To find the interest for more than one term, consider the interest at the end of each term as a deposit on the first day of the next term.

In the following problems, unless otherwise stated, deposits draw interest at 4% per annum, and the interest terms are 3 months, beginning Jan. 1, Apr. 1, July 1, and Oct. 1.

EXAMPLES.

713. 1. On Feb. 5, Mr. J. deposited $200 in a savings bank. How much interest will be placed to his credit on July 1?

Suggestion.—The deposit begins to draw interest April 1. 4% per annum is 1% per quarter.

2. A person is credited with the following deposits: Jan. 1, *1899, $150;* Apr. 1, 1899, $180; and July 20, 1899, $300. What *was the amount* to his credit Jan. 1, 1900?

3. On Mar. 1, 1897, Mr. A. C. Wilson deposited $130; Oct. 15, 1897, $100; Jan. 21, 1898, $160; and June 15, 1898, $40. What will be his credit in bank Jan. 1, 1899?

4. Mr. S. made the following deposits in a savings bank during the year 1899: Jan. 1, $100; Feb. 10, $50; Mar. 17, $40; July 20, $75; Sept. 1, $60; Nov. 5, $85; and Dec. 1, $30. How much was due the depositor Jan. 1, 1900, the interest term being 6 months and computing interest at 4% from the first day of each calendar month?

5. The balance due a depositor Jan. 1, 1903, at a savings bank, was $300. On Jan. 15, he deposited $100; Feb. 20, deposited $60; April 1, deposited $75; May 21, withdrew $80; June 10, deposited $50; July 10, withdrew $60; Aug. 4, withdrew $100; Sept. 1, deposited $200; Sept. 25, deposited $90. What was the balance due the depositor Oct. 1, 1903, interest credited quarterly at 4%?

Date.	Withdrawals.	Deposits.	Balances.
1903			
Jan. 1			300
15		100	400
Feb. 20		60	460
Apr. 1			460
1		Int. 3	463
1		75	538
May 21	80		458
June 10		50	508
July 1			508
1		Int. 4.58	512.58
10	60		452.58
Aug. 4	100		352.58
Sept. 1		200	552.58
25		90	642.58
Oct. 1			642.58
1		Int. 3.53	646.11

EXPLANATION.—Since the interest is credited quarterly it is added to the balance on the first of each quarter. As no interest is allowed on money deposited or withdrawn during the quarter, interest is computed on the smallest balance in the quarter (Art. 712). The smallest balance in the first quarter is $300; and its interest for 3 months at 4% is $3, which is added to the balance at the beginning of the second quarter. The smallest balance during the second quarter is $458, and its interest is $4.58, which is added to the balance at the beginning of the third quarter. The smallest balance during the third quarter is $352.58, and its interest $3.53, which is added to the balance ($642.58) on Oct. 1, making a credit balance of $646.11.

6. The following is a depositor's account upon the ledger of a savings bank: Apr. 1, 1898, $260; Apr. 1, 1898, deposit $200; May 6, *deposit $50*; May 27, withdrawal $60; June 10, deposit

$36; July 14, deposit $70; July 30, withdrawal $75; Aug. 6, deposit $40; Sept. 2, deposit $100; Sept. 20, withdrawal $65. What was due the depositor Oct. 1, 1898, interest at 4%, credited quarterly?

7. On Oct. 1, 1895, a teacher deposited in a savings bank $40; on Nov. 5, he deposited $50; Dec. 10, deposited $75; Dec. 20, withdrew $30; Jan. 15, 1896, deposited $60; Feb. 5, withdrew $25; Mar. 12, deposited $45; Apr. 1, deposited $70; May 30, deposited $100. What was the balance to his credit July 1, 1896, interest at 4% credited quarterly?

8. A depositor had a balance of $534 to his credit at a savings bank on Jan. 1, 1898. Jan. 5, he deposited $250; Jan. 28, deposited $140; Feb. 10, withdrew $100; Feb. 28, deposited $300; Mar. 30, deposited $180; Apr. 12, withdrew $80; Apr. 26, deposited $200; May 10, deposited $120; June 1, deposited $75; June 20, deposited $240; July 1, deposited $250; Oct. 1, deposited $100; Nov. 1, deposited $175; Dec. 7, deposited $125. What was the amount of his credit on Jan. 1, 1899, interest at 4% on average monthly balances, and credited semi-annually?

Suggestion.—The average monthly balance for an interest term is equal to the sum of the smallest balances in each month of a term divided by the number of months in the interest term.

9. A person deposited in a savings bank Jan. 1, 1897, $250; Jan. 20, deposited $60; Feb. 5, deposited $10; Mar. 1, withdrew $50; Mar. 19, deposited $75; Apr. 1, withdrew $30; Apr. 12, deposited $80; May 30, deposited $100; June 5, withdrew $50; June 18, deposited $40. What was the balance due the depositor July 1, 1897, interest at 5% on average monthly balances?

10. Find the balance due James E. Aitkin at the end of the year, in a savings bank which allows 4% interest on average monthly balances, and credits the interest quarterly, his account showing the following entries: Jan. 1, 1900, credit balance $750; Jan. 20, deposit $136; Jan. 30, deposit $225; Feb. 10, withdrawal $75; Feb. 24, deposit $350; Mar. 1, deposit $275; Mar. 24, deposit $400; Mar. 31, withdrawal $35; Apr. 2, withdrawal $134; Apr. 14, deposit $580; May 5, deposit $205; May 23, deposit $417; May 31, withdrawal $600; June 4, deposit $65; June 16, deposit $320; June 30, deposit $156; July 7, deposit $237; July 21, withdrawal $710; Aug. 10, deposit $1200; Aug. 30, withdrawal $125; Sept. 1, withdrawal $75; Oct. 1, deposit $95; Oct. 31, deposit $550; Nov. 9, deposit $187; Dec. 6, withdrawal $215; Dec. 15, withdrawal, $163; Dec. 27, deposit $300.

TAXES.

714. A **Tax** is a sum of money assessed on person or property required to be paid for the support of the government or for other public purposes.

715. A **Property Tax** is a tax assessed on the property of an individual.

1. Property is of two kinds; *Real Estate* and *Personal Property*. *Real Estate* is fixed property, as lands, buildings, etc. *Personal Property* is movable property, as horses, cattle, money, stocks, bonds, merchandise, etc.

2. Taxes are reckoned at a certain rate per cent. upon the assessed or estimated value of the property.

716. A **Capitation** or **Poll Tax** is a fixed sum assessed upon the person of every adult male citizen not exempt by law.

717. An **Assessor** is a public officer chosen to estimate the value of taxable property.

An *Assessment Roll* is a list of the names of the taxable citizens, with the assessed valuation of each one's property, and the amount of his tax.

718. A **Tax Collector** is a public officer chosen to receive and collect the taxes of his collection district.

719. Property taxes may be computed according to the principles of percentage, the assessed value being the *base* and the tax the *percentage*.

Property is usually assessed at less than its real or market value.

EXAMPLES.

720. 1. What is the tax on property whose assessed valuation is \$6500, if the rate of taxation is $\frac{3}{5}\%$?

2. What is the tax on property whose assessed valuation is \$15400, if the rate of taxation is $7\frac{1}{2}$ mills on the dollar?

3. What is the tax on property whose assessed valuation is \$8400, if the rate of taxation is \$1.05 on \$100?

4. If the rate of taxation is 72 cents on \$100, what would be the tax on property assessed at \$10500?

5. A citizen of a certain town has real estate assessed at $12000 and personal property assessed at $2,500. What was his entire tax at 6½ mills on the dollar?

6. Mr. C. has real estate worth $16500 which was assessed at ⅔ of its value. What was his tax at the rate of $1.02 on $100, including the payment for 2 polls at $1.25 each?

7. A tax of $10450 is to be levied on a town whose assessed valuation is $1340000, and which contains 735 taxable polls. The poll tax is $1 each. What is the rate of taxation, and what will be the tax of David Nye whose property is assessed at $8700, and who pays for 2 polls?

Solution.

$10450 − $735 = $9715, amount of property tax.
$9715 ÷ $1340000 = .00725, or 7¼ mills on $1, or 72½¢ on $100.
87 × $.725 = $63.08, Nye's property tax.
2.00, " poll tax.
$65.08, " entire tax.

721. In the assessment of taxes in a town, city, etc., a table is usually prepared by the use of which the labor of finding the tax is greatly reduced. The rate at which this table is computed is .00725.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	$.00725	$10	$.0725	$100	$ .725	$1000	$7.25
2	.01450	20	.1450	200	1.450	2000	14.50
3	.02175	30	.2175	300	2.175	3000	21.75
4	.02900	40	.2900	400	2.900	4000	29.00
5	.03625	50	.3625	500	3.625	5000	36.25
6	.04350	60	.4350	600	4.350	6000	43.50
7	.05075	70	.5075	700	5.075	7000	50.75
8	.05800	80	.5800	800	5.800	8000	58.00
9	.06525	90	.6525	900	6.525	9000	65.25

8. What is E. P. Long's tax who has property in the town assessed at $7650?

Solution.

Tax, by table on $7000 = $50.75
" " " " 600 = 4.35
" " " " 50 = .36
" " " " $7650 = $55.46, Long's property tax.

What is C. W. Gibb's tax whose property is assessed at), and who pays for 3 polls?

What is J. K. Berry's tax whose property is assessed at)0?

gestion.—$43000 are 43 times $1000, or 5 times $8000 + $3000.

The assessed valuation of the property of a town is)000. The estimate of corporate expenses are $7000 for ls, $2500 for streets, $1500 for salaries, $1000 toward the rt of the poor, and $250 for sundry expenses. The town ins 1250 taxable polls at $1.25 each. What will be the tax dollar, and what will be Mr. Best's tax whose property is as- l at $15400, and who pays for 2 polls?

What is the rate of taxation in a town whose taxable prop- is $680000, the net tax being $5301, and the collector's com- on 5%?

DUTIES, OR CUSTOMS.

2. **Duties,** or **Customs,** are taxes levied on imported goods ie support of the Government or the protection of home in- ies, or for both purposes.

3. Duties are of two kinds; *ad valorem* duties and *specific* s.

4. An **Ad Valorem** (*according to value*) **Duty** is a tax levied ertain per cent. of the cost of the goods in the country from ı they are imported, as shown by the invoice.

This duty is computed on the prime cost of the goods; that is, on the goods just ready for shipment and before any charges at all have dded, such as commission, drayage, freight, etc.

Duties are not computed on fractions of a dollar. A fraction of a , if less than one half, is rejected; if over one half, is counted as a dollar.

5. A **Specific Duty** is a tax levied on imported goods at a in sum per pound, ton, gallon, yard, or other measure with- egard to their value.

In computing specific duties the *long ton* (2240 lb.) is used, the hundred t being regarded as 112 lb., and the quarter at 28 lb.

Some *goods are subject* to both a specific and an ad valorem duty.

726. An **Invoice,** or **Shipper's Manifest,** is an itemized list of merchandise shipped or sold and its value in the country where produced.

Invoices must be made out in the weights, measures, and currency of the country from which the goods are imported; the value in United States money of the foreign currency being annually proclaimed for this purpose by the Secretary of the Treasury.

727. Allowances are deductions made before estimating specific duties; as, *Tare*, *Leakage*, *Breakage*, etc.

728. Tare is a deduction made for the weight of a box, bag, cask, etc., containing the goods.

729. Leakage is a deduction made for waste of liquids in casks or barrels.

730. Breakage is a deduction made for waste of liquids in bottles.

731. A **Tariff** is a list of goods showing the rate of duty fixed by law upon the articles of merchandise enumerated.

732. A **Custom House** is a Government building or office established for the purpose of collecting duties, and for the entry and clearance of vessels.

733. A **Port of Entry** is a port at which a custom house is located.

The principal port of entry in the United States is New York, but all of the important seaports are ports of entry.

734. A **Bonded Warehouse** is a building used for storing imported goods until the duties upon them are paid.

1. Goods held for duties are said to be "in bond." For each year in bond 10% extra is charged; and if left for more than three years, the goods are considered as abandoned to the Government, and are advertised and sold at auction to pay for the duties.

2. Goods may be withdrawn from a bonded warehouse for export without the payment of duty; or if paid it will be refunded. The amount of duty thus refunded is called a *drawback*.

735. Smuggling is bringing goods into the country stealthily *to avoid* the payment of duties.

Smuggling is a crime against which the laws prescribe severe penalties.

736. Internal Revenue is the tax levied on goods manufactured and sold in this country for the support of the Government.

This tax is usually collected by means of stamps issued by the Government, and required by law to be affixed to packages containing the articles to be taxed.

737. Ad Valorem Duties are calculated by the principles of percentage in which the Net Value, or Quantity, is the *base*, the Rate Per Cent. Ad Valorem the *rate*, and the Duty the *percentage*.

Reduce foreign currencies to United States money by the table of equivalent values on page 129.

738. 1. What is the duty on imported goods invoiced at £65 8 *s.*, at 20% ad valorem?

$4.8665
65.4
194660
243325
291990
318.26910
.20
63.60, duty.

EXPLANATION.—Since £1 is worth $4.8665, £65 8 *s.*, or £65.4 are worth $318.27. Since the duty is 20%, it is .20 of $318 (cents rejected), or $63.60.

EXAMPLES.

2. What is the duty on imported goods invoiced at £216 15 *s.* at 5% ad valorem?

3. What is the duty, at 30%, upon an importation of 500 dozen kid gloves invoiced at 70 francs per dozen?

4. What is the duty on goods invoiced at 7850 marks, allowing 2% for tare, at 25% ad valorem?

5. What is the duty, at 35%, on an importation of goods from Para invoiced at 5765 milreis?

6. Find the duty, at 24%, on an importation of goods from Yokohama invoiced at 24835 yen.

7. Find the duty, at 40%, on an importation from Spain invoiced at 18760 peseta, allowing 5% for breakage.

8. What is the duty, at 20%, upon a consignment of watches from Geneva invoiced at 12750 francs?

9. The duty upon a certain class of goods is 15%. What will be the duty on 8756 lb. at 45¢ per lb.

10. If the duty on a refined oil is 25%, what will be the amount of duty on 450 barrels of 38 gallons each, invoiced at 40¢ per gallon, allowing *4% for leakage*?

11. What is the duty, at 30% ad valorem, on 60 tons of steel invoiced at 4¼¢ per pound?

12. What is the entire duty on 960 yards of Brussels carpet 30 inches wide, invoiced at 8.25 francs per yard, the ad valorem duty being 25%; and the specific duty 15¢ per square yard?

13. Find the entire duty on an importation of 1200 yards of goods weighing 264 pounds net, and invoiced at 75 cents per yard, the duty being 14¢ per pound and 30% ad valorem?

14. A manufacturer imported 50 bales of wool weighing 320 lbs. each, tare 5%, invoiced at 1 *s.* 6 *d.* per lb. The specific duty was 4¢ per lb., and the ad valorem 10%. What was the entire duty?

15. A merchant imported from Munich an invoice of goods. The packing cost 35 marks and the shipping 42 marks. The entire bill was 3572 marks. What was the duty at 33⅓%?

16. What is the duty at 25% ad valorem, on an importation invoiced at 15875 lira, allowing 8% for breakage?

17. A New York merchant imported 2000 yards of carpet invoiced at 5.75 francs per yard on which a discount of 5% was allowed. The shipping charges were 125 francs, and the commission was 2½%. The net weight was 1526 lb. What was the entire cost of the goods, the specific duty being 12 cents per lb. and the ad valorem duty 25%?

18. J. D. Airy & Co. imported from Lyons a box of silks invoiced at 12750 francs. The shipping charges were 64.25 francs; commission 2%; net weight 1194 lb. What was the entire cost at 14¢ specific duty and 30% ad valorem duty?

19. What is the duty on 20 hhd. of sugar, each weighing 4 cwt., tare 16 lb. per hhd., at the rate of $1¼ per cwt., and 15 hhd. of molasses, 63 gallons each, having a duty of 5¢ a gallon?

20. A merchant imported from Sheffield a box of cutlery invoiced at £185 12 *s.* 6 *d.* on which he was allowed a discount of 10% and 5%. The charges were £6 5*s.*; commission 2½%; the net weight 510 lb. The specific duty was 18¢ per lb., and the ad valorem duty 35%. What was the entire cost of the goods?

INSURANCE.

739. Insurance is a contract of indemnity for loss or damage within a specified time. Insurance is of two kinds, *Property Insurance* and *Personal Insurance.*

740. The **Insurer,** or **Underwriter,** is the party or company undertaking the risk.

741. The **Insured** is the party secured against loss or damage.

742. Insurance companies are chiefly of two kinds, *Stock Insurance Companies* and *Mutual Companies.*

743. A **Stock Insurance Company** is one owned by stockholders who alone share the profits and are liable for the losses.

744. A **Mutual Insurance Company** is one in which the insured share in the gains and losses.

745. The **Policy** is the written contract between the person whose property is insured and the insurance company.

746. A **Valued Policy** is one in which the value of the property insured is specified.

747. An **Open Policy** is one in which the value of the property to be insured is not definitely known when the policy is written.

This form of policy is used by persons who insure goods which are to be conveyed from one place to another, and at different times.

748. The **Premium** is the sum paid for insurance, and is usually a certain rate per cent. of the amount insured.

The rates of premium depend upon the nature of the risk and the length of time for which the policy is issued.

749. The **Term of Insurance** is the period of time covered by the policy.

PROPERTY INSURANCE.

750. Property Insurance is a contract which provides security against loss of property. It includes *Fire Insurance*, *Marine Insurance*, *Live Stock Insurance*, etc.

Among other forms of property insurance may be mentioned *Plate Glass Insurance*, *Steam Boiler Insurance*, *Tornado Insurance*, and *Transit Insurance*.

751. Fire Insurance gives indemnity for loss or damage by fire.

752. Marine Insurance provides indemnity for loss or damage to vessels or their cargoes through the perils of navigation.

753. Live Stock Insurance affords indemnity for loss of horses, cattle, etc., or damage to them, by lightning or other casualty.

754. Fire Insurance Losses are usually adjusted by the insurance company paying the full amount of the loss up to the limit of the policy, unless the policy contain the "*percentage co-insurance* clause" which provides that the company shall pay only such proportion of the loss as the sum insured bears to the full value of the property.

1. Thus, if property worth $9000 were insured for ⅔ of its value, the company in case of loss, under the "percentage co-insurance clause" would pay on ⅔ of the loss; that is, in case of a total loss, the company would pay $6000; in case of a loss of $3000, it would pay $2000, etc.

2. Fire insurance companies may insure property at full value or for more than its value, but according to the terms of the policy the insured can only recover for loss or damage to the extent of the actual loss. They prefer that the insured be interested in its protection to the extent of part of the risk.

755. Certain fire insurance policies contain the following co-insurance clause:

"If at the time of the fire the whole amount of insurance on the property covered by each separate item of this policy on property as described in such item shall be less than 80% of the actual cash value thereof, this Company shall in case of loss or damage be liable for only such proportion of such loss or damage as the amount insured under said item shall bear to the said 80% of the actual cash value of the property covered by such item."

756. Marine Insurance Policies are issued on vessels for a specified time and on cargoes for a certain voyage.

In marine insurance the insurers are usually allowed one-third for the superior value of the new material used in repair of damage.

When a vessel sustains a loss, the insurance company pays only such a *proportion of the loss* as the amount of insurance bears to the total value of *the property.* Marine insurance policies usually contain the "*percentage clause.*"

757. Short Rates are certain rates of premium charged by ɔmpanies when the term of insurance is less than a year. These ıtes are proportionately higher than the rates for a year.

758. Salvage is an allowance made to those who voluntarily aid ı saving a vessel or cargo in times of danger or distress.

759. To Cancel a Policy is to annul the contract between the ısurance company and the holder of the policy.

This may be done at the instance of the company or of the policy holder. When by the former, a *Return Premium* is paid by the company, which ears the same relation to the entire premium as the unexpired portion of ıe term does to the whole term; but when canceled at the request of the olicy-holder, the return premium is only what is left after deducting the short rate premium" for the expired time from the total premium.

760. Computations in property insurance are made in accordnce with the principles of percentage, the Amount Insured being he *base*, the Rate Per Cent. of Premium the *rate*, and the Preıium the *percentage*.

EXAMPLES.

761. 1. A building was insured for \$4500 at a premium of $1\frac{1}{2}\%$. What was the cost of insurance?

2. A house was insured for \$3000 and its furniture for \$500 at $1\frac{1}{4}\%$. What was the cost of insurance?

3. A farmer insured his house valued at \$2400 and his barn valued at \$900 for $\frac{2}{3}$ of their value at 2%. What was the premium?

4. Mr. B. insured his house worth \$3500 for $\frac{4}{5}$ of its value at $1\frac{3}{4}\%$, and his barn valued at \$1800 for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$. What was the cost of his insurance?

5. The loss on a certain property was \$7500. The property was insured for \$2500 in the Ætna, \$3500 in the Franklin, and \$4000 in the Hanover. How much was paid by each company?

6. A grain dealer paid \$270 for the insurance of a cargo of wheat at $1\frac{1}{4}\%$. What was the amount of insurance?

7. At 50 cents a \$100 worth, how much insurance can be secured for \$25?

8. An agent secured a house worth \$8400 for $\frac{3}{4}$ of its value at %, and charged \$1.50 extra for writing the policy. How much was the bill?

[illegible] which one [illegible] and another to [illegible] What was the difference in the cost of the [illegible]

[illegible] was insured for ¾ of its value at 2½%. [illegible] what would have been the actual loss to the [illegible]

[illegible] was insured for $5000 in each of four [illegible] What proportion would the owner receive from each [illegible] a loss of $12000?

[illegible] cost to insure a house worth $8750 for ¾ [illegible]

[illegible] was insured for $6000. It was damaged [illegible] The policy contained the "per- [illegible]" What amount did the company pay?

[illegible] insured in the American Insurance Company [illegible] in the Traders' for [illegible] A loss of $3550 was sustained [illegible] What was the sum paid by each company?

[illegible] was insured for $24000, [illegible] "percentage co-insurance clause." It was damaged [illegible] How much should the company pay?

[illegible] was insured for 80% of its value at [illegible] The ship was lost at sea. How much did the insurance company lose?

A block of buildings was insured for three years for $25000 [illegible] At the end of the first year the policy was canceled at the request of the owner of the real estate. What was the return premium if the short rate for one year was $.75 per $100?

For how much must a consignment of goods valued at [illegible] be insured at a premium of ½% so that, in case of total loss, the cost of the goods and the premium may be covered?

A consignment of 6000 bu. of wheat worth 72¢ per bushel [illegible] for ¾ of its value at 1½% premium. The wheat was [illegible] by fire. What was the loss to the owner?

[illegible] goods valued at $6000 [illegible] damaged [illegible] for 18 months [illegible] policy for [illegible] total [illegible] $100, [illegible]

757. Short Rates are certain rates of premium charged by ompanies when the term of insurance is less than a year. These ates are proportionately higher than the rates for a year.

758. Salvage is an allowance made to those who voluntarily aid n saving a vessel or cargo in times of danger or distress.

759. To Cancel a Policy is to annul the contract between the nsurance company and the holder of the policy.

This may be done at the instance of the company or of the policy holder. When by the former, a *Return Premium* is paid by the company, which ears the same relation to the entire premium as the unexpired portion of he term does to the whole term; but when canceled at the request of the olicy-holder, the return premium is only what is left after deducting the 'short rate premium" for the expired time from the total premium.

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EXAMPLES.

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7. At 50 cents a \$100 worth, how much insurance can be secured or \$25?

8. An agent secured a house worth \$8400 for $\frac{5}{6}$ of its value at %, and charged \$1.50 extra for writing the policy. How much *vas the bill?*

772. The **Beneficiary** is the person or persons to whom the policy is made payable.

773. The **Insured** is the person the loss of whose life is provided against by the policy.

A person may insure his own life or that of another person in whom he has a pecuniary interest as a creditor for instance, or upon whom he depends for support etc. The life of a stranger or one in whom there is no pecuniary interest can not be insured.

774. Life insurance companies may be *Stock*, *Mixed*, *Mutual*, or [illegible] associations.

Stock and mixed companies pay their losses from "reserve funds" or by assessment of the stockholders; Mutual and Co-operative companies from the final premiums of the policy holders.

775. The **Expectation of Life** is the number of years that a person of a given age will probably live.

This is based upon a carefully prepared record of mortality, showing the average number of deaths in every thousand persons of the same age.

776. Rates of Life Insurance are expressed by giving the premium at a certain age on each $1.00 of the amount insured.

1. This is usually shown by means of tables prepared for the purpose.

2. The various life insurance companies explain so fully in their circulars and tables pertaining to the premiums to be paid, the manner of becoming insured, the nature of policies surrendered, etc., and the computations are so simple that few problems involving life insurance need be given.

EXAMPLES.

777. 1. A man at the age of 29 obtained an ordinary life policy for $5000 at $20.30 per $1.00. What annual premium did he pay?

Solution. — $20.30 × 5000 = $101.50, annual premium.

2. What is the annual premium on a life policy of $7500 at $[illegible] per $1.00?

3. A man 35 years of age has taken a 15-year life policy for $[illegible] at $[illegible] per $1.00. Should he live to the expiration of the policy, how much will he have paid in premiums?

4. If a person 30 years of age is insured for $8000, at an annual premium of $[illegible] per $1.00, [illegible] years after 3 [illegible] payments, how much more will he be required to pay than has been paid?

5. A man at the age of 41 [illegible] at policy of $[illegible] at $[illegible] per [illegible] ying the term [illegible] annual premium. What [illegible] of insurance?

RATIO.

777. Ratio is the measure of the relation [illegible]
other of the same kind.

Thus, the ratio of 6 to 3 is 2: the ratio of 3 to [illegible]

778. The **Terms** of a ratio are the two [illegible]

779. The **Antecedent** is the first term [illegible]
nd.

780. The **Consequent** is the second [illegible]
visor.

781. The **Sign** of ratio is the colon (: [illegible]
vision with the line omitted.

Thus, 12 : 3 signifies the ration of 12 to 3, [illegible]
m (12) is the antecedent and the second [illegible]

The ratio may also be written in the [illegible]
ing the numerator and the consequent [illegible]
tio of 12 to 3 may be written 12 : 3, or [illegible].

782. The **Value of a Ratio** is found [illegible]
the consequent.

Thus, 12 : 3 = 4; or $\frac{12}{3} = 4$.

[illegible] us.

783. A **Simple Ratio** is one that [illegible]
d one consequent; as, 8 : 10.

The terms of a ratio must be of the [illegible]

[illegible] *to the*

784. A **Compound Ratio** is [illegible] *means divided*

[illegible] 8 : 5 [illegible]

[illegible] mes divided by

[illegible]

[illegible] terms of a propor-

EXAMPLES.

786. What is the ratio of

1. 4 to 8? 10 to 30? 36 to 9? \$108 to \$36?
2. 6 to 12? 14 to 42? 55 to 11? £132 to £12?
3. 7 to 21? 18 to 6? 64 to 8? \$144 to \$18?
4. 9 to 36? 25 to 5? 42 to 6? 154 ft. to 14 ft.
5. 12 to 24? 48 to 8? 54 to 9? 168 rd. to 21 rd.

What is the ratio of

6. $\frac{5}{8}$ to $\frac{7}{8}$?

Suggestion.—The ratio of $\frac{5}{8}$ to $\frac{7}{8}$ is the same as the ratio of 5 to 7, or $\frac{5}{7}$.

7. $\frac{8}{9}$ to $\frac{4}{9}$? $\frac{7}{11}$ to $\frac{10}{11}$? $\frac{7}{18}$ to $\frac{4}{18}$? $\frac{13}{16}$ to $\frac{15}{16}$?
8. $\frac{16}{25}$ to $\frac{19}{25}$? $\frac{13}{28}$ to $\frac{23}{28}$? $\frac{25}{31}$ to $\frac{15}{31}$? $\frac{9}{47}$ to $\frac{21}{47}$?

What is the ratio of

9. $\frac{2}{3}$ to $\frac{4}{5}$?

Suggestion.—$\frac{2}{3} = \frac{10}{15}$, and $\frac{4}{5} = \frac{12}{15}$; hence, the ratio of $\frac{2}{3}$ to $\frac{4}{5}$ is the ratio of $\frac{10}{15}$ to $\frac{12}{15}$, or of 10 to 12, or $\frac{10}{12}$ or $\frac{5}{6}$. Or, $\frac{2}{3}$ to $\frac{4}{5} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.

10. $\frac{3}{4}$ to $\frac{5}{6}$? $\frac{2}{3}$ to $\frac{3}{5}$? $\frac{4}{7}$ to $\frac{5}{6}$? $\frac{8}{11}$ to $\frac{8}{10}$?
11. $\frac{2}{3}$ to $\frac{3}{4}$? $\frac{3}{8}$ to $\frac{7}{9}$? $\frac{6}{11}$ to $\frac{7}{8}$? $\frac{5}{18}$ to $\frac{5}{6}$?
12. $\frac{4}{5}$ to $\frac{3}{4}$? $\frac{7}{11}$ to $\frac{3}{4}$? $\frac{9}{10}$ to $\frac{6}{7}$? $\frac{12}{18}$ to $\frac{8}{9}$?
13. What is the value of the compound ratio $\left\{\begin{matrix} 5 : 16 \\ 12 : 4 \end{matrix}\right\}$?

Solution.—$\left\{\begin{matrix} 5 : 16 \\ 12 : 4 \end{matrix}\right\} = (5 : 16) \times (12 : 4) = \frac{5}{\cancel{16}_{4}} \times \frac{\overset{3}{\cancel{12}}}{4} = \frac{15}{16}$.

14. Find the value of the ratio $\left\{\begin{matrix} 4 : 9 \\ 7 : 18 \end{matrix}\right\}$.
15. Find the value of the ratio $\left\{\begin{matrix} 12 : 21 \\ 35 : 50 \end{matrix}\right\}$.
16. What is the ratio of 3 ft. 4 in. to 4 ft. 2 in?
17. What is the ratio of 8 gal. 3 qt. to 3 gal. 1 qt.?
18. The antecedent is 72 and the consequent is 9. What is the ratio?
19. The antecedent is 64 and the ratio 12. What is the consequent?
20. The consequent is 35 and the ratio 5. What is the antecedent?
21. *The antecedent is* $\frac{45}{47}$ and the ratio $\frac{3}{5}$. What is the consequent?

PROPORTION.

787. Proportion is an equality of ratios.

Thus, 5 : 10 as 7 : 14 is a proportion.

788. The **Sign** of proportion is the double colon (::). It is written between the ratios to express their equality.

Thus, 5 : 10 : : 7 : 14.

This sign is believed to be a modification of the sign of equality (=) which is sometimes used instead of the double colon. Thus, 5 : 10 = 7 : 14.

The proportion 5 : 10 : : 7 : 14 is read "the ratio of 5 to 10 equals the ratio of 7 to 14," or more briefly, "5 is to 10 as 7 is to 14."

789. The **Terms** of a proportion are the four numbers used in the comparison.

1. The antecedents are the first and third terms, and the consequents are the second and fourth terms. Thus, in the proportion 5 : 10 : : 7 : 14, 5 and 7 are the antecedents, and 10 and 14 the consequents.

The first ratio of a proportion is sometimes called the *first couplet*, and the second ratio the *second couplet*.

790. The **Extremes** of a proportion are the first and fourth terms.

In the proportion given 5 and 14 are the extremes.

791. The **Means** of a proportion are the second and third terms.

In the proportion given 10 and 7 are the means.

792. PRINCIPLES.—1. *The product of the means is equal to the product of the extremes.*

2. *Either extreme is equal to the product of the means divided by the other extreme.*

3. *Either mean is equal to the product of the extremes divided by the other mean.*

It is readily seen from these principles that if three terms of a proportion are given the other may readily be found.

EXERCISES.

Find the unknown term denoted by x in each of the following:

1. $12 : 20 : : 18 : x$.

Suggestion.—$x = \frac{\overset{10}{\cancel{20}} \times \overset{3}{\cancel{18}}}{\underset{2}{\cancel{12}}} = 30.$

2. $8 : 24 : : 11 : x$.
3. $15 : 24 : : 30 : x$.
4. $9 : 45 : : 12 : x$.
5. $x : 10 : : 20 : 5$.
6. $x : 7 : : 16 : 14$.
7. $36 : x : : 144 : 4$.
8. $9 : x : : 6 : 15$.
9. $112 : x : : 81 : 27$.
10. $7 : 42 : : x : 60$.
11. $18 : 6 : : x : 5$.
12. $\frac{3}{5} : \frac{3}{4} : : 24 : x$.
13. \$10 : \$15 : : 300 lb. : x.
14. 6 yd. : 20 yd. : : \$15 : x.
15. $1.35 : 4.05 : : 15 : x$.
16. x : 27 lb. : : \$14 : \$9.
17. 15 men : 20 men : : 18 da. : x.

SIMPLE PROPORTION.

793. A Simple Proportion is an expression of equality of two simple ratios.

Thus, $6 : 15 : : 4 : 10$ is a simple proportion.

794. Simple proportion is employed for the solution of problems in which three of four quantities are given and so related as to form a proportion containing only one unknown quantity which may be readily found by the principles already given (Art. 792).

Simple proportion is sometimes called the "*Single Rule of Three.*"

795. 1. If 6 barrels of flour cost \$30, what will 10 barrels cost?

bar. bar. \$ \$
6 : 10 : : 30 : x

$$\frac{10 \times \overset{5}{\cancel{30}}}{\underset{}{\cancel{6}}} = 50$$

EXPLANATION.—Since the fourth term must be in dollars, we make the third term \$30, because they must form a ratio, and therefore be like quantities. If 6 barrels cost \$30, it is evident that 10 barrels will cost more; hence, we place the greater of the two numbers for the second term and the less for the first. Then solving according to Prin. 2, we find the cost of 10 barrels of flour to be \$50.

RULE.—Make the number which is of the same kind as the required answer the third term.

Make the other two numbers the first and second terms; the greater number the second term when the result is to be greater than the third term, and the less the second term when the result is to be less than the third term.

Multiply the third term by the second and divide by the first.

1. The terms of a couplet must always be of the same denomination.

2. Use cancellation when possible.

3. Other methods of statement and explanation might be given, but the author considers the method shown above as the simplest and most readily understood.

EXAMPLES.

2. If 8 yards of cloth cost $20, how much will 12 yards cost?

3. How much will 21 pounds of sugar cost, if 6 pounds cost 39 cents?

4. If 16 men can do a piece of work in 30 days, how long will it take 24 men to do it?

5. If 27 bushels of wheat cost $18.90, how much will 100 bushels cost?

6. If $200 gain $12 in a year, how much will $650 gain in the same time and at the same rate?

Suggestion.—In this problem the three terms are dollars, but only one term is specifically of the same kind as the required answer. *Dollars gain* are required, hence $12 must be the third term. The two other terms agree as *dollars of principal.*

7. If $350 dollars produce $28 interest in a certain time, how much interest will $1000 produce in the same time?

8. If 21 bushels of wheat can be bought for $15.39, how many bushels can be bought for $262.80?

9. If it require 60 men 104 days to complete a job of work, how many men will it require to complete the work in a month (26 working days)?

10. If $\frac{5}{6}$ of a barrel of flour cost $3.20, how much will 20½ barrels cost?

11. If a man 6 ft. in height cast a shadow 2 ft. 4 in. long, what is the height of a tree that casts a shadow 58 ft. 4 in. long at the same time of day?

12. On property assessed at $6400, the tax is $41.60. What would be the tax on property assessed at $10000?

13. If a garrison of 200 men consume 35 barrels of flour in 8 weeks, how many barrels will it consume in a year?

14. If a 5-cent loaf weigh 14 ounces when flour is worth $4.50 a barrel, how much should it weigh when flour is worth $6 a barrel?

15. A farmer raised 340 bushels of potatoes upon 1 A. 120 sq. rd. of land. At this rate how many bushels could be raised on 5 acres?

16. Mr. M. loaned me $450 for 9 months. For what time should I lend him $600 to fully repay the favor?

17. If 12 men can do a piece of work in 25 days, in how many days can they do the same work with the assistance of 8 other men?

18. A piece of work can be completed by 30 men in 18 days. After working 6 days, 10 men quit. How many days will be required for the rest to finish it?

19. A bankrupt's debts are $7500, and his assets $5000. How much should Mr. Slyder receive, who has a claim of $500?

COMPOUND PROPORTION.

796. A **Compound Proportion** is a proportion in which at least one of the ratios is compound.

Thus, $\left.\begin{matrix} 3 : 6 \\ 5 : 4 \end{matrix}\right\} :: 10 : 16$ is a compound proportion.

797. A compound proportion, as applied to the solution of problems, involves at least two simple proportions. Hence, compound proportion is sometimes called the "*Double Rule of Three.*"

798. PRINCIPLES.—1. *The product of all the terms in the means is equal to the product of all the terms in the extremes.*

2. *Any term in the extremes is equal to the product of the means divided by the product of the other terms of the extremes.*

3. *Any term in the means is equal to the product of the extremes divided by the product of the other terms of the means.*

EXERCISES.

799. Find the unknown term denoted by x in each of the following:

1. $\left.\begin{matrix} 4 : 12 \\ 21 : 14 \\ 9 : 15 \end{matrix}\right\} :: 18 : x.$

Suggestion.—$x = \dfrac{\cancel{12} \times \cancel{14} \times 15 \times \cancel{18}}{\cancel{4} \times \cancel{21} \times \cancel{9}} = 60.$

2. $\left.\begin{matrix} \frac{4}{5} : \frac{5}{6} \\ 2\frac{1}{3} : 5\frac{1}{2} \\ 12\frac{8}{4} : 7\frac{1}{5} \end{matrix}\right\} :: 8\frac{1}{2} : x.$

Suggestion.—$x = \frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{4}} \times 5 \times 11 \times \overset{3}{\cancel{9}} \times \overset{1}{\cancel{17}}}{\underset{1}{\cancel{4}} \times 7 \times \underset{17}{\cancel{51}} \times \overset{1}{\cancel{9}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{5}} \times 2} = \frac{165}{14} = 11\frac{11}{14}.$

3. $\left.\begin{matrix} 4 : 8 \\ 5 : 9 \\ 15 : 20 \end{matrix}\right\} :: 25 : x.$

4. $\left.\begin{matrix} 6 : 14 \\ 18 : 28 \\ 32 : 45 \end{matrix}\right\} :: 72 : x.$

5. $\left.\begin{matrix} 21 : 36 \\ 19 : 27 \\ 34 : 16 \end{matrix}\right\} :: 57 : x.$

6. $\left.\begin{matrix} 8 : 15 \\ 11 : 14 \\ 13 : 12 \\ 18 : 22 \end{matrix}\right\} :: 40 : x.$

7. $\left.\begin{matrix} 5 : 15 \\ x : 9 \\ 7 : 13 \end{matrix}\right\} :: 14 : 8.$

8. $\left.\begin{matrix} 18 : 26 \\ 14 : 16 \\ 20 : x \end{matrix}\right\} :: 9 : 5.$

9. $\left.\begin{matrix} 13 : 21 \\ 9 : 8 \\ 20 : 15 \end{matrix}\right\} :: x : 45.$

10. $\left.\begin{matrix} \frac{5}{8} : 1\frac{1}{4} \\ \frac{7}{12} : \frac{14}{15} \\ \frac{8}{4} : 5\frac{3}{5} \end{matrix}\right\} :: 8\frac{1}{2} : x.$

11. $\left.\begin{matrix} 24 : 20 \\ 15 : x \end{matrix}\right\} :: \left\{\begin{matrix} 19 : 38 \\ 21 : 14 \end{matrix}\right.$

12. $\left.\begin{matrix} \frac{3}{8} : \frac{4}{5} \\ \frac{7}{8} : \frac{9}{10} \end{matrix}\right\} :: \left\{\begin{matrix} 1\frac{1}{4} : 2\frac{2}{3} \\ \frac{15}{16} : x. \end{matrix}\right.$

EXAMPLES.

800. 1. If 5 men can earn \$30 in 6 days of 8 hours each, how much can 17 men earn in 5 days of 10 hours each?

(1) 5 : 17 : : 30 : x, or 102
(2) 6 : 5 : : 102 : x, or 85
(3) 8 : 10 : : 85 : x, or $106\frac{1}{4}$

Or,

(4) $\left.\begin{matrix} 5 : 17 \\ 6 : 5 \\ 8 : 10 \end{matrix}\right\} :: 30 : x$

$$x = \frac{17 \times \overset{1}{\cancel{5}} \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{30}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{6}} \times \underset{4}{\cancel{8}}} = 106\tfrac{1}{4}$$

EXPLANATION.—This problem involves three conditions. The first condition is, If 5 men can earn \$30, how much can 17 men earn? This is solved by proportion (1). The second condition is, If the men can earn \$102 in 6 days, how much can they earn in 5 days? This is solved by simple proportion (2). The third condition is, If the men earn \$85 by working 8 hours a day, how much could they earn by working 10 hours a day? This is solved by proportion (3), which gives \$106¼, or \$106.25.

By inspecting the three proportions it is seen that the values of x in (1) and (2) appear as both antecedent and consequent, and may therefore be omitted. The simple proportions will thus assume the form of the compound proportion (4), which being solved gives the required result, \$106.25.

The problem may be stated at once as in proportion (4) by making the number which is of the same kind as the required answer the third term, and then arranging each couplet with reference to its relation to the desired answer as in simple proportion.

RULE.—*Make the number which is of the same kind as the required answer the third term.*

Arrange each of the other couplets just as if the result depended upon it and the third term.

Divide the product of the means by the product of the given extremes, and the quotient will be the required answer.

Problems in compound proportion may be solved by what is known as the **Cause and Effect** method. They may also be solved by analysis.

The statement of example 1 by cause and effect is as follows:

1st cause.		2d cause.			1st effect.		2d effect.
5 men 6 days 8 hours	:	17 men 5 days 10 hours	::		$30	:	$x.

Analysis.—If 5 men earn $30 in 6 days of 8 hours each, 1 man will earn $\frac{1}{5}$ of $30 or $6 in 6 days of 8 hours each or $1 in 1 day. If 1 man earn $1 in 1 day of 8 hours each, 17 men will earn 17 times $1, or $17, and in 5 days they will earn 5 times $17, or $85. If they earn $85 by working 8 hours a day, by working 10 hours a day they will earn $\frac{1}{10}$ of $85, or $106.25.

2. If 60 men can mow 50 acres of grass in 6 days, how long will it take 40 men to mow 100 acres?

3. If 18 horses eat 24 loads of hay in 8 weeks, how many weeks will it take 20 horses to eat 36 loads of hay?

4. If the freight on 48 barrels of flour for a distance of 56 miles is $6.50, what should be the freight on 75 barrels for 40 miles?

5. If $2500 loaned for 3 yr. 4 mo. produce $500 interest, how much should $4000 produce in 1 yr. 8 mo.?

6. If 15 men build 45 rods of wall in 12 days of 8 hours each, how many men will be required to build 90 rods of wall in 16 days of 10 hours each?

7. How many men in 26 days of 10 hours each, will do 5 times as much work as 13 men can perform in 30 days of 12 hours each?

8. If the capacity of a bin 20 ft. long, 4 ft. wide, and $5\frac{1}{2}$ ft. high is 350 bushels, what is the capacity of a bin 28 ft. long, $4\frac{1}{2}$ ft. wide, and 6 ft. deep?

9. If $1000 are earned by 25 men in 20 days by working 10 *hours a day*, how much can be earned by 60 men in 50 days if they *work 8 hours* a day?

10. If 56 yards of carpet, $\frac{3}{4}$ of a yard wide, will cover a room 21 ft. long and 18 ft. wide, how many yards of carpet, $\frac{5}{6}$ of a yard wide, will cover a room 18 ft. long and 15 ft. wide?

11. The expenses for keeping 13 horses for 9 weeks were $275. At this rate what would be the expenses for keeping 26 horses for 6 weeks?

12. If $500 yield $10.50 interest at 6% in 4 mo. 6 da., at what rate per cent. will $360 yield $7.50 interest in 5 months?

13. If a drove of 75 head of cattle can eat a lot of grain in 20 days, in what time will 3 times the quantity of grain be consumed, if 25 cattle are added when the grain is $\frac{3}{4}$ eaten?

Suggestion.—This problem requires two separate statements.

14. If 32 men can clear 60 acres of ground in 15 days, in what time can they clear 75 acres, if 8 men leave when 40 acres have been cleared?

15. If 27 men in 36 days of 10 hours each dig a ditch 360 rd. long, 6 ft. wide, and 3 ft. deep, of 5 degrees of hardness, how many days of 9 hours each will it take 36 men to dig a ditch 200 rd. long, 8 ft. wide, and 4 ft. deep, if the digging is estimated to be 20% harder?

PARTITIVE PROPORTION.

301. Partitive Proportion is the process of dividing a number into parts which are proportional to other given numbers.

EXAMPLES.

302. 1. Divide 1320 into three parts which shall be to one another as 3, 5, and 7.

$3 + 5 + 7 = 15$
$\frac{1}{15}$ of $1320 = 88$
$\frac{3}{15} = 264$
$\frac{5}{15} = 440$
$\frac{7}{15} = 616$

EXPLANATION.—Since the parts are 3, 5, and 7, the number is divided into $3 + 5 + 7$ or 15 equal parts, of which 3 of these parts, or $\frac{3}{15}$ of 1320, or 264, is the first part; $\frac{5}{15}$ of 1320, or 440, is the second part; and $\frac{7}{15}$ of 1320, or 616, is the third part.

Or, the parts may be found by proportion. Thus, $15 : 3 :: 1320 : x$, for first part; $15 : 5 :: 1320 : x$, for second part; and $15 : 7 :: 1320 : x$, for third part.

2. Divide 1680 into parts proportional to 4, 9, and 11.

3. Divide the number 5292 into three such parts which shall be to one another as 6, 7, and 8.

4. Divide 2600 into three parts which shall be to one another as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Suggestion.—Since fractions having a common denominator are to one another as their numerators, change the fractions to 12ths, and it will be seen that the parts are as 6, 4, and 3. Then solve as in example 1.

5. A father divided $6455 between his two sons in proportion to their ages, which were 9 and 10 years respectively. How much did each receive?

6. A man bequeathed his property to his wife, son, and daughter, in the proportion of 6, 4, and 3, respectively. His entire estate was found to be worth $23725. What was the share of each?

7. A farmer raised in one year 6875 bushels of grain consisting of wheat, oats, and corn. How many bushels of each did he raise if they are as 6, 9, and 10 respectively?

8. Mr. Brown and Mr. Smith bought two farms for which they paid $18750. Mr. Brown paid $2865 more for his farm than Mr. Smith. How much did each pay for his farm?

9. A father divided $7600 among his three sons. He gave $400 more to the oldest son than to the second, and $600 more to the second son than to the third. How much did each of his sons receive?

10. A, B, and C together had $750, of which A had twice as much as B, and B had three times as much as C. How much of the money belonged to each?

11. A stockman owned 275 horses and cattle. How many animals had he of each kind, if 5 times the number of horses equaled 6 times the number of cattle?

12. John and James together earned in one year $850, and $\frac{3}{5}$ of what John earned equaled $\frac{4}{7}$ of what James earned. How much money did each earn?

13. A man sold 3 farms for $14586, and the sums received for each were in the proportion of $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{5}{6}$. How much did he receive for each farm?

14. A, B, and C contributed $2000 toward building a church which is situated $\frac{1}{2}$ of a mile from A, $\frac{2}{3}$ of a mile from B, and $1\frac{1}{4}$ miles from C. It had been agreed by them that the part paid by each should be proportional to the reciprocals of the distances. What was the sum paid by each?

ALLIGATION, OR AVERAGE.

803. Alligation, or **Average,** treats of the mixing or compounding of ingredients of different values to find the average value of the mixture, or to produce a mixture of a given value.

804. Alligation is usually treated under two divisions; namely, Alligation Medial, or Medial Proportion, and Alligation Alternate.

The entire subject of *Alligation*, or *Average*, is called by some authors *Medial Proportion*.

ALLIGATION MEDIAL.

805. Alligation Medial, or **Medial Proportion,** is the process of finding the average value of a mixture when the quantity and value of each ingredient are given.

806. Given the values of several ingredients, to find the average value.

1. A grocer mixed together 30 lb. of rice worth 6 cents per lb, 20 lb. worth 8 cents, and 30 lb. worth 10 cents per lb. What was the value of a pound of the mixture?

$30 \times \$.06 = \$1.80$
$20 \times .08 = 1.60$
$30 \times .10 = 3.00$
$80 \quad \$6.40$
$.08$

EXPLANATION.—30 lb. at 6¢ are worth $1.80, 20 lb. at 8¢ are worth $1.60, and 30 lb. at 10¢ are worth $3. The total value of the 80 lb. is $6.40. Hence, 1 lb. of the mixture is worth 8¢.

2. A confectioner mixed 25 lb. of candy worth 12¢ a lb., 35 lb. worth 15¢ a lb., 45 lb. worth 17¢ a lb, and 50 lb. worth 18¢ a lb. What was the value of a pound of the mixture?

3. A farmer mixed together 40 bu. of oats at 35¢ per bu., 50 bu. of corn at 65¢ per bu., and 75 bu. of rye at 80 per bu. What was the value of a bushel of the mixture?

4. A grocer mixed 45 lb. of tea at 30¢ a lb., 40 lb. at 35¢ a lb., and 20 lb. at 50¢ a lb. Find the value of a pound of the mixed teas.

ALLIGATION ALTERNATE.

807. Alligation Alternate is the process of finding the quantity of ingredients of different values required to produce a mixture of a given value.

Alligation Alternate is a reciprocal proportion in which the results are frequently found to be indeterminate. The method of binding or linking one rate that is less than the mean rate, with one that is greater, has given the name to this branch of Arithmetic.

808. Given the values of the several ingredients and their mean, or average, value, to find the quantity of each.

1. In what proportion must coffees worth 18, 20, 26, and 32 cents a pound be mixed together, so that a pound of the mixture may be worth 24 cents?

		(1)	(2)	(3)	(4	(5	(6)
24	18	$\frac{1}{6}$		4		4	8
	20		$\frac{1}{2}$		1	1	2
	26		$\frac{1}{2}$		2	2	4
	32	$\frac{1}{6}$		3		3	6

EXPLANATION.—The prices of the different kinds are arranged in a vertical column. Then the price of one that is below the average price is linked with one that is greater than the average price, so that in considering them such a proportion of each may be taken as not to gain or lose by selling at the mean, or average, price. In this example 18 cents are linked with the 32 cents.

If we take a pound at 18 cents and sell it at 24 cents, we gain 6 cents, and to gain 1 cent it will take $\frac{1}{6}$ of a pound. If we take a pound worth 32 cents and sell it for 24 cents, we lose 8 cents, and to lose 1 cent it will take $\frac{1}{8}$ of a pound. Now, the loss of 1 cent on $\frac{1}{8}$ of a pound at 32 cents is balanced by the gain of 1 cent on $\frac{1}{6}$ of a pound at 18 cents. In the same manner, we find that if we take $\frac{1}{2}$ of a pound of the 20-cent coffee and mix it with $\frac{1}{2}$ of a pound of the 26 cent coffee, there will be no loss or gain by selling the mixture at 24 cents a pound. Or, in whole numbers (getting rid of the fractions by multiplying the fractions in columns (1) and (2) by the least common multiples of their respective denominators), we may take 4 lb. at 18¢ as often as we take 1 lb. at 20¢, 2 lb. at 26¢, and 3 lb. at 32¢. Or, doubling the proportions in col. 5 to obtain a greater quantity, we get col. 6.

PROOF OF RESULTS.
(as found in col. 5.)

4 ×	$ .18 =	$	.72
1 ×	.20 =		.20
2 ×	.26 =		.52
3 ×	.32 =		.96
10		) $2.40	
			.24

EXPLANATION.—Taking the number of pounds of each kind as found in column 5, at the given price of each we find that the 10 lb. of the mixture cost $2.40, or that the average price is 24 cents, which was to be proved. Or, we can say 10 lb. at $.24, the average price given, equal $2.40, the value of the mixture as found.

1. It may readily be seen that in multiplying col. 5 by 2, 3, 4, etc., an unlimited number of

answers may be found.

4. If we multiply the fractions in col. 1 by 48 the product of the denominators, and those in col. 2 by 8, the product of their denominators we will obtain the results in col. 6. By careful comparison of the results in this column, and also from reason, we find that the quantities used must be taken in an inverse or reciprocal ratio to the gain and loss, that is, we place the difference between 18 and 24, or 6, not opposite the 18, to represent the gain, but opposite the 32, to represent the relative quantity to be taken at that price; and for the same reason we place the difference between 24 and 32, or 8, opposite the 18.

3. In the example given, 18 could be joined to 26, and 20 to 32. This would give a new set of ratios, and as each set of ratios admit of indefinite multiples, it will readily be seen that the number of answers to problems of this character may be infinite.

2. How much tea at 30¢, 45¢, 60¢, and 80¢ per pound may be taken to form a mixture worth 50 cents per pound?

3. A farmer has potatoes worth 60¢, 75¢, 90¢, and $1.00 per bushel. In what proportion may they be mixed to be worth 80¢ per bushel?

4. A druggist formed a mixture from ingredients worth 90¢, $1.20, $1.70, and $1.80 per pound, which he sold at $1.50 per pound. In what proportion may they have been mixed?

5. A dealer in grain mixed oats at 40 cents, corn at 60 cents, and rye at 90 cents per bushel. He sold the mixture at 75 cents per bushel. How many bushels of each may have been taken?

Suggestion.—In this problem both the price of the oats and that of the corn must be compared with the price of the rye.

809. To find the quantity of each ingredient when at least one of the quantities is limited.

1. How much coffee at 15¢ and 18¢ per pound must be mixed with 36 pounds at 30¢ a pound, so that the mixture may be worth 25¢ per pound?

25	15	$\frac{1}{10}$	1	1	$\times 4 =$	4	
	18					20	
	30	$\frac{1}{5}$	2	5		36	
		$\frac{1}{5}$		7			
				9			

$36 \div 9 = 4$

EXPLANATION.—We find as in Art. 808, that 1 lb. at 15¢ must be mixed with 2 lb. at 30¢; and that 5 lb. at 18¢ must be mixed with 7 lb. at 30¢. Or, in other words, as often as we take 1 lb. at 15¢ we must take 5 lb. at 18¢, and 2 + 7, or 9 lb., at 30¢. Since the mixture is to contain 36 lb. of 30-cent coffee, which is 4 times 9 lb., we must take 4 times 1 lb., or 4 lb. at 15¢ and 4 times 5 lb., or 20 lb., at 18 cents.

2. How many pounds of tea at 35¢ and 50¢ must be mixed with 45 *pounds at 75¢, that* the mixture may be worth 60¢ per pound?

3. A grocer mixed coffees at 30¢, 46¢, and 48¢ a pound with 12 pounds at 38 cents, so that the mixture could be sold for 40 cents per pound. How much of each kind did he take?

4. A dealer had 140 bu. of grain that cost him 60¢ per bu., which he mixed with a variety that cost him 80¢ per bu. He sold the mixture at 75¢ per bushel and gained 20% on his investment. How many bushels of the 80-cent grain were required for the mixture?

810. To find the quantity of each ingredient when the total amount of the ingredients is given.

1. A grocer wishing to fill an order for 90 pounds of 30-cent coffee, mixed grades, the prices of which were 14¢, 20¢, and 42¢. How many pounds of each kind did he take?

	(1)	(2)	(3)	(4)	(5)		
30	14	$\frac{1}{16}$		3	·	3	15
	20		$\frac{1}{10}$		6	6	30
	42	$\frac{1}{12}$	$\frac{1}{12}$	4	5	9	45
						18	

$\times 5$

$90 \div 18 = 5$

EXPLANATION.—We find col. 5 as in Art. 809. The sum of the pounds in col. 5 is 18. The required amount is 90 pounds, which are 5 times 18 pounds. Hence, the quantity of each kind is 5 times the result given in col. 5.

2. A merchant having teas worth 25¢, 35¢, 75¢, and $1.00 per pound, prepared a mixture of the grades weighing 115 pounds, and worth 60¢ per pound. How many pounds of each grade did he take?

3. A man had $100 in ten cent pieces which he wished to have exchanged for two cent, five cent, twenty-five cent, and fifty cent pieces. How many of each kind could he get?

4. A grain dealer had oats worth 35¢, corn worth 50¢, and rye worth 85¢ a bushel, which he desired to mix together in such a proportion that he would have 120 bushels of mixed grain worth 65¢ per bushel. How many bushels of each kind of grain were required for the mixture?

5. A dealer bought 100 bushels of corn, timothy seed, and clover seed for $100. He paid $\$\frac{1}{2}$ per bushel for the corn, $3 per bushel for the timothy seed, and $10 per bushel for the clover seed. How many bushels of each did he buy?

6. A man bought 50 animals consisting of lambs, sheep, and calves for $150. He paid $\$1\frac{1}{2}$ each for the lambs, $4 each for the sheep, and $\$10\frac{1}{2}$ each for the calves. How many of each kind did he buy?

GENERAL AVERAGE.

811. Average, commercially considered, is a contribution to a loss suffered by damage done in sea-perils for the general welfare of all parties concerned. It is of two kinds, *General Average* and *Particular Average.*

812. General Average is the apportioned contribution made by all parties concerned in a ship, cargo, and freight, towards a loss voluntarily incurred for the common safety and general benefit.

It is called *general average* because the value of the amount of the loss or expense is assessed upon the value of all interests involved and benefited.

813. Particular Average is the contribution for damage or partial loss happening to the ship, cargo, or freight in consequence of some unavoidable accident, and is borne by the persons whose property is damaged, or by their insurers.

814. Jettison is the throwing overboard of cargo to lighten a ship in stress of weather, or to prevent foundering.

815. Salvage is the compensation allowed to persons who voluntarily assist in saving a vessel, her cargo, or passengers in case of wreck or other imminent danger to life or property.

816. The *Insurers*, or *underwriters*, contribute to the general average such a part of the loss to the owners as the insured value is to the market value of the property insured.

Thus, if the cargo of a vessel is insured for $6000 and its market value is $8000, the underwriters are liable for ¾ of the general average.

817. Before a general average can be declared, it must be shown that there was imminent common danger, that the sacrifice was voluntary and necessary, and that the act was prudent and successful.

818. An **Average Adjuster** is a person who is versed in general average laws, and who apportions the losses and expenses of a general average.

In determining a general average, extracts from the log (log book) of the vessel, the testimony of her officers, a detailed statement of all expenses incurred, with the vouchers of the same, and all other papers relevant to the case are placed in the hands of the adjuster. Sometimes it is a very difficult matter for the adjuster to determine whether certain losses should be made good by a general average or should be charged upon some particular interest (a particular average), and the decision has often resulted in a prolonged litigation.

819. The underlying principle of general average is that a loss incurred for the advantage of all concerned should be borne by each of them in the proportion that his invested interest bears to the total invested interests of all.

820. A General Average loss may include the following:

1. Jettison; damage to cargo by water going down the hatches during jettison; damage by chafing or breaking after jettison; freight on cargo jettisoned.

2. Sacrifices of portions of a vessel, as the cutting away of masts, rigging, etc. One-third of the cost of repairing a vessel is a special charge on the ship, the new work being considered better than the old, and hence, increasing the value of the vessel. The remaining two-thirds only of such cost is included in the general average. No deduction is made for anchors.

3. Expense of floating a stranded vessel, including salvage.

4. Expense of entering a port of refuge, whether for urgent repairs or to avert a common danger.

5. Expense of discharging a cargo to make repairs, warehouse rent, reloading the cargo, etc.

6. Wages and provisions of the crew from the time the vessel deviates from its course until it resumes its voyage.

821. The Contributary Interests and Values are as follows:

1. The ship contributes on its full value before the loss.

2. The cargo contributes on its net market value at the port of destination before the loss.

3. The freight contributes on its full amount, less $\frac{1}{3}$ for the wages, etc., of the crew. In California, New York, Virginia, and some other states, $\frac{1}{2}$ is deducted.

822. To apportion a loss by general average.

1. A vessel valued at $30000, with a cargo worth $50000 *encountered such* a storm that $20000 worth of goods were jettisoned;

and it was found necessary to cut away masts and rigging. The vessel was rescued by a steamer and taken to port for repairs. The salvage amounted to $12000; repairs to the vessel, $3000; wages and provisions for crew from point of deviation, $500; and other expenses $800. The total freight was $5250, of which $2100 was the freight on the goods thrown overboard. Apportion the settlement.

General Average Loss.		Contributary Interest.	
Jettison,	20000	Cargo,	50000
Freight on jettison,	2100	Freight (5250 less ⅓),	3500
Repairs (⅓ of $3000),	2000	Vessel,	30000
Salvage,	12000		
Wages, etc., of crew,	500		
Other expenses,	800		
Total,	37400	Total,	83500

$83500 : $50000 : : $37400 : Cargo's cont. = $22395.21
" : 3500 : : " : Frg't's " = 1567.66
" : 30000 : : " : Vessel's " = 13437.13
$37400 00

Adjuster's Settlement.

(Receives.)		(Pays.)	
Owners of Cargo,	$2395.21	Frgt. Company,	$532.34
Owners of Vessel,	14437.13	Repairs,	3000.
		Salvage,	12000.
		Wages, etc., of Crew,	500.
		Other exps.,	800.
	$16832.34		$16832.34

Explanation of Adjuster's Settlement.—The adjuster finds the total loss subject to general average, which in this example is $37400. He then finds the total contributary interests, which is found to be $83500. Then knowing the value of each contributary interest, it is very easy to find by simple proportion or by fractional parts the contribution of each contributary interest. The adjuster settles with the owners of the cargo by receiving from them their contribution minus the goods thrown overboard, or $22395.21 — $20000 = $2395.21. He settles with the owners of the vessel by receiving from them their contribution plus ⅓ of the cost of the repairs, or $13437.13 + $1000 = $14437.13. He settles with the freight company (the charterers of the vessel) by paying them the freight on the jettison minus their contribution, or $2100 — $1567.66 = $532.34. He pays the full amount of repairs, $3000; salvage, $12000; wages, etc., of crew, $500; and other expenses, $800. Thus, it is seen that the adjuster's settlement account is in balance.

The wages, etc., of the crew paid by the adjuster is their general average portion. He has nothing to do with their regular wages. The adjuster has only to do with *apportioning* the losses and expenses of the general aver-

age. The charterers of the vessel pay their crew their regular wages, receive their freight charges from the owners of the saved portion of the cargo, pay the owners of the vessel for the charter of it, etc., entirely aside from the settlement of the average adjuster.

2. The general average charges occasioned by a storm were found to be $5432.57, and the contributary interests were $97635. What was the rate per cent. of contribution? What was the loss to Mr. Sinker, whose goods were valued at $17850?

3. A ship in her passage from Liverpool to Boston with a cargo valued at $75000, during a storm threw overboard $15000 worth of goods, and cut away masts and rigging. The vessel entered the port of Halifax for repairs. The cost of replacing masts and rigging cut away was $3715. The loss of freight on cargo jettisoned was $1200. The wages and provisions of the crew from the time of deviation from course until ready for sea were $475; other expenses, $725. The total freight (including freight on cargo jettisoned) was $7410. The value of the ship before the loss was $27000. Apportion the loss among the different contributary interests, and state how settlement should be made.

4. What was the rate per cent. of loss to the contributary interests in problem No. 3?

5. A steamer having on board $35000 worth of goods shipped by A, $28745 worth shipped by B, and $30000 worth shipped by C, was obliged to throw overboard during a severe gale $25000 worth of goods, of which $15000 worth belonged to A, and the remainder to B. The repairs of damages purposely done to the vessel to preserve it cost $2400, and the maintenance of the crew during delay was $632; other expenses, $968. The value of the steamer was $60000. The total freight was $8562, of which $2150 was the freight on the jettison. Apportion the settlement.

6. A ship being in distress, threw overboard part of the cargo, cut away the masts, and finally bore away to a port of refuge for repairs in order to complete the voyage. The cost of replacing masts and rigging cut away was $5700, and an anchor cost $225. The value of the cargo was $85000, of which $5000 worth was jettisoned. The total expected freight earnings were $5718, of which $375 was the freight on the goods jettisoned. The expenses of entering port of refuge, unloading, storing, and reloading the cargo were $1500 · wages of crew and provisions during the delay, *$1000.* The adjuster's fee was $150. The value of the vessel was *$45,000.* Apportion the settlement.

PARTNERSHIP.

823. Partnership is the result of a contract, whereby two or more persons agree to combine their capital, skill, or labor for the purpose of a common undertaking and the acquisition of a common profit.

Such an association is usually called a *Firm*, *House*, or *Company*.

824. The **Partners** are the persons associated in the business. They may be *general*, or *active*, *special*, *silent* or *dormant*, and *nominal* partners.

A *general*, or *active*, *partner* is one who takes an active part in the management of the business and who is responsible for the debts of the company to the amount of his entire property.

A *special partner* is one who takes no active part in the business and whose liability is limited only to the amount of his investment, provided that this amount is duly advertised and legally recorded.

A *silent*, or *dormant*, *partner* is one whose name does not appear in the firm, but who nevertheless shares in the profits and thereby renders himself (if discovered) liable for losses.

A *nominal partner* is one whose name appears to the public, but who has invested no capital in the business and receives no profits.

It is a general rule that *each partner is the accredited agent of all the other members of the firm, whether they be active, nominal, or dormant, and has authority as such to bind them, either by simple contracts respecting the goods or business of the firm, or by negotiable instruments circulated in its behalf to any person dealing bona fide.*

825. Articles of Copartnership is the written agreement of the partners, stating the amount of each partner's investment, the period during which the partnership is to exist, the manner in which the gains and losses are to be divided, and all other conditions which may be considered necessary or expedient.

826. The Capital is the money or other property invested in the business.

827. The **Resources**, or **Assets**, of a firm are its entire property including all debts due the firm.

828. The **Liabilities** include all of the debts or other obligations owed by the firm.

829. The **Net Capital** of a firm is the excess of its resources over its liabilities.

The *Present Worth* is the net capital of a firm when closing business.

830. The **Net Insolvency** of a firm is the excess of its liabilities over its resources.

A firm unable to pay its debts in full is said to be insolvent.

831. The **Net Investment** of a partner is the amount of his investments less his withdrawals from the business.

832. The **Net Gain** of a firm is the excess of total gains over total losses for a given period.

833. The **Net Loss** is the excess of total losses over total gains for a given period.

834. A **General Partnership** is one in which each partner is liable for the debts of the firm to the extent of his entire property personal and social.

835. A **Limited Partnership** is one authorized by statute, and not otherwise, in which certain members called special partners are liable only to the extent of the capital contributed by them. Their private property is distinct from the social assets of such limited partnership, and cannot be taken to pay the debts of the firm. Other members, called general partners, remain liable to the creditors for all the debts of the firm, as in general partnerships.

1. In the case of limited partnerships, the laws of the state usually require full and complete articles of copartnership, specifying the general and special partners, the amount invested by each, etc., which articles must be recorded in the Clerk's office of the county. Strict compliance with the local statutes, both in forming the partnership and in conducting the business, is essential to secure the limited liability of the special partners.

2. *A partner* may receive a regular salary in addition to his share of the *profits of a firm*. Such salary must, of course, depend upon the terms of *the articles of* copartnership.

836. Given the investment and the present worth, to find the net gain or net loss.

1. Find the net gain of a business whose net capital at starting was \$6500 and whose present worth is \$8700.

\$8700
6500
\$2200

EXPLANATION—Since the net capital at starting was \$6500, and the present worth is \$8700, the net gain must be the difference between \$8700, the net capital now, and \$6500, the net capital at starting, which is \$2200.

2. Find the net loss of a business whose net investment was \$9500, and whose present worth is \$8350.

\$9500
8350
\$1150

EXPLANATION.—Since the net investment was \$9500 and the present worth is only \$8350, the net loss must be the difference between \$9500 and \$8350, which is \$1150.

RULE.—*Find the difference between the net investment and the present worth. If the present worth is the greater the result will show the net gain; but if the net investment be the greater, the result will show the net loss.*

3. The net capital of a business was \$12600, but it is now worth \$16400. What is the net gain or loss?

4. The net investment of a firm was \$3640. Its present worth is \$4750. What is the net gain or loss?

5. On Jan. 1, 1899, the net capital of a firm was \$6450.80, and on Jan. 1, 1900, it was \$4967.58. What was the net gain or loss?

6. The net capital of a firm at the beginning of a year was \$2375, and at the end of the year \$4780. What was the net gain or loss?

7. The net capital of a firm on Jan. 1, 1899, was \$2740.50. At the end of the year it was insolvent to the extent of \$1498.50. What was the net gain or loss?

8. The insolvency of a firm at the beginning of a year was \$1675, and its net capital at the end of the year was \$2500. What was the net gain or loss?

9. The insolvency of a firm on Jan. 1, 1899, was \$4365.43, and on Jan. 1, 1900, its insolvency was \$2137.50. What was the net gain or loss?

10. If the insolvency of a firm was \$1800, and its present insolvency is \$5000. What is the net gain or loss?

837. Given the items of gain or loss, to find the net gain or net loss.

1. The total gains of a firm are $6750 and the total losses are $2500. What is the net gain?

$6750
2500
$4250

EXPLANATION.—Since the total gains are $6750 and the total losses are $2500, the net gain must be the difference between $6750 and $2500, or $4250.

RULE.—*Find the difference between the total gains and the total losses, and the result will be the net gain or net loss.*

2. What is the net gain or loss of a firm whose total gains are $6120 and total losses $3450?

3. Find the net gain or loss when the total gains are $3490 and the total losses $4500.

4 In a business the gain on Merchandise was $1540 and on Interest and Discount, $75. The Expense account showed a loss of $530, and Furniture and Fixtures, $16. Find the net gain.

5. In a certain business, the Merchandise account showed a gain of $2100, Real Estate $1500, and Commission $650. The Expense account showed a loss of $1160, Furniture and Fixtures $325, and Interest and Discount $40. What was the net gain or loss?

838. Given the net investment, the gains, and the losses, to find the present worth.

1. The net capital invested in a business was $10000. Upon closing, the Loss and Gain account showed the following items of gains: Merchandise, $2400; Interest and Discount, $27; Commission, $315. Losses: Expense, $700; Real Estate, $345. What was the net capital at closing?

LOSS AND GAIN.

Expense,	700		Merchandise	2400			
Real Estate,	345		Int. & Dis.,	27			
Net Gain,	1697		Commission	315			
	2742			2742			

Net investment, $10000
" gain, 1697
" capital, $11697

RULE.—Add the net gain to the net investment, or deduct the net loss from the net investment, and the result will be the present worth.

2. What is the present worth, if the net capital was $3450, and the net gain is $1700?

3. The net capital of a business was $7860. The total gains amount to $3485 and the total losses $1346. What is the present worth?

4. On Jan. 1, 1899, the insolvency of a firm was $670. At the end of the year the total gains were $6157 and the total losses $1531. What was the net capital at the end of the year?

5. A firm commenced business with a capital of $12500. Upon closing, the Loss and Gain account showed a total debit of $4372 and a total credit of $8450. What was the firm's net capital at closing?

6. The net investment of a business was $15000. At the end of a year the Loss and Gain account showed the following debits: Expense, $2400; Interest and Discount, $65; Live Stock, $185. Also the following credits: Merchandise, $4175; Real Estate, $500; Commission, $250. What was the net capital at the end of the year?

7. The insolvency of a firm on Jan. 1, 1899, was $1850. The total gains during the year were $2465.83, and the total losses $3976.24. What was the condition of the business at the end of the year?

839. Given the resources and the liabilities, to find the net capital.

1. The resources of a business were, Cash, $385; Merchandise, $3170; Bills Receivable, $1365; Accounts Receivable, $237. The liabilities were Bills Payable, $950; Accounts Payable, $535. What was the net capital?

Solution.

Resources.		Liabilities.	
Cash,	$385	Bills Pay.,	$950
Mdse.,	3170	Accts. Pay.,	535
Bills Rec.,	1365	Net Capital,	3672
Accts. Rec.,	237		
	$5157		$5157

Rule.—*Find the difference between the resources and the liabilities. The excess of resources over liabilities will be the net capital; the excess of liabilities over resources the net insolvency.*

2. The resources of a firm on commencing business were, Cash, $1850; Merchandise, $4800; Real Estate, $3000; Furniture and Fixtures, $175. The liabilities were, Bills Payable, $1500; and Accounts Payable, $750. What was the net capital?

3. The firm of Adrian & Bayne commenced business with the following resources: Cash, $3700; Merchandise, $8000; Real Estate, $5000. Their liabilities were, Bills Payable, $1475; Personal Accounts, $1250. What was their net capital?

4. At the end of one year the firm, in example 3, had the following resources: Cash, $1800; Merchandise, $9500; Real Estate, $7000; Accounts Receivable, $1920. Their liabilities were, Bills Payable, $2500; Mortgages Payable, $1500; Accounts Payable, $250. What was the net gain or loss?

5. On Jan. 1, 1899, W. H. Snoke and S. S Stambaugh engaged in partnership. Snoke invested $5000 and Stambaugh $6000. The firm's resources at the end of the year consisted of Cash, $2350; Merchandise, $6300; Real Estate, $3500; Bills Receivable, $2360; and Accounts Receivable, $3140. The firm's liabilities were Bills Payable, $840; Accounts Payable, $1100; and Mortgages Payable, $675. What was the net gain or loss at the end of the year, and what was the net capital?

840. Given each partner's investment, the proportion of gains or losses, and the resources and liabilities at closing, to find each one's share of the gain or loss.

1. A and B formed a partnership. A invested $4000 and was to share $\frac{2}{5}$ of the gains or losses, B invested $6000 and to share $\frac{3}{5}$ of the gains or losses. At the close of business their resources were, Cash, $5700; Bills Receivable, $2800; Real Estate, $4000; Accounts Receivable, $1300. Their Liabilities were, Bills Payable, $800; and Accounts Payable, $500. What was each one's share of the gain or loss?

Solution.

Resources.		Liabilities.	
Cash,	$5700	Bills Pay.,	$800
Bills Rec,	2800	Accts. "	500
Real Estate,	4000	A's Investment,	4000
Accts. Rec.,	1300	B's "	6000
		Net Gain,	2500
	$13800		$13800

A's gain equals $\frac{2}{5}$ of $2500, or $1000.
B's " " $\frac{3}{5}$ " " " $1500.

RULE.—*Find the difference between the total resources and the liabilities including the net investment for the net gain or loss.*

Divide the net gain or loss in the proportion agreed upon for each partner's share.

2. A and B formed a partnership and agreed to share the losses equally. A invested \$3600 and B \$3700. At the end of one year their resources were as follows: Cash, \$2000; Merchandise, \$4000; Real Estate, \$2500; Bills Receivable, \$1500; and Accounts Receivable, \$250. Their liabilities were \$1800. What was each one's gain?

3. C and D formed a partnership in which C was to share $\frac{4}{7}$ of the gains or losses. C invested \$5600 and D \$4900. At the close of the year their resources were \$18000 and their liabilities \$4700. What was each one's share of the gain?

4. A, B, and C entered into partnership. A furnished \$4500 and was to share $\frac{3}{10}$ of the gains or losses, B furnished \$5000 and was to share $\frac{1}{3}$ of the gains or losses, and C furnished \$5500 and was to share the remainder of the gains or losses. At the close of the year the firm's resources were as follows: Cash, \$6750; Merchandise, \$8350; Bank Stock, \$5000; and Real Estate \$4400. The liabilities were Bills Payable, \$1700; Mortgages Payable \$2800; and Accounts Payable, \$1100. What was each one's share of the gain?

5. E, F, and G engaged in partnership. E invested \$2500, F \$3000, and G \$3500. E. was to share $\frac{1}{4}$ of the gains or losses, F $\frac{1}{3}$, and G the remainder. During the year E withdrew \$150, F \$200, and G \$250. At the end of the year the assets were, Cash, \$2475.60; Merchandise, \$5240; Bills Receivable, \$1780; and Telephone Stock, \$1800. The liabilities were, Accounts Payable, \$4750. What was each partner's share of the gain or loss?

6. A, B, C, and D formed a partnership, each to share $\frac{1}{4}$ of the gains or losses. A invested \$4000, B \$4200, C \$4150, and D \$4100. During the year A invested \$500, B withdrew \$300, C invested \$200, and D invested \$400. At the close of the year the resources were, Cash, \$2768.40; Mdse., \$1015.87; Bills Rec., \$1483; and Real Estate, \$3600. The liabilities amounted to \$2378. What was the net gain or loss, and what was each partner's interest in the business at the end of the year?

841. To find each partner's share of the gain or loss when in proportion to the investment.

1. A and B entered into a joint speculation. A put in \$500 and B \$700. They gained \$360. What was each one's share of the gain?

A \$5|00
B 7|00
\$12|00

$\frac{5}{12}$ of \$360 = \$150, A's gain.
$\frac{7}{12}$ of \$360 = \$210, B's "

gain, or $\frac{7}{12}$ of \$360, which are \$210.

EXPLANATION.—The entire capital was \$1200. Since A put in \$500, he furnished $\frac{500}{1200}$ or $\frac{5}{12}$ of the capital, and hence should receive $\frac{5}{12}$ of \$360 or \$150. B furnished $\frac{700}{1200}$ or $\frac{7}{12}$ of the capital, and should therefore receive $\frac{7}{12}$ of the

RULE.—*Divide the gain or loss among the partners in proportion to their shares of the capital.*

If preferred each partner's share may be found by proportion. Thus, in Ex. 1, we have

12 : 5 : : \$360 : A's share of the gain; and also
12 : 7 : : \$360 : B's share of the gain.

2. A and B gained in trade \$800. A furnished \$1500 of the capital and B \$2000. What was each one's share of the gain?

3. A, B, and C formed a partnership. A put in \$2000, B \$3000, and C \$5000. They gained \$4000. What was each one's share of the gain?

4. D, E, and F entered into partnership with \$9000, of which D contributed $\frac{1}{3}$, E $\frac{1}{4}$, and F the remainder. They lost \$2400. What was each one's share of the loss?

5. Three men in partnership gained \$9100. What was each one's share of the gain if they were to share in the proportion of 3, 4, and 7?

6. A, B, C, and D formed a partnership. A invested \$6700, B \$5800, C \$7300, and D \$8200. They gained \$5600. What was each one's share of the gain?

7. D, E, and G formed a partnership with a capital of \$25000. D furnished \$7500, E \$8500, and G \$9000. They gained 18¾% of the joint stock. What was each partner's share of the profit?

8. H, K, and L formed a partnership. H invested \$3650, K \$4260, and L \$5390. They lost 33⅓% of their capital the first year and the second year gained 40% of the remaining capital. *What was* each partner's share of the capital at the end of the *second year?*

842. To find each partner's gain or loss when the investments are for different periods of time.

1. A and B in partnership gained $540. A put in $600 for 6 months, and B put in $900 for 8 months. What was each one's share of the gain?

Int. on $600 for 6 mo. = $18, A's interest.
" " 900 " 8 " = 36, B's "
$54, Firm's "
$\frac{18}{54}$ or $\frac{1}{3}$ of $540 = $180, A's gain.
$\frac{36}{54}$ or $\frac{2}{3}$ of $540 = $360, B's gain.

EXPLANATION.—Since the interest on each one's investment, the rate per cent. being the same, depends on the amount invested by each and the period of time the money was in the business, if we find the interest on A's investment for the time it was employed, and also on B's investment for the time it was employed, each one's share of the total gain will be in the same proportion that his interest is to the total interest. A's interest at 6% is $18, and B's $36. The total interest is $540. Hence, A's gain is $\frac{18}{54}$ or $\frac{1}{3}$ of $540 or $180, and B's gain is $\frac{36}{54}$ or $\frac{2}{3}$ of $540 or $360,

RULE.—*Compute the interest on each partner's investments for the time they have been employed in the business; then on the sums withdrawn for the time out of the business, and subtract the debit interest of each partner from his credit interest. Then for each one's share of the gain or loss, take such a part of the entire gain or loss as each partner's credit interest is of the entire credit interest.*

1. The above is believed to be the shortest and most practical method when each shall share in proportion to the average investment for the time in business. Any rate of interest may be used.

2. If preferred, interest need only be calculated to the time of withdrawal, and then on the balance to the time of the second withdrawal or investment, as the case may be, and so on to the end of the time employed, thus finding only credit interest.

2. A and B engaged in partnership. A put in $1500 for 7 months, and B put in $1800 for 10 months. They gained $1425. What was each one's share of the gain?

3. A, B, and C were partners in business. A invested $2000, B $3500, and C $4500. A's money was employed 1 year, B's 10 months, and C's 8 months. They lost $2400. What was each one's share of the loss?

4. A and B were in partnership for one year. A at first put in $4000 and B $3000. At the end of 8 months, A withdrew $1000 and B added $1500 to his investment. At the end of the year they *had gained $5000.* What was each one's share of the gain?

5. On Jan. 1, 1899, Mr. M. commenced business with $8000 capital. On May 1, he took in Mr. A. with $6000 capital, and on July 1, Mr. C. with $10000 capital. At the end of the year they had gained $7000. What was each one's share of the gain?

843. To find each partner's capital at time of closing the accounts when interest is allowed on investments and charged on withdrawals.

1. On Jan. 1, 1898, John Harris and G. G. Groff formed a partnership for one year. It was agreed to share the gains or losses equally, and that each partner receive interest on his investments and pay interest on his withdrawals at 6%. They gained $2500. What was each partner's capital on Jan. 1, 1899? Their accounts at the end of the year were as follows:

JOHN HARRIS.

Dr.						Cr.				
1898					1898					
Apr	1	Cash,		400	Jan.	1	Cash,		5000
July	1	"		500	Sept.	1	"		2500

G. G. GROFF.

1898					1898				
Nov.	1	Cash,		500	Jan.	1	Cash,		4000
					May	1	"		3000

Solution.

JOHN HARRIS.

Int. on $400 for 9 mo. = $18 Int. on $5000 for 1 yr. = $300
" " 500 " 6 mo. = 15 " " 2500 " 4 mo. = 50
$33 $350

Harris' net credit interest = $350 − $33, or $317.

G. G. GROFF.

Int. on $500 for 2 mo. = $5 Int. on $4000 for 1 yr. = $240
" " 3000 " 8 mo. = 120
$5 $360

Groff's net credit interest = $360 − $5, or $355.

$317 + $355 = $672, sum of the net credit interest.

$2500 − $672 = $1828, gain to be divided equally between the partners.

$1828 ÷ $2 = $914, each partner's share of the gain.

Now, we credit each partner's account with his net credit interest and with his share of the gain, and then close the accounts. The "Balance" (*the difference* between the total credits and total debits) of each partner's *account is his* net capital at closing.

The following shows how each partner's account would appear ıen closed at the end of the year, and also how these accounts ɜ re-opened for a continuance of the business:

DR. JOHN HARRIS. CR.

)98					1898				
)r.	1	Cash,		400	Jan.	1	Cash,		5000
ly	1	"		500	Sept.	1	"		2500
c.	31	Balance,		7831	Dec.	31	Interest,		317
							Gain,		914
				8731					8731
					1899				
					Jan.	1	Balance,		7831

G. G. GROFF.

398					1898				
)v.	1	Cash,		500	Jan.	1	Cash,		4000
:c.	31	Balance,		7769	May	1	"		3000
					Dec.	31	Interest,		355
							Gain,		914
				8269					8269
					1899				
					Jan.	1	Balance,		7769

RULE.—*Find the interest on the investments and also on the ithdrawals from their date to the time of settlement. Place the fference between the sums of these interest items on the proper side the account. Then divide the remainder of the gain or loss in e proportion agreed upon, entering a gain on the credit side of e account, or a loss on the debit side, and find the balance.*

2. Henry Bermont, Roy Hill, and David Nye formed a partnerıip on Jan. 1, 1897, for 1 year; the gains and losses to be shared ıually, and interest to be allowed on all investments and charged ı all withdrawals. On Jan. 1, Bermont invested $3000, Hill ł500, and Nye $6000. On Mar. 1, Bermont invested $2500, Hill ł200, and Nye $700. On July 1, Bermont withdrew $1500, Hill ł000, and Nye $800. On Sept. 1, each invested $500. They ıined during the year $4865. What was each partner's capital ı *Jan. 1. 1398?*

3. A and B formed a partnership on Apr. 1, 1902, for 1 year. A invested on Apr. 1, $5000, and B $7000. At the end of 6 months, A withdrew $1000 and B $2000. At the end of the year the net gain was found to be $3600. If the gain was divided in proportion to their average investments, what was each partner's capital on Apr. 1, 1903?

4. Jones and Smith formed a partnership, and agreed to share the gain or loss in proportion to their average investments. On Jan. 1, 1903, each invested $3500. On May 1, Jones added $1500 to his capital, and Smith withdrew $1000. They gained $2400. What was each partner's capital at the end of the year?

5. A. R. Byerly and T. R. Baker formed a partnership on Jan. 1, 1901, for one year, each investing $4000. On Apr. 1, Byerly invested $1000, and on May 1, Baker invested $1500. On July 1, Byerly invested $500, and Baker withdrew $500. At the end of the year, the firm's resources were $18000 and the liabilities $3450. If the gain or loss was divided in proportion to their average investments, what was each partner's share of the net capital on Jan. 1, 1902?

6. A, B, and C formed a partnership on Jan. 1, 1900. It was agreed to allow interest on all investments and to charge interest on all withdrawals at the rate of 6%. A is to share $\frac{3}{12}$ of the gain or loss, B $\frac{4}{12}$, and C $\frac{5}{12}$. On Jan. 1, A invested $8000; on Apr. 2, $1500; and on June 12, he withdrew $1400. B invested on Jan. 1, $11000; on May 5, $3500; and on Aug. 10, he withdrew $2500. C invested on Jan. 1, $10000; on Mar. 10, $4000. He withdrew on July 14, $1800, and on Aug. 25, $1200. They dissolved partnership on Oct. 1, 1900. Their loss was $6972. What was each partner's net credit interest and net capital at closing?

In finding the time in problem 6, count the exact number of days.

7. L. F. Gardner, G. A. Rockwood, and A. C. Macdonald formed a partnership on Jan. 1, 1903, with a capital of $20000, of which Gardner put in $8000, Rockwood $7000, and Macdonald $5000. At the end of 4 months Gardner added $2000 to his investment, and 3 months later he withdrew $3000. Rockwood, at the end of 5 months, added $1000, and 4 months later he added $2000 to his capital. Macdonald, after 2 months in the business, withdrew $1500; but after being in business 6 months he added $4000 to his capital. The net gain of the firm at the end of the *year was $9000.* If the gain was divided according to average in*vestment, what* was each partner's capital on Jan. 1, 1904?

BANKRUPTCY.

844. A **Bankrupt,** or **Insolvent,** is a person who fails in business and is unable to pay his debts.

A person may by due process of law declare himself in a state of bankruptcy, or may be adjudged a bankrupt at the instigation of one or more of his creditors. Firms and corporations when unable to meet their pecuniary obligations are subject to the same legal proceedings as a single individual who is insolvent.

845. The **Assets** of a bankrupt are his entire property including all debts due him.

Available Assets are such as can be converted immediately into cash. *Nominal Assets* are those which cannot be converted into cash, except at a discount. A *debtor* of a bankrupt is a person or firm who owes him.

846. The **Liabilities** of a bankrupt are all of the obligations due by him to his creditors.

A *Creditor* is one to whom the bankrupt is indebted. A *preferred creditor* is one who is paid in full if the assets are sufficient for this purpose. An employee is a preferred creditor.

847. A **Statement,** or **Schedule,** is a list of the assets and liabilities of the bankrupt, giving the names of his creditors and debtors, their place of business or residence, and the sum due each or from each of them.

848. An **Assignee** is a person appointed to take charge of the property of a bankrupt for the purpose of converting it into cash and, after deducting the necessary expenses of the assignment, to pay such a proportion of the liabilities to the creditors as the assets will allow.

When a corporation becomes bankrupt, the person appointed to take charge is called a *Receiver*.

849. A **Dividend** is the portion of the assets which is paid by the assignee to the creditors.

850. A **Discharge** is a decree relieving the bankrupt from further indebtedness to his creditors after he has complied with the full requirements of the insolvent laws.

A bankrupt who receives a *discharge* may begin business anew legally free from all obligations under the former business.

851. Computations in Bankruptcy may be made in accordance with the principles of percentage by regarding the total liability as the *base* and the available assets as the *percentage.*

852. To find each creditor's share of available assets.

1. A bankrupt's assets were as follows: Cash, $1534.27; Merchandise, $2150.43; Bills Receivable, $1050; and Accounts Receivable, $1735.30. His liabilities were the following: Due A. B Hughes & Co., $4250; due G. E. King & Co., $2135; and due C. P. Prime & Co., $1475. The expenses of the assignment were $182. How much did each of the creditors receive?

Solution.

Assets.		Liabilities.	
Cash,	$1534.27	A. B. Hughes & Co.,	$4250
Merchandise,	2150.43	G. E. King & Co.,	2135
Bills Rec.,	1050.	C. P. Prime & Co.,	1475
Accounts Rec.,	1735.30		
Total Assets,	$6470.00	Total Liabilities,	$7860
Expenses,	182.		
Net Assets,	$6288.		

$6288 ÷ $7860 = .80 or 80%, rate of dividend.
$4250 × .80 = $3400, Hughes & Co.'s dividend.
$2135 × .80 = $1708, King & Co 's "
$1475 ÷ .80 = $1180, Prime & Co.'s "

Rule.—*From the total assets deduct the expenses of the assignment, and divide by the total liabilities. The quotient will be the rate of dividend.*

Multiply the amount of each creditor's claim by the rate of dividend.

Each creditor's share of the assets may be found by the following proportion: Total liabilities : Each liability : : Net assets : Each creditor's share of the net assets.

2. A merchant failed with liabilities amounting to $18230. His assets amounted to $12852. How much should each creditor receive on the dollar, the expenses of assignment being $380? How much was allowed L. K. Wise & Co., whose claim was $1500?

3. A firm failed with liabilities amounting to $30000. The assets were, Cash, $1875; Real Estate, $6500; Bills Receivable, $4250; Accounts Receivable, $3475. The expenses of the assignment were $315. How much should D. M. Keefer receive whose claim against the firm was $2350?

4. N. G. Bragg & Co. failed with liabilities amounting to $150000. The assets of the firm amounted to $56750. The assignee's charges were $2800. How much did F. Albert receive who had a claim against the firm for $20000?

5. A leather firm failed with liabilities amounting to $42500. The assets were $27345. If the expenses for settling were $495, how much should D. C. Morrison, F. H. Slyder, and H. A. Bitner receive, whom the firm owed $5263, $6587, and $7345 respectively?

6. Mull & Co. placed in the hands of an assignee, for the benefit of their creditors, a stock of finished goods which sold for $21000, raw material which sold for $2700, and personal property which sold for $3800. Their liabilities were $46420. The assignee's charges were 5% of the assets. How much did each creditor receive on the dollar?

7. The total liabilities of a firm which failed were $38450. The machinery owned by the firm was sold for $15600. The company had on hand manufactured goods which sold for $3150, and raw material which brought $1750. The real estate belonging to the firm consisted of a building and lot which sold for $5400, and a tract of timber land which brought $1175. The assignee also collected on notes and personal accounts $2850. The expenses of the assignment were $925. What dividend was paid to the creditors, and how much did G. W. Hull receive whose claim was $5750?

8. Lane, Root & Co. failed in business. Their liabilities amounted to $80000. The assets of the firm were, real estate, worth $22,000; stock of goods, worth $8000; notes on hand, worth $1500; personal accounts due the firm, $825; and cash, $425. The assignee's charges amounted to 5% on the sum distributed to the creditors. How much could the firm pay on the dollar?

9. Slow & Co. failed, owing John Horst $18750, Samuel Stumbaugh $15000, and Elmer Reynolds $12500. The assets of the firm were $19762, exclusive of the real estate. The expenses of settling were $1850. They owed their employees $2750, which was paid in full. The real estate was sold for $15000. What per cent. of their claims was paid Horst, Stumbaugh, and Reynolds, and *how much did each* receive?

AVERAGE OF ACCOUNTS

853. An **Account** is a record of business transactions, and may contain either debits or credits or both.

854. Average of Accounts, or **Equation of Payments,** is the process of finding the time when sums due at different times may be settled with one payment, without loss of interest to either debtor or creditor.

855. The **Cash Balance** is the sum required to settle an account at a specified time, when part payment has already been made.

856. The **Term of Credit** is the time allowed for the payment of a debt.

All accounts bear legal interest after their term of credit expires.

857. The **Average Term of Credit** is the average time allowed for the payment of two or more sums due at different dates.

858. The **Focal Date** is any assumed date with which the dates of two or more payments are compared to find their average date.

859. The **Equated Time,** or **Average Date,** is the date on which several debts due at different times may be paid in one sum.

SIMPLE AVERAGE.

860. Simple average is the process of averaging accounts containing debit or credit items only.

The process consists in *assuming* a certain date as the date of settlement, and finding what the gain or loss would be to the payer if all the bills were paid by him on that date; then in finding in how many days the total amount of the bills would produce a sum equivalent to this gain or loss of interest. The *true*, or *average*, date of settlement is then found by counting this number or days forward or backward from the *assumed* date.

The interest on bills paid after they are due, should equal the interest on bills paid before their term of credit expires, since the former is a gain to the payer and the latter a loss.

Any date may be assumed as the focal date, or time of settlement, and any rate of interest may be used in making the computations, though the *latest maturity* of any item in the account is perhaps the most convenient *date, and 6%* the most convenient rate.

861. To find the equated time when an account consists of only debits or credits.

1. Find the equated time for the payment of three bills of goods, due as follows: Jan. 15, $50; Mar. 11, $100; May 20, $150.

Jan. 15,	$50,	125 da.	$1.04 interest.
Mar. 11,	100,	70 "	1.17 "
May 20,	150,		
6)$300			$2.21
|.05)$2.21			

44 days before May 20, or Apr. 6.

EXPLANATION.—Assume May 20, the latest due date, as the focal date. If the total amount ($300) of the bills is paid on May 20, there will be a gain of interest to the payer of $2.21. The interest of $300 for 6 days at 6% is $.30, and for 1 day it is $.05. It will take $300 to produce $2.21 as many days as $.05 is contained in $2.21, or 44 days. If, at the assumed date of settlement, there is a gain to the payer of the interest of the entire debt ($300) for 44 days, the date of settlement should be 44 before May 20, or April 6.

RULE.—*Assume the latest due date for the focal date. Compute the interest on each item of the account from the date it is due to the focal date. Divide the total interest by the interest on the total amount of the bills for 1 day, and the quotient will be the number of days to count back from the focal date.*

1. When an item has a term of credit given in days, count the actual number of days to find when due; if given in months, count by months.

2. Use compound subtraction in finding the time when the interval between any due date and the focal date is longer than a year.

3. In dividing for the number of days, if the remainder is less than ½, drop it; if ½ or more, call it a day.

2. What is the average due date of the following bills, each being due at the date given: April 12, $150; May 18, $180; and June 24, $170?

3. On what date could the following bills be paid in one amount without loss of interest to either party: Due Jan. 20, $175; Feb. 12, $130; Mar. 3, $150; and Mar. 30, $240?

4. A merchant sold a customer bills on the basis of net cash as follows; April 6, $180.25; April 24, $200.40; May 18, $160; and May 30, $140.75. On what date in equity could the entire amount be paid?

5. The following bills of goods were purchased on 30 days' credit: June 5, $45; June 24, $60; and July 20, $75. What is the average date of payment?

Suggestion.—When bills are sold on a common term of credit, first find the average date of purchase, and to the result add the common term of credit. When sold on different terms of credit, mature each bill separately and then find the average date of settlement.

6. I bought several bills of goods as follows:

April 7, a bill of $175 on 30 days' credit.
May 20, " " " 215 " 60 " "
June 24, " " " 320 " 3 months' "

On what date may the entire amount be equitably paid?

7. A produce merchant sold Jan. 2, 1900, 80 bbl. flour at $4.20 per barrel on 30 days' credit; Jan. 20, 1900, 50 bbl. apples at $1.75 per barrel on 60 days' credit; Feb. 8, 1900, 40 tubs butter, 48 lb. each, at $.27 per pound on 1 month's credit; Feb. 24, 1900, 75 bbl. Minn. flour at $5.10 per barrel on 2 months' credit; Mar. 5, 1900, 150 bbl. potatoes at $1.25 per barrel on 20 days' credit. What is the equated date at which the entire amount might have been paid?

COMPOUND AVERAGE.

862. Compound Average is the process of finding the mean, or equitable, time for the payment of the balance of an account containing both debits and credits.

863. To find the equated time for the payment of the balance of an account consisting of both debits and credits.

1. What is the equated time for the payment of the balance of the following account?

DR. C. C. GAINES. CR.

1900					1900				
May	7	Mdse, 30 da.,		450	July	18	Cash,		300
June	22	" 60 "		360	Sept.	17	Note, 30 da.,		180
Aug.	10	" 3 mo.,		240					
June	6, $450	157 da., . .	$11.78		July	18, $300	115 da., . .	$5.75	
Aug	21, 360	81 " . .	4.86		Oct.	17, 180	24 " . .	.72	
Nov.	10, 240								
	$1050		$16.64			$480		$6.47	
	480		6.47						
	$570		$10.17		Int. on $570 for 1 da. = $.095.				

$10.17 ÷ $.095 = 107 days back from Nov. 10, or July 26, 1900, is the *equated time* for payment.

EXPLANATION.—Assume Nov. 10, the latest due date of either side of the account, as the focal date. If the balance ($570) of the account is settled Nov. 10, there will be a gain to C. C. Gaines of the interest ($16.64) on the debtor items due before Nov. 10, and a loss to him of the interest ($6.47) on all the credit items paid before Nov. 10. Hence, if settlement is made Nov. 10, there is a net gain of interest to C. C. Gaines of $10 17. The interest on the balance of the account ($570) for 1 day is $ 095; and to produce $10.17 interest it will take as many days as $.095 are contained times in $10.17, or 107 days. If at the assumed date of settlement (Nov. 10), there is a gain to C. C. Gaines of the interest of $570 for 107 days, the true date of settlement should be 107 days earlier than Nov. 10, or July 26.

Had the credit interest been the greater, C. C. Gaines would have had the right to keep the balance of the debt until the interest upon it would equal the credit interest. In that case the number of days would be counted forward from the focal date.

RULE.—*Compute the interest on each item from the time it is due to the latest due date of any item in the account. Find the total debit interest and the total credit interest; and divide their difference by the interest on the balance of the account for 1 day, and the quotient will be the number of days to count backward or forward from the focal date.*

If the balance of the account and the balance of the interest are in favor of the same side, count back; if on opposite sides, count forward.

2. Find the average date of maturity for the balance of the following account:

DR. G. E. RHODES. CR.

1898					1898				
Apr.	4	Mdse.,		475	May	8	Cash,		450
May	20	"		650	June	12	"		100

3. When may the balance of the following account be paid without loss to either party?

DR. R. P. HYTE. CR.

1899				1899			
Jan.	2	Mdse., 30 da.,	256	Feb.	10	Cash,	250
Feb.	12	" 30 "	420	Mar.	20	"	495
Mar.	18	" 60 "	375		30	"	145

4. Find the equated time for the payment of the balance of the following account:

DR. D. O. DECK & Co. CR.

1897				1897					
June	10	Cash,	350	Apr.	4	Mdse., 30 da.,	450	80
	20	Note, 2 mo.,	200		20	" 2 mo.,	364	75
July	12	Bank Draft,	360	May	14	" 3 "	287	40
				June	24	" 60 da.	560	35

5. What is the balance of the following account, and when will it begin to draw interest?

DR. JAMES FLOYD. CR.

1900					1900				
Jan.	4	Mdse., 30 da.,	302	50	Jan.	20	Cash,	275	
Mar.	12	" 60 "	412	60	Apr.	30	"	500	
Apr.	20	" $485, 6% off,	***	**	May	18	Check,	125	
June	9	" $270, 10 & 5% off,	***	**					

6. What was the balance due on the following account, Sept. 3, 1900, interest allowed at 6%?

DR. G. W. GRADLEY. CR.

1900					1900				
Feb.	5	Mdse, 30 da.,	1340		Mar.	1	Cash,	1250	
Apr.	10	" 60 "	2400		June	16	"	1500	
June	1	" 30 "	600		July	14	"	1150	
	30	" 10 "	280						

Suggestion.—Find the true date for the settlement of the account, and then compute the interest on the balance of the account from this date to Sept. 3. The interest added to the balance of the account will give the sum due.

ACCOUNTS CURRENT.

864. An **Account Current** is a written statement of the business transactions between two houses, showing the balance due on a fixed date. The amount due is called the *Cash Balance.*

An Account Current is a transcript of the ledger account, with the addition of certain details taken from the books of original entry when deemed necessary, and arranged in a different form.

Interest is reckoned or not, according to the custom of the business or the agreement between the parties. When interest is not charged, the balance due is the difference between the two sides of the ledger account.

865. The **Adjustment** of an account is the determining of the balance due at a specified date.

In adjusting an account we find *what balance is due* at a *specified date;* in *averaging* an account, we find *at what date the balance is due.*

866. Accounts Current are sometimes averaged as in the Average of Accounts. This is done either to fix a date for settlement *in the future*, or to find the time from which to compute interest on the balance when the average date is *past.*

867. 1. What is the cash balance, including interest at 6%, on the following account, Jan. 1, 1901, and what is the average date of settlement? (Study with the account Art. 868.)

DR. A. B. MOSS. CR.

1900				1900			
Sept.	3	Balance,	500	Sept.	27	Cash,	400
Oct.	22	Mdse., 30 da.,	450	Oct.	13	Note, 30 da.,	100
Nov.	26	" 30 "	300		23	Cash,	200

868. Account Current and Interest Account.

DR. A B MOSS, Troy, N. Y., in account with C. RILL, Albany, N. Y., CR
Interest to Jan. 1, 1901, at 6%.

		Due		Time.	Int.		Amt.		Date.			Due.		Time.	Int.		Amt	
									1900									
3	Balance,	Sept.	3	120	10		500		Sept.	27	Cash,	Sept.	27	96	6	40	400	
22	Mdse., 30 da.,	Nov.	21	41	3	08	450		Oct.	13	Note, 30 da.,	Nov.	12	50		83	100	
26	" 30 "	Dec.	26	6		30	300			23	Cash,	Oct.	23	70	2	33	200	
									1901									
1	Bal. of Int.,						3	82	Jan.	1	Bal. of Int.,				3	82		
											" " & %						553	82
					13	38	1253	82							13	38	1253	82
1	Balance,						553	82										

$3.82 ÷ $.092 = 42.

The average date of settlement is 42 days back of Jan. 1, 1901, or Nov. 20, 1900.

EXPLANATION—We first find the due date of each item of the account and compute the interest on each item from its due date to the date of settlement, Jan. 1, 1901. The total interest on the debit side of the account is $13.38, and on the credit side $9.56. The balance of the interest ($3.82) is therefore in favor of the debit side, or is due Mr. Rill. Since both the balance of the account ($550) and the balance of interest ($3.82) are on the debit side of the account, the cash balance due Jan. 1, 1901, is $550 plus $3.82, *or* $553.82.

If the balance of interest had been on the credit side of the account, the cash balance due would have been $550 minus $3.82, or $546.18.

To find the average date, we simply divide the balance of the interest by the interest on the balance of the account ($550) for 1 day, and the quotient gives us 42 days to count back from Jan. 1, 1901, or Nov. 20, 1900.

1. If an item falls due after the date of settlement, the interest on such item should be transferred to the interest column on the opposite side of the account.

2. Use compound subtraction in finding the time when over a year.

3. In computing interest on the items, consider only the whole number of dollars by dropping the cents when less than half a dollar, and regarding the cents as a dollar when equal to or greater than half a dollar. In adding the items, however, the exact figures must be taken.

4 The Cash Balance may also be found by first finding the equated date of the payments, and computing the interest on the balance due at that time, from the equated date to the specified time; or if the equated date is later than the date of settlement, discount the balance of the account for the intervening time.

2. What was the cash balance on the following account, Jan. 1, 1899, interest at 6%?

Dr. JOHN ROX. Cr.

1898				1898			
July	1	Balance,	650	Aug.	4	Cash,	475
Sept.	15	Mdse.,	340	Oct.	28	"	290
Nov.	20	"	480				

3. Find the cash balance of the following account, May 1, 1898, interest at 6%.

Dr. A. B. DIAMOND. Cr.

1898					1898				
Jan.	1	Balance,	896	23	Jan.	30	Net Proceeds,	648	37
	24	Mdse.,	785	40	Mar.	8	" "	560	84
Feb.	10	Draft, J. H. G.,	950	78	Apr.	20	Cash,	300	

4. Find the cash balance of the following account, Sept. 1, 1900, interest at 6% ; also the average date.

Dr. R. C. ROSH in % current with T. K. LAKE & Co. Cr.

1900				1900			
Apr.	2	Balance,	1500	Apr.	25	Mdse., 30 da.,	1200
	20	Draft $850,	1480	May	21	" 60 da.,	850
June	12	" $895,	785	June	14	" 2 mo.,	560
Aug.	15	Note, 3 mo.,	600				

5. Find the cash balance due on the following account, Apr. 1, 1904, interest at 6%.

DR. EDWARD BROOKS in % current with E. RICE. CR.

1903				1903				
June	1	Mdse., 10 da.,	800	July	15	Cash,		500
Aug.	20	" 30 "	600	Sept.	12	Note, 30 da.,		700
Oct.	26	" 2 mo.,	2000	Nov.	2	Cash,		1500
Dec.	7	" 60 da.,	1200					

6. Find the cash balance of the following account, Jan. 1, 1901, interest at 6%.

DR. D. C. BOX in % current with JNO. ROE & CO. CR.

1899				1899				
May	1	Balance,	1000	June	15	Mdse., 2 mo.,		800
Oct.	12	Draft,	700	Sept.	6	" 30 da.,		900
Nov.	21	"	600	1900				
1900				Feb.	8	" 60 "		1150
Jan.	4	Check,	500	Mar.	30	" 3 mo.,		725
Apr.	20	Draft,	800	May	11	" 30 da.,		1080
Sept.	27	"	1600	Aug.	30	" net		400

AVERAGING ACCOUNT SALES.

869. When a consigment of goods is received to be sold on commission, the consignee usually pays the freight charges and other necessary items of expense, and frequently makes cash advances to the consignor, charging interest on the same or retaining the net proceeds sufficiently long to offset the accrued interest on the advances.

870. To find the date of payment of the net proceeds of a consignment.

RULE.—*Find the average date of the sales as in simple average. Then take this date as the date for the total sales, and of the commission (and guaranty, if any). Place the charges and their dates on the debit side, the total sales and their average date (as found) on the credit side, and average the account.*

Another method is to date the commission on the last date of sales, and equate the account in the ordinary way, taking the charges as debits and the sales as credits. While this method is simpler than the other it is not so exact, and hence the examples should be solved by the former method.

1. What are the net proceeds of the following account sales, and when was it due?

ALBANY, N. Y., Sept. 19, 1900.

ACCOUNT SALES OF FLOUR,

SOLD FOR ACCOUNT OF D. J. FAUST.

1900					
Jan.	5	160 bbl.	@ $4.20	* * * * *	
Feb.	20	120 "	" 4.25	* * * * *	
Mar.	30	100 "	" 4.30	* * * * *	
Apr.	12	75 "	" 4.40	* * * * *	* * * * * *
		Charges.			
Jan.	2	Freight,		75 60	
	2	Cartage,		12 50	
Feb.	24	Remittance,		1000	
		Commission 4%,		* * * *	* * * * * *
		Net Proceeds,			* * * * *

2. Find the net proceeds of the following account sales, and the date when due.

BALTIMORE, MD., Dec. 21, 1899.

ACCOUNT SALES OF APPLES,

SOLD FOR ACCOUNT OF E. C. GRACE.

1899					
Oct.	12	180 bbl.	@ $1.75		
	30	150 "	" 1.80		
Nov.	17	240 "	" 1.90		
Dec.	15	130 "	" 2.00		
		Charges.			
Oct.	10	Freight,		64 70	
	11	Drayage,		21	
	15	Storage,		13 80	
		Commission 3¾%,			

3. Find the date for settlement of the following account sales, *and the cash* balance on Sept. 1, 1900, at 6%.

CLEVELAND, O., Sept. 1, 1900.

.CCOUNT SALES OF PORK,
SOLD FOR ACCOUNT OF H. J. LEAN & Co.

00				
y	14	25 bbl. @ $12.25, 10 da.,		
	30	40 " " 12.50, 30 da.,		
y	18	50 " " 12.35, 20 da.,		
g.	1	35 " " 11.85, Cash,		
		Charges.		
y	1	Frgt. and drayage,	126	40
	20	Remittance,	450	
		Commission 3%,		
		Guaranty 2% (credit items),		

uggestion.—First find the equated date for the payment of the net pro-ls; then add the interest to the net proceeds for the intervening time ı the equated date to the time of settlement.

. Find the date for settlement of the following account sales, the cash balance on Apr. 1, 1904, at 6%.

PHILADELPHIA, PA., Apr. 1, 1904.

ıCCOUNT SALES OF FLOUR AND GRAIN,
SOLD FOR ACCOUNT OF POPE & SAXE.

04						
.	5	150 bbl. F. F.,	@	$4.50		
).	16	200 " J. S.,	"	4.75		
	29	100 " P. B.,	"	5.		
r.	12	50 " S. F.,	"	4.80		
	25	500 bu. Wheat,	"	.90		
	31	1000 " "	"	1.		
		Charges.				
.	2	Freight,			135	
	4	Drayage,			40	
).	20	Remittance,			1200	
r.	15	Storage,			25	
		Commission 3½%,				
		Guaranty 1½%,				

GENERAL REVIEW.

871. 1. A house which cost $4750 was sold for $5600. What was the gain per cent.?

2. Find the cost of a bill of hardware listed at $850 with discounts 15%, 10%, and 5%.

3. How much must be received for a lot of grain which cost $3500 to gain 2½%?

4. A man sold a house for 10% less than it cost him. If he had sold it for $5175 he would have gained 15%. For how much did he sell it?

5. A bought goods in New York, costing $442.38. Transportation, casing, etc., adds 5% to the first cost. At what price must the goods be sold to gain 25% on the full cost?

6. The gross proceeds of a sale were $475. The total expenses were storage $8.50, drayage $10.75, and commission 3%. What were the net proceeds of the sale?

7. A grocer mixed 20 lb. tea at 40¢ with 30 lb. at 50¢. At what price per pound must he sell the mixture to gain 25%?

8. If it require 164 rd. of fencing for a rectangular field 50 rd. long, how many acres does it contain?

9. A cistern 8 ft. in diameter and 13 ft. deep lacks 2 feet of being full of water. How many gallons of water in the cistern?

10. What is the premium for insuring a house valued at $3600 for ⅔ of its value when the rate of premium is 1⅝%?

11. A farmer bought a yoke of oxen for $150, which was 25% less than their real value, and sold them for 25% more than their real value. What was his gain per cent.?

12. What is the average width of a tapering board 23¾ in. wide at one end and 17⅝ in. wide at the other end?

13. If the board in problem 12 is 24 ft. long, how many square *feet does it* contain?

14. At the beginning of the year a firm's resources were as follows: Cash, $2350; Mdse., $8970; Bills Rec., $3240; and Accts. Rec., $1750. The firm's liabilities were Bills Pay., $2325; and Accts. Pay., $3425. At the end of the year the firm's resources were as follows: Cash, $3700; Mdse. $7540; Bills Rec., $2500; and Accts. Rec., $1200. The firm's liabilities at this time were, Bills Pay., $500; and Accts. Pay., $1475. How much did the firm gain or lose during the year?

15. A rectangular lot 100 ft. long and 60 ft. wide is surrounded by a close board fence 7 ft. high. What was the cost of the boards at $14 per M.?

16. What is the interest upon a note for $750 for 90 days at 7½%?

17. A certain note for $960 and dated April 1, 1890, was paid on June 24, 1891. What was the amount of the note, interest at 6%?

18. A pile of wood 140 ft. long, 6 ft. high, and 4 ft. wide was bought for $60. What was the gain if the wood was afterward sold at $3.60 per cord?

19. A farmer has a bin 8 ft. long, 5 ft. 9 in. deep, and 4 ft wide. How many bushels of wheat (approximate measure) will it hold?

20. A crib 20 ft. long, 14 ft. high, and 3½ ft. wide will hold about how many bushels of unshelled corn?

21. At $17 per M, what will be the cost of the planking for a barn floor 36 ft. by 20 ft., the planking to be 2½ in. thick?

22. A coal dealer bought a car load of coal, 33000 lb., at a cost of $4.10 per long ton, which he retailed at 26¢ per hundredweight. What was his gain on the car load if there was a loss of 3% in handling the coal?

23. Two houses were bought for $5600 each. The purchaser sold the one at a gain of 20% and the other at a loss of 20%. Did he gain or lose by the transaction?

24. Two houses were sold for $5600 each, the seller having gained 25% on the one house and lost 25% on the other. Did he gain or lose in the sale of the two houses?

25. A man received $15 a month ground rent for real estate in a certain city. If money was worth 6%, what was the value of the *property*?

26. A merchant bought goods listed at $2500 for 20% and 10% off, and sold the goods at an advance of 15% on the list price What was the gain?

27. Jan. 14, 1904, Geo. Smith & Bro., bought of Messrs. Brown & Co., a bill of goods amounting to $21475.83, on 6 months time, with the privilege of discounting the whole or a portion of the bill, upon the following terms: 7% for 10 days; 6% for 30 days; and 4% for 60 days. Jan. 24, Smith & Bro., paid $10000; Feb. 13, they paid $5000; and Mar. 13, they paid $3000. What was due on the account at the end of 6 months?

28. A commission merchant sent in net proceeds $384, after deducting $16 charges. What rate of commission did he charge, if $4 was paid for storage?

29. The gable ends of a house are each 36 ft. wide, and the perpendicular height of the ridge above the eaves is 12 ft. How many feet of boards will it take to cover both gable ends?

30. If a staff 5 ft. high cast a shadow 8 ft. long, what is the height of a steeple the shadow of which is 190 ft.?

31. What must be the face of a note which when discounted for 90 days at 6% will yield $500?

32. Portland, Me., is 70° 15′ 18″ west longitude and Chicago, Ill., is 87° 35′ west longitude What is their difference in time?

33. What will be the cost of excavating a cellar 32 ft. long, 28 ft. wide, and 5½ ft. deep at 75¢ per cu. yd.?

34. How many bricks 8 in. by 4 in. will be required to pave a yard 25 ft. long and 20 ft. wide?

35. A banker's assets are $8760, and his liabilities $10512. What per cent. can he pay his creditors?

36. If $1500 are gained by selling goods at a profit of 16⅔%, what would be received by selling the goods at a loss of 16⅔%?

37. The selling price was $325, and the rate of gain 25%. What was the cost?

38. A merchant sold a bill of goods for $1260, and lost 20%. Had he sold ⅝ of the goods at 15% profit and the remainder at 12½% profit, what would he have gained?

39. If a merchant buys goods billed at $1500, 10% and 15% off *the list, and* sells them at 5%off the list price, what per cent. profit *does he make*?

40. A merchant bought an invoice of grain, which, including $1\frac{1}{8}\%$ commission, cost \$5050.62½, and paid \$15.25 for freight. He sold the grain at a profit of 15% on its first cost, and invested the proceeds in sugar, which he sold at 5% profit, receiving in payment a note legally due in 48 days. This note he had discounted at 6% at bank. What was the cost of the grain, and how much were his profits?

41. What must be the depth of a circular cistern 8 ft. in diameter to hold 300 barrels of water of 31½ gallons each?

42. At what per cent. above the manufacturer's price must a wholesale merchant mark goods that after allowing a discount of 20% and 5% he may still have a profit of 25%?

43. A grocer bought at auction 200 cases of fine wines for \$2800. What must he ask for it per case, so that he may discount 20% on asking price, allow 10% of sales to be bad, pay $2\frac{1}{2}\%$ commission for selling, and still make a profit of 100% on cost?

44. What is the present worth of \$500 due in 1 yr. 8 mo. when money is worth 6%?

45. A certain cylinder is 4 ft. in diameter and 10 ft. long. What is its entire surface?

46. A merchant bought 20 pieces of cloth, each containing 25 yards at \$4.25 per yard, on a credit of 6 months. and sold them at \$5.50 per yard on a credit of 4 months. What was his cash gain, money being worth 6%?

47. The true discount on a certain sum of money for 4 mo. 12 da. at 6% is \$10.40. What is the sum?

48. The list price of an organ was \$90. An agent bought it at 25% off and sold it for \$84. What was his gain per cent.?

49. When railroad stock, at 10% advance, is selling at \$110 per share, how many shares may be bought for \$2750, and what will be the amount of premium?

50. A merchant directed his broker to purchase 50 shares of Bank stock, par value \$100, which was selling at 8% premium. What was the cost if the brokerage was $\frac{3}{4}\%$?

51. Find the cost of 400 shares Union Pac. stock at $118\frac{3}{8}$, brokerage $\frac{1}{8}\%$.

52. I sold 150 shares Col. & Southern railroad stock at $89\frac{1}{4}$, brokerage $\frac{1}{8}\%$. *How* much did I realize from the sale?

53. Mr. K. sold 500 shares stock ($50) at 47⅝ and with the proceeds bought Mich. Cent. at 131⅜. How many shares did he buy, and what was his surplus, brokerage on both transactions?

54. What is the duty on 600 yd. of brussels carpet, 27 in. wide, invoiced at 6*s.* 6*d.* per yd., when the specific duty is 30¢ per sq. yd. and the ad valorem 25%?

55. How much insurance at 1⅛% can be obtained for $85.50 premium?

56. The Dr. side of Mdse. account is $48000; Cr. side, $24500; Inventory, $25000. Required, the per cent. gain or loss on sales.

57. If the net earnings of a company with a capital of $200000 are $19500, what rate of dividend may be declared?

58. An importer owed a bill in London of £520 8*s.* 6*d.* What amount would settle the bill, exchange quoted at $4.86⅝?

59. A, B, and C were partners. A invested $3500, B $4000, and C $4500. At the end of the year their resources were $18970 and their liabilities $1720. What was each partner's share of the gain?

60. Having bought goods to the amount of $1680 cash, I gave my 60-day note in settlement. What was the face of the note, discount at 6%?

61. What is the present worth of a debt for $1005, of which $475 is to be paid in 10 months, and the remainder in 1 year and 3 months, the rate of interest being 6%?

62. How much would it cost to paint the dome of a hall, in the form of a hemisphere 50 ft. in diameter, at 35¢ a sq. yd.?

63. What are the bank discount and proceeds of a note of $1500, dated Apr. 1, 1901, payable in 90 days, and discounted May 10, 1901, at 6%?

64. A man offered a bill of goods for $1800 on 6 mos. credit, or for the present worth of that sum for cash. I accepted the latter offer, and obtained the money at a bank for the same time at 6%. Did I gain or lose, and how much?

65. How many yards of carpet ¾ yd. wide will be required to cover the floor of a room 20 ft. long and 18 ft. wide?

66. If the average depth of a stream is 18 in., average width 10 *ft. and its* current velocity 1½ ft. per second, how many barrels *per hour does* the stream yield?

67. What is the difference between the true discount and the bank discount on $1000 for 90 da. (no grace) at 6%?

68. Two concentric circles have diameters of 30 ft. and 60 ft., respectively. What is the area of the space enclosed between the circumferences?

69. A can do a piece of work in 20 days, B in 25 days, and C in 30 days. In what time can they together do the work?

70. If it cost $300 to fence a square field at 75¢ per rod, what will it cost to plow the field at $1.50 per acre?

71. A merchant sells goods at retail 30% above cost, and at wholesale 10% less than the retail price. What is his gain per cent. at wholesale?

72. A coal merchant sold 15% of three-sevenths of his interest for $3000, and for the balance of his entire interest he received a note payable in 90 days. If he sold the balance of his interest on same rates as first sale, what was the face of the note?

73. Mr. Brown sold lumber on commission at 5%, and invested the net proceeds in dry goods at 2% commission. His whole commission was $70. What was the value of the lumber, and the dry goods?

74. How many bricks, each 8 in. × 4 in. × 2 in. will be required for 720 cu. ft. of masonry, allowing 12½% for the mortar?

75. What is the face of a bill which can be bought on London for $840, when exchange is at $4.86¾?

76. At $20 per M, what will be the cost of lumber for a circular tank 10 ft. in diameter and 8 ft. high, the bottom to be of 3 inch and the sides of 2 inch planks?

77. Add seven thousand four and three hundred twenty-seven millionths to sixty thousand fifty and seventeen ten-millionths.

78. A wagon-bed is 4 ft. wide and 1¾ ft. deep. How long must it be approximately, to hold 50 bu. of unsacked wheat?

79. A piece of timber is 25 ft. long, 3 ft. wide, and 2½ ft. thick. How many feet of boards can be sawed from it, allowing 20% for waste in sawing?

80. How many barrels of water fell on a roof 40 ft. × 36 ft. during a two-inch rain?

81. Hardluck & Co. failed with the following resources and liabilities:

Resources.		Liabilities.	
Cash, ...	986.50	Mortgages Pay., . .	6940
Mdse., ...	12650.70	Bills Pay., ...	20500
Bills Rec., . .	7500.	Accts. Pay., ...	37478
Accts. Rec., . .	8972.65		

After reserving $1000 to pay for insolvency proceedings, how much can be paid on the dollar to the general creditors?

82. Mr. G. sold 16⅔% of his interest in a factory for $2500. What is the value of his remaining interest, on the same basis?

83. A merchant invested 12½% of his capital in a lot, 20% of it in erecting suitable buildings thereon, 33⅓% of it in a stock of hardware, and had $2640 remaining. What was his whole capital?

84. $1500. READING, PA., June 20, 1898.

On demand I promise to pay to the order of J. H. Leidig, Fifteen Hundred Dollars, for value received, without defalcation, with interest at 6%.

GUY OLIVER.

The above note had the following indorsements:

Sept. 20, 1898, . . .	$150	Jan. 1, 1900, . . .	$300
Apr. 30, 1899, . . .	200	Oct. 5, 1900, . . .	175

Find the amount due Dec. 20, 1901, by the U. S. rule for partial payments.

85. I paid a premium of $122.50 for insurance on my store building valued at $6000 and the store goods valued at $15000, the building and goods taken at ⅘ of their value. At the same rate what should I pay for insuring a block of buildings valued at $25000 for ¾ of its value?

86. A commission merchant received a consignment of 500 bbl. of flour, and advanced $200 cash, paid $50 freight, and $15 drayage. He sold 200 bbl. at $4.75; 120 bbl. at $4.50; and the remainder at $4.25 per barrel. His commission for selling was 2½%. How much was still due the consignor?

87. On June 24, 1901, the following invoices were bought: $400 on 3 months' credit; $500 on 6 months' credit; and $700 on 8 *months' credit.* What time should be allowed for the payment of *the whole indebtedness* at one time?

88. What principal will yield $80 interest in 1 yr. 3 mo. 18 da. at 6%?

89. In what time will $425 produce $63.75 interest at 5%?

90. Add the following fractions, and reduce the result to a decimal, correct to five places: $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{9}{16}$, $\frac{16}{24}$.

91. Reduce each fraction in the preceding example to a decimal, correct to five places, and find their sum.

92. Find the accurate interest on $1345.75 from Jan. 1, 1904, to May 10, 1904, at 7%.

93. A wagon-bed 10 ft. 6 in. long, 3 ft. 9 in. wide, and 1 ft. 9 in. deep is filled with wheat. What is the weight of the load?

94. A note of $1000 was given May 12, 1900, bearing interest at 6%, on which were the following indorsements: Sept. 5, 1900, $100; Nov. 12, 1900, $250; Dec. 20, 1900, $300.

Find the balance due Jan. 1, 1901, by the Merchants' rule.

95. Find the equated date of settlement of the following account:

W. C. BARTOL

1901				1901			
Apr.	4	Mdse.,	360	June	20	Cash,	400
June	20	"	520	July	15	"	150
Aug.	7	"	275	Nov.	1	"	200

96. A man bought 80 shares of bank stock at 89⅛, and sold the same at 104½, brokerage ⅛% in each case. What was his gain?

97. A taxpayer owned real estate assessed at $7500, and personal property assessed at $3500, in a township in which the rate of taxation was $1.75 on the $100. What was the amount of his taxes?

98. What is the rate of taxation when the valuation of the property is assessed at $17500, and the tax is $262.50?

99. If oranges are bought at 30¢ per dozen and sold at 4¢ a piece, what is the gain per cent.?

100. The cost of a draft on Berlin was $313.50, when exchange was $.95 for 4 marks. What was the face of the draft?

101. A commission merchant wishes to remit the net proceeds of a shipment by a bank draft. Total sales, $7000; commission, 3½%; exchange 1½% premium. How large a draft will he send?

102. A merchant in Boston owes £275 10*s.* 8*d.* in Manchester, Eng., and wishes to remit the amount by draft at 60 days. Exchange is \$4.80¼, and brokerage ⅛%. What will the draft cost?

103. I bought an exchange on Antwerp, at short sight, for 8460 francs, at 5.21¼. What was the cost?

104. A window is 45 ft. from the ground. What must be the length of a ladder to reach the window when the foot of the ladder is placed 12 ft. from the house?

105. A tree was broken 36 ft. from the ground, and fell so that the top struck 40 ft. from the foot, the other end resting upon the stump. What had been the height of the tree?

106. A depositor is credited with the following deposits at a savings bank: Jan. 1, 1900, \$500; June 20, 1900, \$200; Aug. 10, 1900, \$50; Mar. 6, 1901, \$300. No withdrawals. What was his credit on Jan. 1, 1902, if the interest rate was 4%, and interest credited quarterly?

107. What is the compound interest of \$500 for 3 yr. 5 mo. 20 da. at 6%, interest compounded annually?

108. If 5 men, working 8 hours a day, can plow 50 acres in 8 days, in how many days can 9 men, working 10 hours a day plow 60 acres?

109. Find the cash balance of the following account on Jan. 1, 1904, money being worth 6%.

T. E. PHILLIPS.

1903				1903			
Mar.	4	Mdse., 1 mo.,	600	May	30	Cash,	500
Apr.	20	" 30 da.,	350	July	12	"	200
June	16	" 2 mo.	400	Oct.	1	"	300
Sept.	5	" 60 da.,	150				

110. What per cent. of a yard is 90% of a meter?

111. A rectangular lot is 100 ft. long and 50 ft. wide. What is the distance around it in meters?

112. The distance between two places is 8 kilometers. What is the distance in miles?

113. A farmer has a field containing 6½ acres. How many hektares does it contain?

114. A rank of wood is 12 ft. long, 4 ft. wide, and 6 ft. high. How much is the rank worth at 75¢ per stere?

115. A pail containing $1\frac{1}{2}$ gallons of water has how many liters of water in it?

116. The nickel five-cent piece weighs 5 grams. How many Troy grains does it weigh?

117. A bar of iron weighed 60 pounds. What was its weight in kilos?

118. The distance from Poughkeepsie to New York is 75 miles. What is the distance in kilometers?

119. A real estate agent sold five Sections of land in the state of Kansas. What was the area of this land in square kilometers?

120. In digging a cellar Mr. Strong removed 200 cubic yards, or loads, of earth. How many cubic meters of earth did he remove?

121. What is the par value of 4562 guilders, or florins, of the Netherlands?

122. What is the par value of a draft on Paris for 123645.40 francs?

123. How much must be paid in U. S. money for 2500 milreis of Brazil?

124. What is the value in U. S. money of 1200 gold pesos of Cuba?

125. What is the value in U. S. money of 570 crowns of Denmark?

126. A merchant exchanged $5150 for rubles of Russia. How many rubles did he obtain?

127. How many colons of Costa Rica should be received for $465?

128. What is the face of a bill of exchange on Liverpool, that costs $1194.99, when exchange is 4.88?

129. What will be the face of a 3 days' bill of exchange on London, that can be bought for $5964.13, exchange $4.86\frac{1}{2}$?

130. What will be the face of a bill of exchange on Berlin, that costs $1500, when exchange is at $.96?

131. I sent an agent $1000 to invest in apples at 60¢ a barrel, allowing him a commission of 4%. He paid 15¢ apiece for the barrels and 20¢ a barrel for transportation. How many barrels of apples did he buy?

132. How many bricks of average size will be required to build the walls of a house 50 ft. long, 35 ft. wide, and 24 ft. high, the walls to be 13 in. thick, allowing 450 sq. ft. for doors and windows?

133. S. H. Crown bought of J. D. Soley merchandise amounting to $950.80, terms 30 days, and gave in payment his interest bearing note at 90 days. What was the face of the note?

134. The capital stock of a certain railroad is $2875000, and its debt is $75000. Its gross earnings for the year 1903 were $500000. and its expenses $265000. If the company paid expenses and interest on its debt, at 6%, and reserved $500 as surplus, what dividend would a stockholder receive who owned 25 shares, par value $100?

135. A manufacturing company fails in business. Its whole indebtedness amounts to $150000. Its effects consist of buildings and machinery, worth $20000; stock of goods, worth $30000; notes on hand, valued at $15000; and cash, $5000. The assignee's charges will amount to 5% on the amount distributed to the creditors. How much will the company pay on a dollar, and what will be A. E. Janes' dividend on a claim of $2700?

136. At 15¢ per copy for printing, 12¢ per lb. for paper weighing 70 lb. per ream, and 20¢ per copy for binding, what would be the cost of an edition of 20000 copies of a 12 mo. book of 408 pages?

137. What is the value of a rectangular block of fine gold 10 in. long, 4 in. wide, and 3 in. thick, the specific gravity of gold being 19.258, when 23.22 grains of gold are worth $1?

138. If a man purchases 150 U. S. 4's at 120⅜, 70 railroad 5's at 103⅛, and 50 railroad 6's at 115½, through a broker who charges ⅛%, what rate of income does he receive on the total investment?

139. How many square yards are there in the surface of the sidewalk around a city block 360 ft. by 420 ft. on the building line, if the sidewalk is 15 ft. wide?

140. What per cent. do I make by purchasing flour at $4.50 per bbl. cash, and selling the same for $5.25 per bbl. on 3 mos. credit, money being worth 6%?

141. I bought a lot of coffee at 12¢ per pound. Allowing that the coffee will fall short 5% in roasting and weighing it out, and that 10% of the sales will be bad debts, for how much per pound *must I sell it,* to make a clear gain of 14% on the cost?

142. I have a balance of $4210.12, belonging to a Liverpool ıerchant, which I am instructed to invest in English exchange, ıd remit. The rate of exchange is 4.82. I charge ½ of 1% com- ıission for investing. What is my commission, and what is the ıce of the bill?

143. A Liverpool cotton factor sold 200 bales of cotton weigh- g 90000 lbs., at 8¼*d.* per lb. He charged 2½% commission for ılling. What was his commission, and what were the net pro- ıeds? How large a bill in dollars did we receive if exchange in ondon was 49*d.* to the dollar?

144. A and B are partners. A invested $5000, and B $3000, aking the capital of the firm $8000. They sell ⅛ of the business ı C for $8000. How shall A and B divide the $8000 received ·om C, so that each may retain a ⅛ interest in the business with- ut increasing or diminishing the original capital of the firm?

145. A and B formed a partnership on Jan. 1, 1900, for 1 year; ıe gains and losses to be shared equally. Interest was allowed t 6% on all sums invested and withdrawn. On Jan. 1, A invested 5000; on Apr. 2, $1500. He withdrew $600 on June 1, and $400 n Sept. 19. On Jan. 1, B invested $7500; on May 1, $1200. He vithdrew $1000 on Mar. 1, and $500 on July 2. They gained luring the year $4713. What was each partner's capital on Jan. ., 1901?

ANSWERS.

TO WRITE NUMBERS IN FIGURES.

Page 12, Art. 20.

2. 120.
3. 145.
4. 250.
5. 428.
6. 777.
7. 808.
8. 1250.
9. 3125.
10. 6542.
11. 10716.
12. 13201.
13. 17360.
14. 30400; 42600.
15. 64520; 70394.
16. 420000; 835100.
17. 301600; 350000.
18. 758208; 79005.
19. 900009; 75075.
20. 35023000; 27142240; 16015000.
21. 40000000; 38500204; 38701280.
22. 312040400; 516000000.
23. 28600000; 325030440; 355000275.
24. 213317412625.
25. 600005000095.

Page 13.

26. 100100000000.
27. 50050050050.
28. 70340000707.
29. 3000204605025.
30. 700007007700070.

TO WRITE UNITED STATES MONEY.

Page 13, Art. 25.

1. $8.15; $13.07; $20.05.

Page 14.

2. $45.40; $37.17; $50.10.
3. $200.30; $900; $80.801.
4. $365.435; $77.03.
5. $650.65; $39.205.
6. $1200.317; $2005.
7. $5310.87; $40.082.
8. $15400.06; $28.039.
9. $42705.35; $60000.
10. $90090.095; $2000.022.

ROMAN NOTATION.

Page 15, Art. 28.

XVII.
DIII.
CCCXIV.
DCCCXXXIX.
XLIV.

LXXXV.
MLX.
MCLXXI.
MCCCLXV.
CXXVII.

DCXVIII.
CCCCX.
DXV.
LIX.
DCCLIV.
DCXXI.

MDCCCLIX.
MCDLXIII.
MDCCLXXVI.
CCXV.
MM.
MMMCI.

DCCLVI.
LXVII.
DCCCXXXI.
DCCXXXV.
DCCCCLXXVIII.

$\overline{\text{V}}$IDCCXCII.
CCCXLII.
MMMMDCLXXV.
$\overline{\text{V}}$CCCXLIV.
I$\overline{\text{X}}$DCCCCXCIX.

ADDITION.

Page 21, Art. 41.

3. 457.
4. 665.
5. 898.
6. 885.
7. $9.77.
8. 10.75.
9. 875.
10. 1286.
11. 2123.
12. $20.80.
13. $15.39.
14. 22.38.
15. 11391.
16. 14244.
17. 12887.
18. $132.15.
19. $226.14.
20. $215.23.
21. 43876.
22. 45011.
23. 44021.
24. 56715.
25. $554.31.
26. $650.03.
27. 23833.
28. 16763.
29. 29152.
30. 32033.
31. $616.57.
32. $114.74.
33. 26483.
34. 17511.
35. 18160.

Page 22.

36. 23000.
37. 13215.
38. 22967.
39. 33216.
40. 14940.
41. 5207016.
42. 48835805.
43. 555969914.
44. 5727189821.
45. 1087124.
46. 12571810.
47. 12 929073.
48. 1193597553.
49. 297221.

Page 23.

50. 21921068.

Art. 42.

2. $973.
3. 1380.
4. $850.
5. $5200.
6. $2750.
7. $3635.
8. 2321644.
9. 48444949.
10. $25100.
11. 183690.

Page 24.

12. $4290.
13. $11250.
14. $11640.
15. $226.40.
16. $9353.65.
17. $7007.90.

Page 25, Art. 44.

2. 266463.
3. 303788.
4. 237112.
5. 280811.
6. 268747.
7. 320194.

Page 26.

8. 506215.
9. 602386.
10. 175574.
11. 116049.
12. 518768.
13. 545397.

Page 35, Art. 58.

1. 120.
2. 143.
3. 104.
4. 109.
5. 92.
6. 127.
7. 113.
8. 165.
9. 143.
10. 188.
11. 222.
12. 282.
13. 236.
14. 253.
15. 196.
16. 218.

Page 36, Art. 60.

1. $11.58.
2. $10.79.
3. $26.66.
4. $17.20.
5. $78.50.
6. $76.80.
7. $78.76.
8. $110.79.
9. $23.80.
10. $20.95.
11. $27.15.
12. $131.05.
13. $109.18.
14. $30.53.
15. $39.65.
16. $113.20.

Page 37, Art. 61.

2. 52649.
3. 18649.
4. 91295.
5. 12151.
6. 16487.
7. 47423.
8. 72211.
9. 999906.
10. 167110.
11. 893024.
12. 24685.
13. 128973.
14. 56450.
15. 22416.
16. $32587.46.

SUBTRACTION.

Page 41, Art. 69.

3. 24.
4. 42.
5. 42.
6. 34.
7. 44.
8. 112.
9. 213.
10. 231.
11. 212.
12. 243.
13. 321.
14. 244.
15. 253.
16. 322.
17. 362.
18. 462.
19. 153.
20. 420.
21. 205.
22. 233.
23. $1.85.
24. $1.90.
25. $2.33.
26. $4.09.
27. $4.93.
28. 1322.
29. 2383.
30. 2886.
31. 2978.
32. 3805.
33. $11.07.
34. $23.65.
35. $21.15.
36. $19.32.
37. $19.09.
38. $13.16.
39. $18.05.
40. $28.05.
41. $23.66.
42. $22 17.
43. 15889.
44. 30735.
45. 14825.
46. 54571.
47. 13683.

Page 42.

49. 2153.
50. 2328.
51. 1757.
52. 2122.
53. 1655.
54. 1592.
55. 1644.
56. 34797.
57. 19174.
58. 20242.
59. $2765.36.
60. $3599.80.
61. $2843.53.
62. $2768 55.
63. $1816.04.
64. 66564566.
65. 76765762.
66. 51972615.
67. 21826200.
68. 83606709.

Art. 70.

2. 475A.
3. $918.
4. $4258.
5. 67 yr.
6. 46 yr.

Page 43.

7. $46606.
8 1738627
9. $6000.
10. $13605.
11. $1123.
12. $3689.
13. $12.43.
14. $17475.
15. $960.15.
16. $939.60.
17. $1827.15.
18. $7260.68.

MULTIPLICATION.

Page 46, Art. 79.

2. 1038.
3. 2288.
4. 1366.
5. 4710.
6. 2190.
7. 3241.

Page 47.

8. $21.75.
9. $13.44.
10. $43.38.
11. $41.58.
12. 53.76.
13. 80.19.
14. 940.
15. 1269.
16. 3505.
17. 3852.
18. 3096.
19. 2940.
20. 5643.
21. 8074.
22. 6996.
23. 21605.
24. 36701.
25. 38712.
26. 32984.
27. 45872.
28. 51555.
29. 75207.
30. 74916.
31. 95414.
32. 153792.
33. 297144.
34. 206855.
35. 392945.
36. 424854.
37. 664328.
38. 515578.
39. 749936.
40. 759330.

Art. 80.

2. $47.50.

.
ɔu. in.
)0.

ft.
30.
50.
ft.
.

Art. 83.

103400;
000.
; 214000;
000.
; 190500;
000.
; 407400;
000.
; 298400;
000.
; 522400;
000.
; 297000;
000.
; 499800;
000.
; 656800;
000.
; 514800;
000.

26. 171646.
27. 385154.
28. 594439.
29. 274822.
30. 738528.
31. 123300.
32. 181272.
33. 148559.
34. 478514.
35. 375270.
36. 631720.
37. 589899.
38. 394542.
39. 725754.
40. 957726.
41. 284745.
42. 888528.
43. 2285400.
44. 1998384.
45. 2455008.
46. 1597140.
47. 2715660.
48. 3846570.
49. 5851242.
50. 7950043.
51. 72984934.
52. 139878400.
53. 363709314.
54. 147859285.
55. 204837084.
56. 378756000.
57. 1309050120.
58. 1633007250.
59. 2015293800.
60. 4896638120.
61. $1011109.40.
62. $1836195.56.
63. $3353933.40.
64. $4822096.44.
65. $4837267.84.
66. $6903110.
67. $5202999.54.
68. $4905544.88.
69. $3286687.13.
70. $5486353.30.

Page 51, Art. 84.

1. 14000.
2. 15300.
3. $146.40.
4. 725.
5. $1820.
6. $6956.
7. 266240.
8. $701250.
9. 1663200.
10. 2433600.
11. 396838.
12. 163500.
13. $787.
14. 12312.
15. 44640.
16. $19890.
17. $729.
18. 13052400.

DIVISION.

Art. 99.

17. 2788.
18. 857.
19. 628.
20. 1185.
21. 1477.
22. 889.
23. 925.
24. 925.
25. 762.
26. 547.
27. 672.
28. 1393.
29. 670.
30. 1034.
31. 1046.
32. 1192.
33. 725.
34. 1476.
35. 1596.
36. $921\frac{1}{3}$.
37. $1078\frac{5}{8}$.
38. $900\frac{1}{9}$.
39. 926.
40. 1009.

41. $1224\frac{1}{8}$.
42. $854\frac{1}{7}$.
43. 1022.
44. $963691\frac{1}{7}$.
45. $427345\frac{5}{8}$.
46. 102052.
47. 81043.
48. $730361\frac{1}{8}$.
49. $72.08.
50. $57.78.
51. $102.14.
52. $187.53.
53. $112.34.
54. $108.85.
55. $133.16.
56. $81.55.
57. $142.47.
58. $91.30.
59. $155.72.
60. $83.75.
61. $72.10.
62. $82.06.

Art. 100.

1. 291.
2. 244.

Page 57.

3. 235.
4. 144.
5. 200.
6. $834.
7. 121.
8. $914.
9. 38.
10. $2435.
11. $24.50.

Page 58, Art. 103.

20. $217\frac{4}{5}$.
21. $62\frac{1}{40}$.
22. $127\frac{1}{4}$.
23. $116\frac{27}{70}$.
24. $114\frac{23}{25}$.
25. $77\frac{7}{40}$.
26. $83\frac{23}{90}$.
27. $179\frac{67}{200}$.
28. $104\frac{7}{80}$.
29. $107\frac{143}{500}$.
30. $120\frac{48}{60}$.
31. $694\frac{483}{700}$.
32. $109\frac{77}{400}$.
33. $104\frac{78}{900}$.
34. $142\frac{781}{8000}$.
35. $136\frac{547}{800}$.
36. $127\frac{581}{1500}$.
37. $105\frac{1681}{4000}$.
38. $623\frac{627}{7000}$.
39. $91\frac{379}{8000}$.
40. $63\frac{28}{1800}$.

Page 60, Art. 104.

42. 12.
43. 14.
44. 34.
45. 24.
46. 31.
47. 36.
48. 31.
49. 28.
50. 23.
51. 27.
52. 225.
53. 184.
54. 213.
55. 218.
56. 312.
57. $136\frac{20}{21}$.
58. $69\frac{18}{21}$.
59. $62\frac{28}{51}$.
60. $121\frac{1}{2}$.
61. $144\frac{38}{51}$.
62. $133\frac{22}{51}$.
63. $96\frac{55}{71}$.
64. $67\frac{28}{77}$.
65. $80\frac{72}{91}$.
66. $214\frac{25}{39}$.
67. $198\frac{13}{21}$.
68. $148\frac{36}{68}$.
69. $154\frac{5}{16}$.
70. $537\frac{59}{72}$.
71. $832\frac{51}{62}$.
72. $870\frac{33}{41}$.
73. $420\frac{17}{46}$.
74. $1429\frac{36}{43}$.
75. $1392\frac{34}{53}$.
76. $1152\frac{56}{73}$.
77. $800\frac{20}{21}$.
78. $865\frac{1}{18}$.
79. $858\frac{19}{21}$.
80. $1050\frac{16}{47}$.
81. $1399\frac{9}{23}$.
82. $2150\frac{35}{38}$.
83. $2763\frac{7}{15}$.
84. $2613\frac{46}{55}$.
85. $3097\frac{37}{65}$.
86. $4189\frac{41}{75}$.
87. $4721\frac{39}{85}$.
88. $3028\frac{32}{95}$.
89. $11428\frac{34}{47}$.
90. $11586\frac{53}{56}$.
91. $14556\frac{13}{49}$.
92. $7326\frac{53}{87}$.
93. $8026\frac{12}{89}$.
94. $8684\frac{4}{97}$.
95. $7439\frac{8}{38}$.
96. $131139\frac{33}{101}$.
97. $115086\frac{127}{201}$.
98. $114967\frac{201}{301}$.
99. $90138\frac{33}{407}$.
100. $237553\frac{95}{105}$.
101. $139081\frac{23}{511}$.
102. $121846\frac{59}{170}$.
103. $116832\frac{143}{304}$.
104. $59393\frac{382}{821}$.
105. $44164\frac{34}{785}$.
106. $66182\frac{199}{852}$.
107. $267814\frac{77}{219}$.
108. $425569\frac{601}{987}$.
109. $711554\frac{127}{899}$.
110. $877420\frac{70}{111}$.
111. $342813\frac{113}{305}$.
112. $421239\frac{89}{374}$.
113. $164783\frac{625}{1067}$.
114. $198677\frac{1763}{3240}$.
115. $141708\frac{958}{1721}$.
116. $198019\frac{792}{1069}$.
117. $93643\frac{2062}{6087}$.

Art. 105.

2. 32.
3. 123.
4. 172.

Page 61.

5. 40.
6. 24.
7. 75.

8. 27.
9. 345.
10. 42.
11. 79.
12. 32.
13. 476.
14. 7500.
15. 3007.
16. 1601.
17. 6036.
18. 36; 215 sq. mi.
19. 139.

EXPRESSIONS CONTAINING PARENTHETICAL FORMS.

Page 62, Art. 107.

3. 54.
4. 60.
5. 35.
6. 50.
7. 12.
8. 16.
9. 50.
10. 4.
11. 47.
12. 0.
13. 100.
14. 16.
15. 546.

ARITHMETICAL ANALYSIS.

Page 63, Art. 110.

2. 18¢.
3. 48¢.
4. 40¢.
5. $160.
6. $66.
8. $15.
9. 12¢.
10. $35.
11. 120.
12. $6.

Page 64.

14. 44.
15. 24.
16. 60.
17. $16.
18. 450.

REVIEW.

Page 64, Art. 111.

1. $4600.
2. $1000.
3. $81.
4. $6.92.
5. $362.
6. 1000.
7. $9400.
8. $126.55 gain.
9. 10 yr.
10. 50.

Page 65.

11. 46.
12. 335.
13. 4000.
14. 15.
15. 294.
16. 26.
17. $444.
18. 135.
19. $423.36.
20. $203.40.

FACTORING.

Page 68, Art. 126.

2. 2, 2, 3, 7.
3. 2, 2, 3, 3, 3.
4. 2, 2, 3, 3, 7.
5. 5, 5, 5.
6. 2, 2, 2, 2, 3, 5.
7. 2, 2, 2, 3, 13.
8. 2, 2, 107.
9. 2, 2, 2, 2, 2, 2, 2, 2, 2.
10. 2, 3, 3, 5, 7.
11. 2, 2, 7, 43.
12. 2, 2, 5, 3, 3, 13.
13. 2, 1637.
14. 2, 2, 2, 2, 271.
15. 2, 2, 2, 631.
16. 2, 2, 3, 3, 5, 5, 7.
17. 5, 1487.
18. 2, 2, 1531.
19. 3, 3, 7, 7, 13.
20. 2, 3, 1081.
21. 2, 2, 2, 2, 2, 151.
22. 7, 1051.
23. 3, 3, 607.
24. 3, 3, 5, 181.
25. 2, 2, 5, 5, 5, 19.

CANCELLATION.

Page 69, Art. 129.

2. 60.
3. 84.
4. 168.
5. 432.
6. 196.
7. 100.
8. 60.
9. 168.
10. 48.
11. $43\frac{1}{2}$.
12. $1\frac{1}{4}$.

13. 222.
14. 400.
15. $34\frac{2}{7}$.
16. 1001.
17. 450
18. $1\frac{2}{3}$.
19. $\frac{24}{78}$.
20. $\frac{98}{185}$.

Page 70, Art. 130.

2. 6.
3. 45.
4. $54.
5. $66\frac{2}{3}$.
6. 21.
7. 9¢.
8. 18.
9. 20¢.
10. 28.

GREATEST COMMON DIVISOR.

Page 71, Art. 134.

2. 18.
3. 21.
4. 24.
5. 4.
6. 4.
7. 10.

Page 72, Art. 135.

2. 96.
3. 19.
4. 105.
5. 312.
6. 21.
7. 1.
8. $\frac{5}{7}$.
9. $\frac{4}{5}$.
10. $\frac{8}{11}$.
11. $\frac{8}{18}$.
12. $\frac{5}{3}$.
13. $\frac{18}{21}$.
14. $\frac{18}{32}$.
15. $\frac{2479}{3946}$.
16. 40 A. 1st tract 6; 2d, 8; 3d, 11.

LEAST COMMON MULTIPLE.

Page 73, Art. 140.

2. 360.
3. 1680.
4. 2376.
5. 900.
6. 224.
7. 1260.
8. 9288.
9. 36480.
10. 84.
11. 440.
12. 180.
13. 405.
14. 600.
15. 2275.
16. 720.

Page 74, Art. 141.

2. 8379.
3. 12852.
4. 43977.
5. 2394.
6. 8925.
7. 4140.
8. 30132.
9. 41943.
10. 187245.
11. 30 days.
A 45 times.
B 40 times,
C 36 times.
12. 260975.

REDUCTION OF FRACTIONS.

Page 77, Art. 158.

2. $\frac{88}{5}$.
3. $\frac{89}{4}$.
4. $\frac{77}{6}$.
5. $\frac{55}{4}$.
6. $\frac{125}{8}$.
7. $\frac{53}{3}$.
8. $\frac{104}{5}$.
9. $\frac{199}{7}$.
10. $\frac{283}{9}$.
11. $\frac{131}{3}$.
12. $\frac{311}{6}$.
13. $\frac{445}{7}$.
14. $\frac{899}{12}$.
15. $\frac{1259}{15}$.
16. $\frac{2012}{21}$.
17. $\frac{1962}{25}$.
18. $\frac{5298}{41}$.
19. $\frac{5089}{75}$.
20. $\frac{4919}{82}$.
21. $\frac{7995}{91}$.
22. $\frac{9899}{99}$.
23. $\frac{987}{8}$.
24. $\frac{2361}{11}$.
25. $\frac{8639}{25}$.
26. $\frac{15521}{81}$.
27. $\frac{581185}{88}$.
28. $\frac{99255}{88}$.
29. $\frac{96091}{100}$.

Page 78.

30. $\frac{60}{4}$; $\frac{120}{6}$; $\frac{860}{10}$; $\frac{800}{12}$; $\frac{510}{15}$; $\frac{900}{20}$; $\frac{1250}{25}$; $\frac{1200}{20}$.

Art. 159.

2. $4\frac{1}{6}$.
3. $6\frac{4}{5}$.
4. $6\frac{6}{7}$.
5. $7\frac{1}{8}$.

19. $\frac{156}{228}$.

Page 79, Art. 168.

2. $\frac{3}{4}, \frac{2}{3}$.
3. $\frac{2}{3}, \frac{4}{5}$.
4. $\frac{4}{5}, \frac{9}{11}$.
5. $\frac{4}{5}, \frac{5}{7}$.
6. $\frac{2}{3}, \frac{2}{3}$.
7. $\frac{3}{4}, \frac{3}{4}$.
8. $\frac{3}{4}, \frac{10}{11}$.
9. $\frac{28}{31}, \frac{31}{34}$.
10. $\frac{34}{37}, \frac{37}{39}$.
11. $\frac{8}{9}, \frac{43}{45}$.
12. $\frac{92}{99}, \frac{40}{43}$.
13. $\frac{47}{49}, \frac{7}{8}$.
14. $\frac{8}{9}, \frac{4}{5}$.
15. $\frac{2}{8}, \frac{3}{5}$.
16. $\frac{13}{14}, \frac{34}{38}$.
17. $\frac{7}{11}, \frac{5}{6}$.
18. $\frac{91}{122}, \frac{3}{4}$.
19. $\frac{104}{133}, \frac{47}{51}$.
20. $\frac{3}{4}, \frac{4}{7}$.
21. $\frac{5}{6}, \frac{6}{7}$.
22. $\frac{6}{7}, \frac{7}{8}$.
23. $\frac{5}{7}, \frac{7}{9}$.
24. $\frac{4}{5}, \frac{5}{7}$.
25. $\frac{7}{8}, \frac{8}{9}$.

Page 80, Art. 168.

2. $\frac{6}{12}, \frac{8}{12}, \frac{9}{12}$.
3. $\frac{8}{12}, \frac{10}{12}, \frac{5}{12}$.
4. $\frac{8}{24}, \frac{9}{24}, \frac{4}{24}$.
5. $\frac{12}{16}, \frac{8}{16}, \frac{5}{16}$.
6. $\frac{18}{30}, \frac{21}{30}, \frac{20}{30}$.
7. $\frac{18}{24}, \frac{12}{24}, \frac{3}{24}$.
8. $\frac{12}{15}, \frac{10}{15}, \frac{4}{15}$.
9. $\frac{8}{36}, \frac{27}{36}, \frac{2}{36}$.
10. $\frac{30}{40}, \frac{16}{40}, \frac{35}{40}$.
11. $\frac{35}{42}, \frac{18}{42}, \frac{3}{42}$.
12. $\frac{6}{16}, \frac{5}{16}, \frac{4}{16}$.
13. $\frac{9}{30}, \frac{12}{30}, \frac{25}{30}$.
14. $\frac{24}{56}, \frac{35}{56}, \frac{20}{56}$.
15. $\frac{8}{36}, \frac{27}{36}, \frac{5}{36}$.
16. $\frac{8}{32}, \frac{12}{32}, \frac{1}{32}$.
17. $\frac{9}{30}, \frac{24}{30}, \frac{14}{30}$.
18. $\frac{6}{40}, \frac{32}{40}, \frac{35}{40}$.
19. $\frac{11}{44}, \frac{12}{44}, \frac{18}{44}$.
20. $\frac{35}{105}, \frac{21}{105}, \frac{15}{105}$.
21. $\frac{20}{90}, \frac{63}{90}, \frac{60}{90}$.
22. $\frac{14}{30}, \frac{25}{30}, \frac{18}{30}$.
23. $\frac{6}{28}, \frac{16}{28}, \frac{5}{28}$.
24. $\frac{21}{24}, \frac{22}{24}, \frac{13}{24}$.
25. $\frac{75}{100}, \frac{56}{100}, \frac{65}{100}$.
26. $\frac{16}{48}, \frac{18}{48}, \frac{20}{48}, \frac{21}{48}$.
27. $\frac{24}{36}, \frac{30}{36}, \frac{8}{36}, \frac{21}{36}$.
28. $\frac{6}{30}, \frac{10}{30}, \frac{21}{30}, \frac{8}{30}$.
29. $\frac{45}{60}, \frac{54}{60}, \frac{50}{60}, \frac{35}{60}$.
30. $\frac{36}{84}, \frac{56}{84}, \frac{66}{84}, \frac{15}{84}$.
31. $\frac{100}{120}, \frac{72}{120}, \frac{78}{120}, \frac{85}{120}$.
32. $\frac{30}{140}, \frac{68}{140}, \frac{52}{140}, \frac{98}{140}$.
33. $\frac{96}{120}, \frac{100}{120}, \frac{110}{120}, \frac{105}{120}$.
34. $\frac{112}{144}, \frac{81}{144}, \frac{88}{144}, \frac{40}{144}$.
35. $\frac{315}{525}, \frac{105}{525}, \frac{154}{525}, \frac{350}{525}$.
36. $\frac{270}{860}, \frac{315}{860}, \frac{330}{860}, \frac{312}{860}$.
37. $\frac{90}{126}, \frac{56}{126}, \frac{42}{126}, \frac{45}{126}$.
38. $\frac{20}{55}, \frac{5}{55}, \frac{10}{55}, \frac{33}{55}$.
39. $\frac{21}{60}, \frac{36}{60}, \frac{12}{60}, \frac{40}{60}$.
40. $\frac{45}{90}, \frac{40}{90}, \frac{20}{90}, \frac{34}{90}$.

ADDITION OF FRACTIONS.

Art. 170.

10. $1\frac{17}{60}$.
11. $2\frac{13}{77}$.
12. $2\frac{1}{16}$.
13. $2\frac{89}{90}$.
14. $\frac{10}{13}$.
15. $3\frac{18}{25}\frac{13}{20}$.
16. $15\frac{7}{12}$.
17. $30\frac{13}{20}$.
18. $34\frac{1}{2}$.
19. $41\frac{3}{4}$.
20. $23\frac{1}{24}$.
21. $19\frac{73}{105}$.
22. $113\frac{17}{60}$.
23. $130\frac{1}{46}$.

24. $418\frac{143}{180}$.
25. $1021\frac{43}{112}$.

Page 82, Art. 171.

1. $419\frac{7}{8}$.
2. $644\frac{5}{16}$.
3. $43\frac{11}{20}$.
4. $143\frac{13}{32}$.
5. $147\frac{19}{56}$.
6. $38\frac{41}{120}$.
7. $28\frac{127}{320}$.
8. $146\frac{7}{12}$.
9. $145\frac{55}{72}$.
10. $107\frac{51}{160}$.

SUBTRACTION OF FRACTIONS.

Page 83, Art. 173.

4. $\frac{6}{17}$
5. $\frac{1}{6}$.
6. $\frac{8}{63}$.
7. $\frac{7}{40}$.
8. $\frac{87}{72}$.
9. $\frac{13}{182}$.
10. $\frac{16}{105}$.
11. $\frac{29}{84}$.
12. $\frac{41}{144}$.
13. $\frac{7}{60}$.
14. $\frac{37}{100}$.
15. $\frac{167}{504}$.
16. $\frac{9}{56}$.
17. $\frac{88}{975}$.
18. $\frac{26}{81}$.
19. $\frac{85}{616}$.
20. $\frac{8}{20}$.
21. $\frac{13}{70}$.
22. $2\frac{1}{2}$.
23. $3\frac{11}{12}$.
24. $12\frac{24}{35}$.
25. $33\frac{25}{36}$.
26. $13\frac{48}{78}$.
27. $20\frac{47}{56}$.
28. $9\frac{59}{84}$.
29. $12\frac{41}{42}$.
30. $81\frac{999}{1100}$.

Art. 174.

1. $14\frac{3}{4}$.
2. $\frac{13}{40}$.
3. $\frac{93}{160}$.
4. $21\frac{11}{80}$.

Page 84.

5. $58\frac{5}{8}$.
6. $56\frac{33}{40}$.
7. $81\frac{7}{8}$.
8. $\frac{3}{8}$.

MULTIPLICATION OF FRACTIONS.

Page 84, Art. 176.

2. $2\frac{1}{3}$
3. $2\frac{1}{2}$.
4. $2\frac{2}{11}$.
5. $1\frac{1}{3}$.
6. $6\frac{1}{2}$.
7. $1\frac{7}{17}$.
8. $8\frac{4}{7}$.
9. $19\frac{1}{2}$.
10. $26\frac{3}{5}$.
11. $50.
12. $3\frac{1}{2}$.
13. $60.
14. $39.

Page 85, Art. 178.

3. 9.
4. 15.
5. 16.
6. 30.
7. $11\frac{2}{5}$.
8. $36\frac{3}{4}$.
9. 32.
10. 70.
11. $61\frac{9}{29}$.
12. $13\frac{1}{2}$.
13. $6400.

Page 86, Art. 179.

2. $\frac{1}{2}$.
3. $\frac{7}{10}$.
4. $\frac{1}{6}$.
5. $\frac{22}{35}$.
6. $\frac{6}{17}$.
7. $\frac{4}{9}$.
8. $\frac{2}{3}$.
9. $\frac{3}{8}$.
10. $\frac{5}{12}$.
11. $\frac{2}{3}$.
12. $3\frac{1}{3}$.
13. $7\frac{1}{3}$.
14. 2.
15. 40.
16. $11\frac{1}{4}$.
17. 10.
18. $\frac{16}{27}$.
19. $\frac{1}{3}$.
20. $\frac{7}{15}$.
21. $\frac{45}{804}$.
22. $\frac{5}{11}$.
23. $3\frac{11}{15}$.
24. $\frac{7}{9}$.
25. $11\frac{1}{4}$.
27. 78.
28. $76\frac{1}{2}$.
29. $102\frac{3}{8}$.
30. $148\frac{4}{5}$.
31. 302.
32. 969.
33. 1029.
34. 1598.
35. $964\frac{3}{5}$.
36. $858\frac{2}{3}$.
37. 1388.
38. 2607.
40. 92.
41. 99.
42. 170.
43. 235.
44. 354.
45. 200.
46. 332.

5.
2.
1.
0.
$1\frac{4}{15}$.

87.

$3\frac{3}{4}$.
$8\frac{3}{8}$.
$1\frac{3}{16}$.

56. $391\frac{11}{15}$.
57. $702\frac{2}{3}$.
58. $665\frac{1}{2}$.
59. $1551\frac{1}{4}$.
60. $1729\frac{7}{20}$.
61. $2514\frac{1}{4}$.
62. $2586\frac{3}{8}$.
63. $6008\frac{19}{21}$.
64. $16972\frac{13}{14}$.

Art. 180.

1. $\$45\frac{1}{2}$.
2. $\$150$.
3. $\$131\frac{1}{4}$.
4. $\$13\frac{9}{16}$.
5. $\$8079$.
6. $\$12\frac{31}{32}$.
7. $\$5.25$.
8. $\$65\frac{7}{8}$.
9. $\$41\frac{7}{16}$.

DIVISION OF FRACTIONS.

88, Art. 182.

$\frac{7}{5}$.
$\frac{8}{5}$.
$\frac{1}{5}$.
$\frac{9}{40}$, or $22\frac{1}{2}¢$.
$\frac{4}{5}$, or $16¢$.

89, Art. 183.

$\frac{7}{8}$.

)8.
$17\frac{2}{5}$.
$56\frac{2}{9}$.
$72\frac{4}{28}$.
1.
3.

184.

1.
$\frac{5}{3}$.

$\frac{1}{8}$.
$\frac{4}{5}$.
$\frac{20}{77}$.

11. $1\frac{1}{4}$.
12. $1\frac{629}{1296}$.
13. $\frac{13}{18}$.
14. $\frac{2}{9}$.
15. $\frac{3}{22}$.
16. $\frac{9}{100}$.
17. $7\frac{1}{2}$.
18. $18\frac{2}{3}$.
19. $11\frac{1}{5}$.
20. $\frac{7}{8}$.
21. $1\frac{1}{17}$.
22. $1\frac{2}{7}$.
23. $1\frac{19}{66}$.
24. $3\frac{3}{4}$.
25. $1\frac{3}{7}$.

Page 90.

27. $3\frac{7}{10}$.
28. $4\frac{5}{24}$.
29. $4\frac{1}{8}$.
30. $5\frac{11}{20}$.
31. $6\frac{12}{35}$.
32. $6\frac{17}{42}$.
33. $5\frac{17}{24}$.
34. $6\frac{11}{36}$.
35. $11\frac{11}{24}$.
36 $9\frac{13}{40}$.
37. $16\frac{5}{7}$.
38. $10\frac{23}{24}$.
40. $7\frac{3}{7}$
41. $7\frac{11}{13}$.
42. $6\frac{6}{13}$
43. $6\frac{9}{17}$.
44. $5\frac{17}{19}$.
45. $10\frac{19}{17}$.
46. $7\frac{1}{15}$.
47. $19\frac{18}{28}$.
48. $8\frac{7}{10}$.
49. $6\frac{41}{44}$.
50. $13\frac{29}{49}$.
51. $11\frac{17}{47}$.
53. $9\frac{22}{27}$.
54. $6\frac{94}{129}$.
55. $5\frac{29}{880}$.
56. $10\frac{36}{833}$.
57. $10\frac{118}{279}$.
58. $12\frac{141}{224}$.
59. $10\frac{226}{295}$
60. $12\frac{28}{207}$.
61. $28\frac{118}{244}$.
62. $25\frac{898}{415}$.
63. $17\frac{237}{262}$.
64. $19\frac{191}{2096}$.

Page 91, Art. 185.

1. 9.
2. 84.
3. 4.
4. $60¢$.
5. $\$15\frac{3}{4}$.
6. $\$1.20$.
7. $\$75$.
8. 66.
9. $\$27\frac{3}{4}$
10. $\$90\frac{3}{4}$.
11. $\$8.33\frac{1}{3}$
12. $\$56\frac{1}{4}$.
13. 15.
14. $\$6250$.
15. $\$320$.
16. $27\frac{1}{2}$.

COMPLEX FRACTIONS.

Page 92, Art. 186.

3. $\frac{1}{3}$.
4. $\frac{23}{48}$.
5. $\frac{2}{4}$.
6. $\frac{14}{15}$.
7. $1\frac{1}{2}$.
8. $\frac{1}{3}$.
9. $1\frac{5}{6}$.
10. $\frac{5}{74}$.
11. $\frac{c}{24}$.
12. $\frac{8}{48}$.
13. $\frac{448}{1125}$.
14. $1\frac{1}{89}$.

DECIMAL FRACTIONS.

Page 95, Art. 196.

2. .4; .7; .02; .05; .09.
3. .12; .14; .006; .207; .0015.
4. .206; .0372.
5. .000525; .0036; .00021.
6. 75.5; 32.032; 340.005.
7. 1000.001; 2063.00026; 450.000681.
8. .0032; .000123; .406; .02147; .0006347; .004004.
9. .34572; .0023465; .000000047; .00001035; .0000000005.
10. .7.
11. .21.
12. .136.
13. .1456.
14. 7.6.
15. 37.65.
16. 100.04.
17. 49.009.
18. .3.
19. .30.
20. .300.
21. .3000.
22. .3.
23. .03.
24. .003.
25. .0003.

Page 98, Art. 200.

2. .25.
3. .2.
4. .66$\frac{2}{3}$.
5. .75.
6. .4.
7. .42$\frac{6}{7}$.
8. .16$\frac{2}{3}$.
9. .6.
10. .83$\frac{1}{3}$.
11. .44$\frac{4}{9}$.
12. .625.
13. .77$\frac{7}{9}$.
14. .36$\frac{4}{11}$.
15. .58$\frac{1}{3}$.
16. .53$\frac{1}{3}$.
17. .77$\frac{3}{11}$.
18. .75.
19. .25.
20. 6.25.
21 12.5.
22. 14.66$\frac{2}{3}$.
23. 3.0625.
24. .2775.
25. 39.38.

Art. 201.

2. $\frac{3}{20}$.
3. $\frac{9}{20}$.
4. $\frac{27}{60}$.
5. $\frac{16}{25}$.
6. $\frac{3}{4}$.
7. $\frac{1}{8}$.
8. $\frac{59}{250}$.
9. $\frac{69}{200}$.
10. $\frac{19}{40}$.
11. $\frac{79}{125}$.
12. $\frac{9}{200}$.
13. $\frac{61}{200}$.
14. $\frac{1}{200}$.
15. $\frac{9}{40}$.
16. $\frac{1}{1000}$.
17. $\frac{1067}{5000}$.
18. $\frac{3}{80}$.
19. $\frac{13}{2000}$.
20. $\frac{1}{2500}$.
21. $\frac{101}{2000}$.
23. $\frac{5}{8}$.
24. $\frac{1}{8}$.
25. $\frac{3}{16}$.
26. $\frac{3}{8}$.
27. $\frac{5}{12}$.
28. $\frac{23}{40}$.
29. $\frac{5}{8}$.
30. $\frac{223}{300}$.
31. $\frac{97}{800}$.
32. $\frac{11}{80}$.
33. $\frac{3}{400}$.
34. $\frac{3}{160}$.
35. $4\frac{1}{8}$.
36. $10\frac{5}{8}$.
37. $12\frac{2}{8}$.
38. $15\frac{7}{80}$.

ADDITION OF DECIMALS.

Page 99, Art. 202.

2. 7.61.
3. 38.491.
4. 60.307.
5. .711.
6. 11.0541.
7. 11.644.
8. 10.97.
9. 8.999.
10. 5.4772.
11. 20.15.
12. 901.09.

3. 2627.507.
4. $170.87.
5. $254.3125.
16. $265.17.
17. 340.11254.
18. 3825.015122.
19. 37.607.
20. 24.2625.

SUBTRACTION OF DECIMALS.

age 100, Art. 203.
2. .218.
3. .1627.
4. .641.
5. 2.164.
6. 2.236.
7. 17.4565.
8. 50.7654.
9. 567.255.
10. 100.0695.
11. $16.69.
12. $18.22.
13. 51.085.
14. $40.75.
15. $67.375.
16. 24.005985.
17. 4.498.
18. $1117.24.
19. 310.925.
20. 217766.34.
21. $1352.625.

MULTIPLICATION OF DECIMALS.

age 101, Art. 204.
2. .0345.
3. .84.
4. 28.416.
5. .0013.
6. 32.991.
7. .74725.
8. .0000624.
9. .03037.
11. 41321.
12. 21234.
13. 1435.8.
14. 3689.7.
15. 75403.5.
16. 22510.2.
17. 219.15652.
18. 25000.25.
19. $46.53.
20. $35.37.
21. $5.16.

DIVISION OF DECIMALS.

age 102, Art. 205.
3. 3.6022+.
4. 453.2353+.
5. 3.21.
6. 4170.
7. 3.72.
8. 1.2.
9. 12000.
0. .012.
1. .12.
2. 176.3.
3. 2.521.
4. .59.
5. 9.3.
6. 135.
17. 387100.
18. 27.4.
19. 40.8.
20. 33272.7272+.

Page 103.

22. 6.1.
23. 48.9.
24. .921.
25. .08935.
26. .002134.
27. .354671.
28. 3.17714.
29. .0153795.
30. .152202.
31. 56.7891.
32. 34.
33. 12.
34. 1.88228.
35. 24.12734+.
36. 21.91751+.
37. 160.
38. 145.
39. 8.25.
40. 15.7.
41. 750.
42. 395.
43. 31.5.
44. $90.70.

SHORT PROCESSES.

'age 105, Art. 213.
2. 54700.
3. 103400.
4. 186200.
5. 212400.
6. 67800.
7. 25410.
8. 10625.
9. 6210.
10. 90560.
11. 36300.
12. 215400.
13. 117700.
14. 628800.
15. 112000.
16. 2131000.
17. 864000.
18. 171866⅔.
19. 1072500.
20. 401100.
21. 634900.
22. 360500.
23. 674800.
24. 566000.

25. 892750.
26. 784666⅔.
27. 1989000.
28. 1390800.

Page 108, Art. 224.

3. 462627.
4. 741076.
5. 810532.
6. 6741252.
7. 5864248.
8. 60879072.
9. 46892142.
10. 763139692.
11. 509782280.
12. 895535568.

Page 109, Art. 228.

3. 166509.
4. 355740.
5. 1178744.
6. 2714364.
7. 3192231.
8. 7449150.
9. 6675032.
10. 14611580.
11. 22326984.
12. 37580064.

Page 112, Art. 238.

2. $31.75.
3. $13.95.
4. $52.70.
5. $52.93.

Page 113.

6. $228.96.
7. $73.39.
8. $110.

Art. 239.

2. $19.89.
3. $46.41.
4. $130.65.
5. $114.32.
6. $93.22.
7. $12.
8. $103.20.
9. $286.88.

Page 114, Art. 240.

2. 12.19.
3. 28.67.
4. 23.40.
5. 13.41.
6. $18.
7. $16.25.
8. $23.25.
9. $15.91.
10. $4.90.
11. $17.76.

BILLS AND ACCOUNTS.

Page 119, Art. 263.

1. $71.94.

Page 120.

2. $80.13.
3. $39.71.
4. $2837.

Page 121.

5. $2257.25.
6. $23.55.
7. $7.80.
8. $172.30.
9. $36.25.
10. $70.53.

REDUCTION OF DENOMINATE NUMBERS.

MEASURES OF VALUE.

Page 125, Art. 284.

2. 76d.
3. 115d.
4. 138d.
5. 534d.
6. 3692d.
7. 5963d.
8. 363 far.
9. 598 far.
10. 835 far.
11. 6122 far.
12. 11263 far.
13. 20927 far.

Page 126.

15. 3 far.
16. 3¾ far.
17. 8d.
18. 5d. ¾ far.
19. 15s.
20. 16s. 8d.
21. 11s. 8d.
22. 18s. 8d.
24. 3 far.
25. 3.44 far.
26. 2d. 3.52 far.
27. 6d. 2.88 far.
28. 4d. 2 far.
29. 18s.
30. 12s. 6d.
31. 17s. 6d.

Art. 285.

2. £19 16s. 8d.
3. £24 11s. 4d.
4. £29 14s. 7d.
5. £342 7s.
6. £30 19s. 4d.
7. £317 5s.
8. £18 7s. 6d. 3 far.
9. £27 11s 6d. 3 far.
10. £45 8s. 3d. 2 far.

Page 127.

12. £$\frac{3}{80}$.
13. £$\frac{7}{160}$.
14. £$\frac{1}{24}$.
15. £$\frac{1}{384}$.
16. £$\frac{7}{2640}$.
17. £$\frac{11}{2880}$.
18. £$\frac{1}{3840}$.
19. £$\frac{1}{1920}$.
20. £$\frac{1}{1280}$.
22. £ 26875.
23. £ .6302+.
24. £ .9489+.
25. £3.2229+.

:7.4281+.
:20.7302+.
:35.999—.

) 127, Art. 286.

:30.54.
:98.97.
:148.
:221.30.
:59.53.
:101.19.

287.

:55 11s. 4+d.

3. £64 15s. 2d. 3— far.
4. £97 14s. 7+d.
5. £127 8s. 5d. 1+ far.
6. £110 9s. 6d. 3— far.
7. £152 1s. 1d. 1+ far.
8. £377 6d. 2—far.
9. £1021 15s. 2d. 3— far.

Page 130, Art. 291.

1. $38.60.
2. $175.
3. $93.72.
4. 2590.67.
5. $35.70.
6. 1013.95—
7. 150.
8. 40.
9. 1000.
10. $2010; $2433.25.
11. $62.89.
12. $12.78.
13. $102.20.
14. $193.
15. $113.51.

MEASURES OF WEIGHT.

) 133, Art. 302.

:319 gr.
:740 gr.
:2332 gr.
:06597 gr.
.7 pwt. 12 gr.
) oz. 12 pwt.
:oz. 7 pwt. 14⅔ gr.
:33 gr.
:075 gr.
:6652 gr.
.44350 gr.
:ʒ 6 ʒ 1 ℈ 4 gr.
:332 oz.

14. 11615 oz.
15. 19431 oz.
16. 123845 oz.
17. 268936 oz.
18. 7 cwt. 50 lb.
19. 84 lb.
20. 13 cwt. 50 lb.
21. 3 oz.
22. 31 lb. 10 oz. 10 pwt.
23. 2 ʒ 2 ʒ.
24. 15 ℔.
25. 1 ℔ 8 ʒ 4 ʒ 1 ℈.
26. 59 ℔ 9 ʒ 4 ʒ.
27. 1 ℔ 5 ʒ 3 ʒ 10 gr.
28. 1 cwt. 35 lb.
29. 6 cwt. 11lb. 4 oz.
30. 2 T. 16 cwt. 40 lb.
31. 2 T. 8 lb. 2 oz.
32. 42 bar.
33. 48 bar. 180 lb.
34. 35 bar. 60 lb.
35. $\frac{1}{400}$ lb.
36. $\frac{5}{4608}$ ℔.
37. $\frac{7}{256000}$ T.
38. .005729+oz.
39. 456597+ ℔.

MEASURES OF EXTENSION.

) 135, Art. 309.

'1 ft.
.39 ft.
.79 ft.
:04 ft.
:7500 ft.
:0534 ft.
.37 in.
.64 in.
:12 in.
:501 in.
:63511 in.
:06016 in.
25 l.
:87 l.
:15 l.
:60 l.

17. 2093 l.
18. 123075 l.
19. 3 yd. 2 ft.
20. 3 yd. 1 ft. 3¾ in.
21. 3 yd. 5⅐ in.
22. 213 rd. 5 ft. 6 in.
23. 142 rd. 3ft. 8 in.
24. 149 rd. 5 ft. 6 in.
25. 2 rd. 12½ l.
26. 2 rd. 10 l.
27. 2 rd. 20 l.
28. 1 ft. 11.4 in.
29. 152 rd.
30. 2 rd. 12.5 l.

Page 136.

31. 7 rd. 1 yd. 1 ft. 6 in.
32. 20 ch. 2 rd. 10 l.
33. 311 rd. 8 ft. 3 in.
34. 1 mi. 276 rd. 2 yd.
35. 1 mi. 131 rd. 14 ft. 6 in.
36. 83 ch. 3 rd
37. 1 mi. 300 rd.
38. 13 mi. 234 rd. 5 yd.
39. 1 mi. 66 rd. 3 yd. 1 ft.
40. $\frac{8}{165}$ rd.
41. $\frac{19}{1440}$ yd.
42. $\frac{7}{42240}$ mi.
43. $\frac{1}{120}$ mi.
44. .833⅓ yd.
45. .82828+rd.

SURFACE MEASURES.

Page 138, Art. 318.

1. 651 sq. in.
2. 1116 sq. in.
3. 7056 sq. in.
4. 425736 sq. in.
5. 4705344 sq. in.
6. 31384944 sq. in.
7. 3 sq. yd. 3 sq. ft. 30 sq. in.
8. 4 sq. yd. 2 sq. ft. 8 sq. in.
9. 10 sq. rd. 4 sq. yd. 1 sq. ft. 72 sq. in.
10. 24 sq. rd. 22 sq. yd. 8 sq. ft.
11. 144 sq. rd. 28 sq. yd.
12. 48 A.
13. 120 A.
14. 5 sq. rd. 295 sq. l.
15. 10 sq rd.
16. 22 sq. yd. 6 sq. ft. 108 sq. in.
17. 6 sq. ft. $86\frac{2}{9}$ sq. in.
18. 9 sq. rd. 375 sq. l.
19. 14 sq. rd.
20. 10 sq. yd. 3 sq ft. 133.38 sq. in.
21. 10 sq. rd. 500 sq. l.
23. $\frac{1}{868}$ sq. rd.
24. $\frac{5}{242}$ sq. rd.

Page 139.

25. $\frac{1}{1944}$ sq. yd.
26. $\frac{17}{8200}$ A.
27. $\frac{8}{256000}$ sq. mi.
28. .17851 + sq. yd.
29. .25 sq. ch.
30. .65 A.

MEASURES OF VOLUME.

Page 140, Art. 323.

1. 20976 cu. in.
2. 1676160 cu. in.
3. 150336 cu. in.
4. 69132 cu. in.
5. 2 cu. yd. 23 cu. ft. 1564 cu. in.
6. 4 cu. yd. 15 cu. ft. 998 cu. in.
7. 17533 cu. yd. 9 cu. ft.
8. 5068 cd. 1 cd. ft. 12 cu. ft.
9. $\frac{4}{185}$ cu. yd.
10. .78125 cd.

MEASURES OF CAPACITY.

Page 142, Art. 329.

1. 23 gi.
2. 30 gi.
3. 406 gi.
4. 575 gi.
5. 390 m.
6. 465 m.
7. 2705 m.
8. 28220 m.
9. 47805 m.
10. 524675 m.
11. 39 pt.
12. 53 pt.
13. 63 pt.
14. 186 pt.
15. 333 pt.
16. 569 pt.

Page 143.

17. 6 gal. 3 qt.
18. 24 gal. 2 qt. 1 gi.
19. 28 gal. 2 qt. 1 gi.
20. 3 O 4 f ʒ 7 f ʒ.
21. 7 O 13 f ʒ.
22. 3 f ʒ 7 f ʒ.
23. 357 bu. 2 pk.
24. 67 bu. 6 qt.
25. 136 bu. 2 pk. 3 qt.
26. $\frac{3}{82}$ gal.
27. $\frac{5}{256}$ bu.
28. .6875 bu.
29. 4.875 bu.
30. 5.609375 bu.

MEASURES OF TIME.

Page 145, Art. 334.

1. 11415 sec.
2. 19472 sec.
3. 45955 sec.
4. 1121 hr.
5. 1308 hr.
6. 1628 hr.
7. 19 hr. 12 min.
8. 15 hr.
9. 20 hr. 20 min.
10. 3 da. 19 hr. 12 min.
11. 3 mo. 3 wk. 1 da. 16 hr.
12. 10 mo. 1 wk. 4 da. 16 hr.
13. 11 mo. 10 da. 3 hr.
14. 1 yr. 1 mo. 2 da. 6 hr.
15. 6 da. 15 hr. 29 min. 28 sec.
16. 9 da. 10 hr. 2 min. 22 sec.
17. 11 da. 13 hr. 46 min. 40 sec.

Page 146.

18. $2\frac{1}{160}$ da.
19. $\frac{5}{56}$ wk.
20. .00059+da.
21. .00193+wk.
22. .00307—da.

CIRCULAR MEASURE.

147, Art. 338.
5210″.

2. 134312″.
3. 145701″.
4. 6° 45′ 50″.
5. 20° 44′ 40″.

MISCELLANEOUS MEASURES.

148, Art. 344.
17.28.
100.

3. $7.75.
4. $.35.
5. 8 bl. 1 bdl. 4 qr. 18 sh.

MEASURES OF HEAT.

149, Art. 347.
° R.

4. 192° R.
5. $-39\frac{4}{9}$° C.

ADDITION OF DENOMINATE NUMBERS.

150, Art. 350.
152 19s. 9d.
2 lb. 5 oz. 11 wt.

4. 107 T. 5 cwt. 75 lb. 5 oz.
5. 13 yr. 11 mo. 28 da.
6. 75 cd. 63 cu. ft. 1442 cu. in.
7. 51 mi. 65 rd. 5 yd. 1 ft. 7 in.
8. 1260 mi. 33rd 5 yd. 9 in.

SUBTRACTION OF DENOMINATE NUMBERS.

151, Art. 352.
36 12s. 2d.
75 14s. 10d.
5 lb. 10 oz. 16 wt. 21 gr.
)4 T. 2 cwt. 84). 14 oz.

6. 25 mi. 246 rd. 2 yd. 2 ft. 3 in.

Art. 353.

2. 7 yr. 7 mo. 8 da.
3. 4 yr. 7 mo. 16 da.
4. 8 yr. 10 mo. 25 da.

Page 152.

5. 84 yr. 3 mo. 11 da.
6. 67 yr. 9 mo. 22 da.
8. 78 da.
9. 584 da.
10. 354 da.

MULTIPLICATION OF DENOMINATE NUMBERS.

152, Art. 355.
131 18s. 8d.
1227 14s. 1 d. far.

4. 92 lb. 9 oz. 13 pwt. 3 gr.
5. 188 bu. 2 qt.
6. 305 hhd. 30 gal. 2 qt. 1 pt.
7. 396 bu. 3 pk. 4 qt.
8. 139 cwt. 4 lb.
9. 131 hhd. 39 gal.
10. 4 lb. 4 oz. 19 pwt.

DIVISION OF DENOMINATE NUMBERS.

153, Art. 357.
7 13s. 5d.
15 9s. 7d.
3 hhd. 4 gal. 2 t. 1 pt. $3\frac{7}{9}$ gi.

5. 17 lb. 4 oz. 6 pwt. 11 gr.
6. 145 bu. 2 pk. 6 qt. $1\frac{3}{8}$ pt.
7. 31 mi. 98 rd. 12 ft. $4\frac{1}{2}$ in.
8. 3° 9′ $10\frac{5}{6}$″.
10. 14.
11. 20.
12. 45.

LATITUDE, LONGITUDE, AND TIME.

156, Art. 361.
1° 2′ 36″.
3′ 31″.
5° 23′ 27″.
3° 30′ 54″.

7. 87° 26′ 45″.
8. 124° 7′ 15″.

Art. 362.

2. 4 hr. 16 min. $21\frac{1}{3}$ sec.
3. 1 hr. 20 min. $4\frac{1}{5}$ sec.
4. 5 hr. 58 min. $0\frac{2}{15}$ sec.
5. 10 hr. 57 min. 32 sec. A.M.

Page 157.

6. 3 hr. 3 sec. P.M.

Art. 363.

2. 78° 19′ 15″.

3. 102° 41′ 54″.
4. 10° 33′ 45″ W.
5. 88° 19′ 30″ E.
6. 82° 21′ 30″ W.
7. 123° 54′ 15″ W.
8. 139° 10′ 30″ E.
9. 47° 30′ W.

THE METRIC SYSTEM.

Page 161. Art. 378.

14. 76.13^{m}.
15. 452.3^{l}.
16. 7865.004^{g}.
17. $15.0302^{sq\ m}$.
18. $4.015043005^{cu\ m}$.
19. 78060.045^{g}.
20. $7513.2416^{sq\ dm}$.

Page 162, Art. 380.

2. $1242.35^{d\ m}$.
3. 513628^{cg}.
4. 3.21546^{l}.
5. 4.12637^{Kl}.
6. 2.41673^{Hm}.
7. 3.412678^{Mg}.
8. $.023456^{sq\ m}$.
9. $51342.647^{sq\ dm}$.
10. $4.134563^{cu\ m}$.
11. $4186745^{cu\ cm}$.

Page 163, Art. 382.

1. .51839—lb.
2. 189.7396 yd.
3. 932.055 mi.
4. 58.305 sq. yd.
5. 173.587+A.
6. 163.5 cu. yd.
7. 90.6^{g}.
8. 36.576^{m}.
9. 35.24^{Hl}.
10. 4082.373+kilos.
11. 132.475^{l}.
12. 66.06^{l}.

Page 164, Art. 383.

1. $39.38
2. $3.41.
3. $18.28.
4. $2.11.
5. $63 60.
6. $2.16.
7. $200.
8. Former 12 + ¢ cheaper.
9. $299.15.
10. $402.35.
11. $.8361^{sq\ m}$.
12. $916\frac{2}{3}^{m}$.
13. 7200^{T}.
14. 1950^{Kg}.
15. (*a*) inch; (*b*) pound; (*c*) yard; (*d*) quart; (*e*) mile.

INVOLUTION AND EVOLUTION.

INVOLUTION.

Page 165, Art. 388.

2. 512; 1331; 2744; 13824.
3. 144; 225; 324; 12.25; 1764.
4. 729; 2197; 4913; 29.791; .166375.
6. $\frac{4}{9}$; $\frac{16}{25}$; $\frac{216}{343}$; $\frac{1331}{9261}$.
8. 3^{9}.
10. 7^{8}; 8^{9}; 12^{8}; 15^{12}.

Page 166, Art. 390.

2. $30^{2} + 2\ (30 \times 6) + 6^{2}$.
3. $40^{2} + 2\ (40 \times 2) + 2^{2}$.
4. $50^{2} + 2\ (50 \times 8) + 8^{2}$.
5. $60^{2} + 2\ (60 \times 7) + 7^{2}$.
6. $70^{2} + 2\ (70 \times 1) + 1^{2}$.
7. $80^{2} + 2\ (80 \times 3) + 3^{2}$.
8. $90^{2} + 2\ (90 \times 5) + 5^{2}$.
9. $80^{2} + 2\ (80 \times 7) + 7^{2}$.

Art. 392.

2. $10^{3} + 3\ (10^{2} \times 5) + 3\ (10 \times 5^{2}) + 5^{3}$.
3. $10^{3} + 3\ (10^{2} \times 8) + 3\ (10 \times 8^{2}) + 8^{3}$.
4. $20^{3} + 3\ (20^{2} \times 5) + 3\ (20 \times 5^{2}) + 5^{3}$.
5. $30^{3} + 3\ (30^{2} \times 4) + 3\ (30 \times 4^{2}) + 4^{3}$.
6. $40^{3} + 3\ (40^{2} \times 5) + 3\ (40 \times 5^{2}) + 5^{3}$.
7. $50^{3} + 3\ (50^{2} \times 8) + 3\ (50 \times 8^{2}) + 8^{3}$.
8. $70^{3} + 3\ (70^{2} \times 8) + 3\ (70 \times 8^{2}) + 8^{3}$.
9. $80^{3} + 3\ (80^{2} \times 5) + 3\ (80 \times 5^{2}) + 5^{3}$.

SQUARE ROOT.

Page 169, Art. 402.

3. 36.
4. 76.
5. 84.
6. 98.
7. 121.

04.
75.
99.
9.15.

12. 18.02.
13. .00125.
14. 712.86.
15. $\frac{16}{21}$.
16. $\frac{25}{32}$.
17. $\frac{37}{49}$.
18. $\frac{216}{256}$.
19. 3.4641.
20. 6.0124.

CUBE ROOT.

171, Art. 406.
5.
2.
5.
3.
6.
6.
5.
64.
16.

12. 507.
13. 8.57.
14. 96.2.
15. .42.
16. .093.
17. .125.
18. 1 8171.
19. $\frac{8}{12}$; .8549; $\frac{27}{88}$.
20. 3.4369.

Page 172, Art. 407.
22. 345.
23. 703.
24. 743.
25. 859.
26. 1234.
27. 4567.
28. 58.13.
29. 7.583.
30. .0962.

PRACTICAL MEASUREMENTS.

PLANE SURFACES.

175, Art. 426.
00 sq. ft.
20 sq. rd.
0 A
0 rd.
2 A.
393.533 + sq m.
4^{Ha}.

429.
60 sq. ft.
0 sq. yd.
0 rd.

176, Art. 432.
80 sq. ft.
99 sq. ft.
A.
52 sq. ft.

177, Art. 434.
0.642 + sq. ft.
A. 29.693 + sq. d.
A. 118.847 — q. rd.

178, Art. 439.
8 ft.
3.216 + rd.

5. 226.274 + rd.
6. 26.076 + ft.
7. 23.469 + ft.

Art. 442.
1. 238 sq. in.
2. 5.156 + A.
3. 150 sq. ft.

Page 179, Art. 445.
1. 6.883 + sq. rd.
2. 25 A.
3. 285.147 — A.

Art. 448.
1. 270 sq. ft.
2. $36\frac{3}{4}$ A.

Page 180, Art. 451.
1. 37.6992 ft.
2. 9.9484 ft.
3. 31.83 ft.

Page 181.
4. 100 rd.
5. 15 ft.
6. 491.904 + rev.

Art. 453.
1. 2.182 — sq. ft.
2. 490.875 sq. ft.
3. 1017.8784 sq. ft.
4. 3.1416 sq. rd.
5. 71.6175 A.
6. A's field, 160 rd.; and area, 10 A. B's field, 125.664 rd.; and area, 7.854 A.

Page 182, Art. 455.
1. 21.110 + ft.
2. 70.897 + rd.
3. 14.272 + rd.

Art. 456.
1. 17.724 ft.
2. 16.926 ft.

Page 183, Art. 459.
1. 7.071 + in.
2. 12.727 + in.

Art. 461.
1. 84.8232 sq. ft.
2. 235.62 sq. ft.

Page 184, Art. 462.
2. $17.17.
3. $9.37.
4. $43.87.

Art. 463.

2. $128.09.
3. $88.07.

Page 185.

4. $256.20.

Art. 464.

2. $31.50.
3. $25.50; $23.80.
4. 60 yd.
5. $40.

Page 186, Art. 465.

2. $1.80.
3. $4.14.
4. $4.40.
5. $15.50.
6. $13.10.

SURFACE OF SOLIDS.

Page 188, Art. 477.

1. 45 sq. ft.

Page 189.

2. 87.9648 sq. ft.
3. 350 sq. ft.
4. 227.766 sq. ft.

Art. 478.

1. 180 sq. ft.

2. $16.67.
3. 615.7536 sq. ft.
4. 452.3904 sq. ft.
5. 464 sq. ft.

Art. 479.

1. 640 sq. ft.

Page 190.

2. 285.8856 sq. ft.

3. 250 sq. ft.
4. 145.299 sq. ft.

Art. 480.

1. 452.3904 sq. in.
2. 314.16 sq. in.
3. 198.9375 sq. in.
4. $11.04.

VOLUME OF SOLIDS.

Page 191, Art. 482.

1. 90 cu. ft.
2. 1740 cu. ft.
3. $290.28.
4. 2006.
5. 13 ft.

Art. 483.

1. 960 cu. in.
2. 1 cu. ft. 156.96 cu. in.
3. 653.4528 cu. ft.

Page 192, Art. 484.

1. $133\frac{1}{3}$ cu. in.

2. 402.1248 cu. ft.
3. $133\frac{1}{3}$ cu. ft.
4. 2565.64 cu. ft.
5. 77.942 + cu. ft.

Art. 485.

2. 6840 cu. ft.
3. 74.613 cu. ft.

Page 193, Art. 486.

1. 268.0832 cu. in.
2. 113.09—cu. in.
3. 458.15 cu. in.
4. 235.62 lb.

Art. 487.

1. 5.196 + in.
2. 7.358 — in.

Page 194, Art. 488.

1. 2199.12 cu. in.
2. 1270.5 cu. in.

Art. 492.

1. $3\frac{3}{4}$ cd.
2. $47.25.
3. $61\frac{1}{4}$ cd.
4. $53.13.
5. $4429.69.

BOARDS AND TIMBER.

Page 195, Art. 495.

1. 14 ft.
2. $17\frac{1}{2}$ ft.
3. $21\frac{1}{3}$ ft.
4. $18\frac{2}{3}$ ft.
5. 15 ft.
6. 21 ft.
7. 20 ft.
8. $19\frac{1}{2}$ ft.
9. $7.65.
10. $80.

Art. 496.

1. $18\frac{2}{3}$ ft.

Page 196.

2. 45 ft.
3. 56 ft.
4. 150 ft.
5. $168.

Art. 497.

2. $27\frac{2}{9}$ cu. ft.
3. 28 cu. ft.
4. 36 cu. ft.

Page 197, Art. 499.

2. 240 ft.

3. $116\frac{4}{27}$ ft.
4. $310\frac{1}{3}$ ft.
5. $326\frac{2}{3}$ ft.; 288 ft.

Art. 500.

2. $27\frac{2}{9}$ cu. ft.
3. $70\frac{1}{12}$ cu. ft.
4. $239\frac{1}{81}$ cu. ft.

Page 199, Art. 505.

2. 76.8 cu. ft.
3. 513.76 cu. ft.

STONEWORK AND BRICKWORK.

200, Art. 509.

)$\frac{10}{33}$ perches.
40.91.
35.33.

4. $51\frac{7}{99}$ perches.
5. $118.11.

Page 201, Art. 511.

2. 54327 bricks.
3. $517.23.
4. $166.23.
5. $22.69.

CAPACITY OF BINS, CISTERNS, CASKS, ETC.

202, Art. 514.

).29 — bu.
2.32 + bu.
66 — ft.
364.50.
28.89 — bu.
132 — bu.

203, Art. 518.

$79\frac{4}{77}$ gal.
$14\frac{2}{11}$ gal.
7.5 — bbl.
7.32 — bbl.
55.18 — bbl.

6. 33.51 — ft.
7. 200^{HI}.
8. 675'.
9. 8 hr. 28 min. 55.7 + sec.

Page 204, Art. 520.

1. 56.59 — gal.
2. 15.99 — gal.
3. 41.62 + gal.

Page 205, Art. 522.

1. $109\frac{1}{5}$ bu.
2. $172\frac{4}{5}$ bu.
3. 525 gal.
4. 67.32 bbl.
5. 38 784 perches.
6. 72.72 perches.
7. 171.467 + T.
8. 20 T.
9. $3\frac{1}{2}$ T.
10. 9 T.
11. $16\frac{2}{3}$ T.
12. $5\frac{5}{8}$ T.
13. $29\frac{17}{27}$ T.
14. $255.56.
15. $113.38.
16. $25\frac{25}{27}$ T.

PERCENTAGE.

207, Art. 531.

)8; $\frac{2}{25}$.
)9; $\frac{9}{100}$.
12; $\frac{3}{25}$.
25; $\frac{1}{4}$.
48; $\frac{12}{25}$.
49; $\frac{49}{100}$.
50; $\frac{1}{2}$
32; $\frac{31}{50}$.
35; $\frac{17}{20}$.
90; $\frac{9}{10}$.
)6$\frac{2}{3}$; $\frac{1}{15}$.
)7$\frac{1}{2}$; $\frac{3}{40}$.
12$\frac{1}{2}$; $\frac{1}{8}$.
13$\frac{1}{3}$; $\frac{2}{15}$.
22$\frac{1}{2}$; $\frac{9}{40}$.
24$\frac{1}{2}$; $\frac{49}{200}$.
33$\frac{1}{3}$; $\frac{1}{3}$.
37$\frac{1}{2}$; $\frac{3}{8}$.
62$\frac{1}{2}$; $\frac{5}{8}$.
83$\frac{1}{3}$; $\frac{5}{6}$.
.
.15; $1\frac{3}{20}$.

23. 1.20; $1\frac{1}{5}$.
24. 1.45; $1\frac{9}{20}$.
25. 1.50; $1\frac{1}{2}$.
26. 1.85; $1\frac{17}{20}$.
27. $1.37\frac{1}{2}$; $1\frac{3}{8}$.
28. $2.62\frac{1}{2}$; $2\frac{5}{8}$.
29. $3.12\frac{1}{2}$; $3\frac{1}{8}$.
30. $4.16\frac{2}{3}$; $4\frac{1}{6}$.
31. $.00\frac{1}{2}$; $\frac{1}{200}$.
32. $.00\frac{1}{4}$; $\frac{1}{400}$.
33. $.00\frac{3}{4}$; $\frac{3}{400}$.
34. $.00\frac{2}{5}$; $\frac{1}{250}$.
35. $.00\frac{9}{10}$; $\frac{9}{1000}$.
36. $.00\frac{11}{20}$; $\frac{11}{2000}$.
37. $.00\frac{13}{25}$; $\frac{13}{2500}$.
38. $.00\frac{19}{25}$; $\frac{19}{2500}$.
39. $.00\frac{33}{50}$; $\frac{33}{5000}$.
40. $.00\frac{3}{5}$; $\frac{3}{500}$.

Art. 533.

2. $66\frac{2}{3}$%.
3. 75%.
4. $33\frac{1}{3}$%.
5. 80%.
6. $87\frac{1}{2}$%.
7. $18\frac{2}{11}$%.
8. $66\frac{2}{3}$%.
9. $71\frac{3}{7}$%.
10. 40%.
11. 90%.
12. $35\frac{5}{17}$%.
13. 76%.
14. 1%.
15. 2%.
16. 125%.

Page 210, Art. 536.

3. $2.32.
4. $20.55.
5. $208.
6. $24.
7. $15.61.
8. $114.10.
9. $127.73.
10. $14.72.
11. $234.
12. 128 A.

13. 207 A.
14. 1638 cattle.
15. $4600.
16. $2660.
17. 360 bu.
18. $28.03.
19. 600 bu.
20. $100.
21. $720.

Page 211, Art. 537.

3. $15.
4. $28.
5. $83.08.
6. $15.
7. $1400.
8. $1230.
9. $848.
10. $3500.

Page 212.

11. $1120.
12. $8000.
13. $1200.
14. $5833.33.
15. $14000.
16. $18000.
17. $5400.
18. $3013.04.
19. $10000.
20. $8000.
21. $3200.

Page 213, Art. 538.

3. 25%.
4. $66\frac{2}{3}$%.
5. $71\frac{21}{29}$%.
6. $31\frac{1}{4}$%.
7. $57\frac{1}{7}$%.
8. $50\frac{2}{11}$%.
9. $38\frac{7}{26}$%.
10. 200%.
11. $66\frac{2}{3}$%.
12. 35%.
13. J. $47\frac{4}{13}$%; S. $52\frac{9}{13}$%.
14. 10%.
15. $12\frac{1}{2}$%.

Page 214.

16. 15%.
17. $53\frac{1}{3}$%.
18. 40%.
19. 50%.
20. $6\frac{2}{3}$%.
21. 63%.

REVIEW.

Page 215, Art. 542.

1. $9500.
2. $2925.
3. 40%.
4. $2850.
5. $1500.
6. $750.

Page 216.

7. $24000.
8. $190.
9. $8305.08.
10. 20%.
11. 22%.
12. $4221.
13. $3\frac{1}{2}$%.
14. $8736.
15. $3492.39.
16. $440.
17. $1028\frac{4}{7}$%.
18. $26608.70.
19. $1326.15.
20. 1st, 25%; $18.75.
2d, 35%; $26.25.
3d, 40%; $30.

PROFIT AND LOSS.

Page 219, Art. 550.

2. $3.47.

Page 220.

3. $350.63.
4. $530.74.
5. $517.50.
6. $5698.
7. $4.50.
8. 18¢.
9. $3233.23.
11. $570.
12. $68.75.
13. $5700.
14. $3.50.

Page 221.

15. 30¢.
16. $3600.
18. $5\frac{167}{217}$%.
19. $13\frac{113}{250}$%.
20. 10%.
21. $33\frac{1}{3}$%.
22. 25%.
23. $1.20.
24. $.96.
25. $3450.
26. $75.
27. $4740.93.
28. $134.40.
29. $3.36.

Page 222.

30. $3.75.
31. $12\frac{1}{2}$%.
32. $2356.
33. 25%.
34. $1418.63.
35. $1.95.
36. $36 32.
37. 90¢.
38. $12\frac{1}{2}$%.
39. $33\frac{1}{3}$%.
40. No gain or loss.
41. Lost, $800.
42. $6800.

MARKING GOODS.

23, Art. 554.

) t.
50.
$\frac{d}{a}$.
) a.

7. $\frac{o\ t}{f\ o\frac{c}{a}}$

8. a. n h.

Page 224.

9. $\frac{m}{m.\ w\ n}$; $\frac{m.\ s\ n}{e.\ s\ s}$; $\frac{e.\ s\ n}{s.\ o\ s}$.

10. h. a y.

Page 225, Art. 556.

3. $1.76.
4. $2.99.
5. $.63.
6. $8.25.
7. $1.50.
8. $4.20.

TRADE DISCOUNT.

26, Art. 560.

38.
39.36.
36.87.
4.60.
$\frac{3}{8}$%.
in, $10.
.26.25.
)60.

11. 47½%.

Page 227.

12. $6.
13. $850.

Art. 561.

2. $1.50.
3. $20.
4. $25.
5. $12.50.
6. $2.
7. $65.
8. $636.64.

Page 228.

9. $2000.
11. 30%.
12. 16⅔%

COMMISSION.

29, Art. 571.

..
3.25.
26.
..45.
25; $11025.
).47.
59.65.

30.

325.
)90.
)0 bu.

13. $5260.
14. $291; $9215.
16. $1300.
17. $2150.
18. 3000 bu.
19. $60.
21. $2560.
22. $1.80.

Page 231.

23. $9.75.
25. 2½%.
26. 2½%.
27. 2%.
28. 4½%.
29. 4%.
30. $497.37.
31. $355.99.
32. $12.
33. $825.75.

Page 232.

34. $1270.29.
35. $2552.62.
36. $643.36.
37. $377.87.
38. $6376.81.

INTEREST.

36, Art. 589.

t.
20.75.
26.
3.20.
78.24.
59.60.
)8.77.
41.

10. $42.36.
11. $55.65.
12. $139.65.
13. $120.07.
14. $144.93.
15. $583.
16. $335.

Page 237.

17. $139.20.
18. $359.33.
19. $1592.55.
20. $523.02.

Page 238, Art. 591.

2. $71.
3. $41.85.
4. $72.75.
5. $38.08.
6. $35.60.

7. $120.41.
8. $92.82.
9. $161.03.
10. $98.13.
11. $74.81.
12. $35.53.
13. $89.51.

Page 239.

14. $17.93.
15. $97.88.
16. $822.40.
17. $510.42.
18. $441.12.
19. $871.77.
20. $280.92.

Page 240, Art. 592.

3 $4.80.
4. $17.85.
5. $4.29.
6. $7.22.
7. $7.20.
8. $3.62.
9. $303.75.
10. $228.09.

Page 242, Art. 595.

3. $ .23.
4 $ .40.
5. $1.19.
6. $4.20.
7. $4.80.
8. $9.05.
9. $7.50.
10. $1.09.
11. $.57.
12. $19.
13. $49.
14. $63.
15. $185.65.
16. $101.59.
17. $5.46.
18. $3.82.
19. $59.22.
20. $170.33.
21. $5.16.
22. $5.51.

Page 243, Art. 596.

2. $1.30.
3. $4.96.
4. $2.74.
5. $5.18.
6. $12.43.
7. $13.87.
8. $6.80.
9. $61.22.
10. $6.87.
11. $10.52.
12. $50.79.
13. £2 6s. 8.87d.
14. £4 12s. 10 + d.
15. £6 13s. 10.11 — d.

Page 244, Art. 597.

2. $ .05.
3. $ .25.
4. $9.82.
5. $17.83.
6. $9.82.
7. No difference in this ex.
8. $.83.
9. $59.22.

Page 245, Art. 598.

2. 4%.
3. 6%.
4. 7%.
5. 5½%.
6. 6%.

Page 246, Art. 599.

2. 3 yr.
3. 4 yr.
4. 3 yr. 6 mo.
5. 4 yr. 8 mo.
6. 3 yr. 6 mo. 15 da.
7. 2 yr. 10 mo. 24 da.

Art. 600.

2. $327.
3. $440.
4. $322.96.
5. $300.
6. $860.
7. $6000.

Page 248, Art. 605.

2. $578.60.
3. $425.04.
4. $735.
5. $588.98.
6. $320.94.
7. $882.88.
8. $191.25.
9. $64.05.
10. $1230.18.

Page 249, Art. 607.

2. $43.26.
3. $78.81.
4. $135.38.

Page 250.

6. $1508.52.
7. $1034.64.
8. $1790.81.
9. $238.73.
11. $2087.64.
12. $1803.10.
13. $806.11.
14. $342.70.
15. $845.82.
16. $230.67.
17. $744.43.

Page 258, Art. 623.

1. $566.68.
2. $959.50.
3. $1136.33.

Page 259.

4. Dec. 5; $265.31
5. Apr. 1, 1900.
6. $6885.

PARTIAL PAYMENTS.

Page 262, Art. 627.

2. $340.48.
3. $892.50.
4. $1281.24.
5. $772.44.
6. $1148.95.

Page 263.

7. $2351.82.
8. $2833.49.

rt. 628.

1. $337.95.

2. $671.15.
3. $480.79.
4. $401.94.
5. $743.93.

TRUE DISCOUNT.

age 264, Art. 632.

?. $500.
?. $700; $140.
4. $347.47; $13.03.
5. $393.61; $32.14.

Page 265.

6. $497.75.
7. $1000.
8. $145.34.
9. 1st, $30.49.
10. $241.90.
11. Gain $1.73.
12. $6458.22.
13. $608.78.
14. $119.73.

BANK DISCOUNT.

age 267, Art. 637.

?. $1.40.
4. $4.51.
5. $3.63.
?. $1.71.
9. $9.33.
?. $174.12.
0. $737.05.
1. $567.97.
2. $917.34.
?. $641.09.
?. July 1, '98; $492.42.
4. $734.37.
5. $7.20; $792.80.

16. Dec. 11, '98; $1205.41.
17. Oct. 2, '00; $579.65.
18. $32.

Page 268.

19. Oct. 21, '03; 78 da.; $690.90.
20. Mar. 6, '04; 28 da.; $995.33.
21. Aug. 30, '04; 89 da.; $271.27.
22. Nov. 30, '04; 90 da.; $446.88.
23. Oct. 28, '04; 118 da.; $1548.46.

Page 269.

24. Feb. 10, '05; 101 da.; $855.85.
25. July 2, '04; 64 da.; $2008.35.

Art. 638.

2. $507.61.
3. $2424.24.
4. $1000.
5. $600.
6. $457.63.

STOCKS AND BONDS.

age 277, Art. 676.

?. $220.
?. $607.50.
4. $239.25.
5. $19.50.
?. $453.13.
7. $641.25.
?. $4220.
9. $3400.
0. $3101.25.
1. $2387.50.
?. $11992.50.
?. $4725.
4. $2733.75.
5. $4370.63.
6. $22815.
8. $6187.50.

age 278.

9. $3843.38.
20. $2603.13.
21. $2122.50.
22. $3609.38.
23. $10573.75.
24. $12853.75.
25. $16170.25.
26. $7910.
27. $1816.88.
28. $630.
29. $50012.50.
30. $32513.13.
31. $3\frac{1}{2}$%.
32. $4\frac{1}{2}$%.
33. $3\frac{3}{4}$%.
34. $11287.50.

Page 279.

36. 28 shares; $56.50 over.
37. 42 shares; $46.75 over.
38. 73 shares; $106.88 over.
39. 93 shares; $37.38 over.
40. 162 shares; $58.50 over.
41. 34 shares; $47.75 over.
42. 208 shares.
44. $300.
45. $400.
46. $500.
47. $252.
48. $400.

Page 280.

50. $12218.75.
51. $15518.75.
52. $28781.25.
53. $31612.50.
54. $46050.

56. 8%.
57. $6\frac{4}{11}$%.
58. $6\frac{1}{7}\frac{6}{8}\frac{2}{3}$%.
59. $5\frac{1}{7}$%.
60. 5%.
61. $80.
62. $87.50.
63. $150.

Page 281.

65. $59\frac{7}{8}$.
66. $87\frac{2}{8}$.
67. $137\frac{3}{8}$.
69. $3\frac{1}{1}\frac{3}{4}$%.
70. 5%.
71. $3\frac{3}{8}$%.
72. 5%.

Page 282.

74. 110.
75. $95\frac{5}{11}$ or $95\frac{1}{2}$.
76. 124.
77. $35140; $2360.
78. 5% bonds at 90.
79. $6\frac{1}{3}$%.
80. $6\frac{1}{1}\frac{8}{9}\frac{8}{7}$%.
81. $26000.
82. $4000.
83. $40383.
84. $795.
85. $6\frac{1}{2}$%.
86. $840.
87. $51450.
88. $5\frac{9}{2}\frac{5}{6}\frac{5}{1}$%.

Page 283.

89. $6\frac{2}{4}\frac{6}{8}\frac{6}{9}$%.

Art. 677.

2. Gain, $320.50; Bal. due, $2320.50.
3. Bal. remitted, $3045.94; Profit, $545.94.

Page 284.

4. Bal. due, $6084.86; Gain, $1084.86.
5. $5147.75.

Art. 678.

2. $302.33.
3. Bal. due, $3451.31; Profit, $451.31.

EXCHANGE.

Page 287, Art. 690.

2. $160.20.
3. $300.75.
4. $495.82.
5. $107.73.
6. $273.97.
7. $365.66.
8. $426.06.
9. $799.

Page 288.

10. $555.80.
12. $1197.
13. $551.13.
14. $1479 38.
15. $1321.88.
16. $431.09.
18. $460.68.
19. $997.51.
20. $1500.
21. $2007.53.

Page 289.

23. $686.93.
24. $589.58.
25. $1000.
26. $3636.76.
27. $2248.65; $2228.97.
28. $5961.25.
29. $375.94.
30. $7126.20.

Page 293, Art. 698.

3. $729.38.
4. $2168.57.
5. £742 5s. 4d. 1 far.
6. $902.37.
7. $2625.65.
8. $576.92.
9. $96.25.
10. $283.50.
11. $2381.25.
12. $2852.43.
13. $2407.50.
14. $1074.45.
15. $6692.62.

Page 294.

16. 6563.30 francs.
17. £3596 3s. 11d. 2 far.
19. $110\frac{9}{9}\frac{7}{7}\frac{0}{8}$ or 111.
20. $76\frac{8}{9}\frac{8}{7}$ or $76\frac{4}{7}$.
21. $109\frac{7}{2}\frac{7}{0}$ or $109\frac{1}{4}$.
22. $73\frac{4}{5}\frac{9}{0}\frac{3}{0}\frac{3}{0}$ or 74.
23. $191\frac{2}{8}\frac{6}{0}\frac{3}{0}$ or $191\frac{1}{4}$.
24. $77\frac{9}{1}\frac{9}{0}$ or 78.
25. $1753.75.
26. 29857.83 francs.

BANKS AND BANKING.

Page 297, Art. 708.

1. 5 persons; $50000.
2. $200000; $100.
3. $75000; $75000.
4. $18750.
5. $1438 27.
6. *$168855.*
7. *24%.*
8. $600.
9. $3750; $4875.

Page 298, Art. 713.

1. $2.
2. $644.54.

Page 299.

3. $449.09.
4. $449.36.
6. $565.36.

Page 300.

7. $389.63.
8. $2576.30
9. $493.32.
10. $4420.18.

TAXES.

age 301, Art. 720.

1. $39.
2. $115.50.
3. $88.20.
4. $75.60.

Page 302.

5. $94.25.
6. $114.70.

Page 303, Art. 721.

9. $46.21.
10. $317.41.
11. $7\frac{81}{1000}$ + mills; Mr. Best's tax, $110.78.
12. $.008\frac{7}{84}$.

DUTIES, OR CUSTOMS.

age 305, Art. 738.

2. $369.25.
3. $2026.50.
4. 457.75.
5. $1101.80.
6. $2968.32.
7. $1376.
8. $492.20.
9. $591.
10. $1641.50.

Page 306.

11. $1710.
12. $502.25.
13. $306.96.
14. $1162 80.
15. $277.33.
16. $704.75.
17. $2895.73.
18. $3427.83.
19. $143.25.
20. $1184.09.

INSURANCE.

age 309, Art. 761.

1. $67.50.
2. $43.75.
3. $44.
4. $69.25.
5. Ætna, $1875; Franklin, $2625; Hanover, $3000.
6. $21600.
7. $5000.
8. $36.50.

age 310.

9. $18.75.
10. $3678.40.
11. $3000.
12. $94.79.
13. $2400.
14. American, $960; Orient, $1320; Traders' $1560; Delaware, $2040.
15. $16000.
16. $33590.80.
17. $250.
18. $3500.
19. $500.04.
20. $108.

Page 311.

21. $7500.
22. $50416.67; $60000.

Page 312, Art. 776.

2. $243.53.
3. $8611.20.
4. $6370.
5. $1392.

RATIO.

age 314, Art. 786.

1. $\frac{1}{2}$; $\frac{1}{8}$; 4; 8.
2. $\frac{1}{2}$; $\frac{1}{8}$; 5; 11.
3. $\frac{1}{8}$; 3; 8; 8.
4. $\frac{1}{4}$; 5; 7; 11.
5. $\frac{1}{2}$, 6; 6; 8.
7. $\frac{7}{10}$; $1\frac{3}{4}$; $\frac{13}{15}$.
8. $\frac{16}{19}$; $\frac{13}{28}$; $1\frac{2}{3}$; $\frac{3}{7}$.
10. $\frac{9}{10}$; $1\frac{1}{9}$; $\frac{24}{35}$; $2\frac{14}{33}$.
11. $\frac{8}{9}$; $\frac{27}{56}$; $\frac{48}{77}$; $\frac{6}{13}$.
12. $1\frac{1}{15}$; $\frac{28}{33}$; $1\frac{1}{20}$; $1\frac{1}{26}$.
14. $\frac{14}{81}$.
15. $\frac{2}{5}$.
16. $\frac{4}{5}$.
17. $2\frac{9}{13}$.
18. 8.
19. $5\frac{1}{3}$.
20. 175.
21. $1\frac{220}{329}$.

PROPORTION.

age 316, Art. 792.

2. 33.
3. 48.
4. 60.
5. 40
6. 8.
7. 1.
8. $22\frac{1}{2}$.
9. $37\frac{1}{8}$.
10. 10.
11. 15.
12. 27.
13. 450 lb.
14. $50.
15. 45.

16. 42 lb.
17. 24 da.

Page 317, Art. 795.

2. $30.
3. $1.37.
4. 20 da.
5. $70.
6. $39.
7. $80.
8. $358\frac{4}{57}$ bu.
9. 240 men.
10. $78.72.
11. 150 ft.
12. $65.

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13. $227\frac{1}{2}$ bar.
14. $10\frac{1}{2}$ oz.
15. $971\frac{3}{7}$ bu.
16. $6\frac{3}{4}$ mo.
17. 15 da.
18. 18 da.
19. $\$333\frac{1}{3}$.

Page 319, Art. 799.

3. 120.
4. $367\frac{1}{2}$.
5. $65\frac{41}{119}$.
6. $107\frac{9}{18}$.
7. $87\frac{3}{4}$.
8. $6\frac{19}{26}$.
9. $41\frac{11}{14}$.
10. $203\frac{7}{75}$.
11. 24.
12. $\frac{243}{418}$.

Page 320, Art. 800.

2. 18 da.
3. $10\frac{4}{5}$ wk.
4. $7.25.
5. $400.
6. 18 men.
7. 90 men.
8. $601\frac{4}{11}$ bu.
9. $4800.

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10. 36 yd.
11. $366.67.
12. 5%.
13. $56\frac{1}{4}$ da.
14. $21\frac{2}{3}$ da.
15. $35\frac{5}{9}$ da.

Art. 802.

2. 280; 630; 770.
3. 1512; 1764; 2016.

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4. 1200; 800; 600.
5. $3057.63; $3397.37.
6. Wife, $10950; Son, $7300; Daughter, $5475.
7. Wheat, 1650 bu.; Oats, 2475 bu.; Corn, 2750 bu.
8. Brown, $10807.50; Smith, $7942.50.
9. 1st son, $3000; 2d son, $2600; 3d son, $2000.
10. A, $450; B, $225; C, $75.
11. 150 horses; 125 cattle.
12. John, $450; James, $400.
13. 1st farm, $4590; 2d farm, $4896; 3d farm, $5100.
14. A, $960; B, $720; C, $320.

ALLIGATION, OR AVERAGE.

Page 323, Art. 806.

2. $16\frac{2}{31}$¢.
3. $64\frac{26}{38}$¢.
4. $35\frac{5}{7}$¢.

Page 325, Art. 808.

2. 3 lb. at 30¢; 2 lb. at 45¢; 1 lb. at 60¢; 2 lb. at 80¢.
3. 1 lb. at 60¢; 2 lb. at 75¢; 1 lb. at 90¢; 1 lb. at $1.00.
4. 1 lb. at 90¢; 2 lb. at $1.20; *3 lb. at $1.70; 2 lb. at $1.80.*
5. 3 bu. oats; 1 bu. corn; 8 bu. rye.

Art. 809.

2. 15 lb. at 35¢; 30 lb. at 50¢.

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3. 9 lbs. at 30¢; 15 lb. at 46¢; 3 lb. at 48¢.
4. 20 bu.

Art. 810.

2. 40 lb. at 25¢; 15 lb. at 35¢; 25 lb. at 75¢; 35 lb. at $1.00.
3. 500 ten ct. pcs.; 300 five ct. pcs.; 100 twenty-five ct. pcs.; 100 fifty ct. pcs.
4. 20 lb. at 35¢; 40 lb. at 50¢; 60 lb. at 85¢.
5. 94 bu. corn; 1 bu. timothy seed; 5 bu. clover seed.
6. 33 lambs; 12 sheep; 5 calves.

GENERAL AVERAGE.

ıge 330, Art. 822.

'. $5\frac{564}{1000}+\%$; $993.17.
'. Cargo's contrib. int., $13940.07;
Freight's contrib. int., $918.18;
Ship's contrib.int.,$5018.42.
Ship pays $6256.75 ;
Owners of Cargo rec. $1059.93 ;
Freight Co. rec. $281.82 ;
Repair Co. rec. $3715 ;
Crew, etc., rec. $475 ;
Other exp., rec. $725.
$18\frac{58674}{100000}+\%$.
5. A's contrib. int., $6661.84 ;
B's contrib. int , $5471.27 ;
C's contrib. int., $5710.15 ;
Freight's contrib. int., $1086.45 ;
Steamer's contrib. int., $11420.29.
C pays $5710.15 ;
Steamer pays $12220.29 ;
A rec. $8338.16 ;
B rec. $4528.73 ;
Freight Co. rec. $1063.55 ;
Repair Co. rec. $2400 ;
Crew rec. $622 ;
Other exp., rec. $968.
6. Cargo's contrib. int., $7654.39;
Freight's cont. int., $343.28 ;
Ship's cont. int., $4052.33.
Cargo pays $2654.39 ;
Ship pays $5952.33 ;
Freight Co. rec. $31.72 ;
Repair Co. rec. $5700 ;
Owners of Anchor rec. $225;
Crew, etc., rec. $1000 ;
Other exp., rec. $1500 ;
Adjuster, rec. $150.

PARTNERSHIP.

age 333, Art. 836.

3. Gain, $3800.
4. Gain, $1060.
5. Loss, $1483.22.
6. Gain, $2405.
7. Loss, $4239.
8. Gain, $4175.
9. Gain, $2227.93.
0. Loss, $3200.

'age 334, Art, 837.

2. Gain, $2670.
3. Loss, $1010.
4. Gain. $1069.
5. Gain, $2725.

'age 335, Art, 838.

2. $5150.
3. $9999.
4. $3956.
5. $16578.
6. $17275.
7. Insol. $3360.41.

'age 336, Art. 839.

2. $7575.
3. $13975.
4. Gain, $1995.
5. Net gain, $4035 ;
Net cap., $15035.

Page 337, Art. 840.

2. $575.
3. C's gain, $1600 ;
D's gain, $1200.
4. A's gain, $1170 ;
B's gain, $1300 ;
C's gain, $1430.
5. E's loss, $463.60 ;
F's loss, $618.13 ;
G's loss, $772.67.
6. Net loss, $10760-.73 ;
A's cap.,$1809.82;
B's cap.,$1209.82;
C's cap.,$1659.82;
D's cap.,$1809.81.

Page 338, Art. 841.

2. A's gain, $342.86;
B's gain, $457.14.
3. A's gain, $800 ;
B's gain, $1200 ;
C's gain, $2000.
4. D's loss, $800 ;
E's loss, $600 ;
F's loss, $1000.
5. 1st, $1950 ;
2d, $2600 ;
3d, $4550.
6. A's gain, $1340 ;
B's gain, $1160 ;
C's gain, $1460 ;
D's gain, $1640 ;
7. D's profit, $1406-.25 ;
E's profit, $1593-.75 ;
G's profit, $1687-.50.
8. H's cap., $3406-.66 ;
K's cap , $3976 ;
L's cap., $5030-.67.

Page 339, Art. 842.

2. A's gain, $525; B's gain, $900.
3. A's loss, $606.32; B's loss, $884.21; C's loss, $909.47.
4. A's gain, $2558-.14; B's gain, $2441.86.

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5. M's gain, $3294-.12; A's gain, $1647-.06; C's gain, $2058-.82.

Page 341, Art. 843.

2. Bermont's cap., $6071.34; Hill's cap., $6811.33; Nye's cap., $8082.33.

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3. A's cap., $5542.86; B's cap., $7057.14.
4. Jones' capital, $6472.73; Smith's capital, $3427.27.
5. Byerly's net cap., $7576.92; Baker's net cap., $6973.08.
6. A's net cr. int., $383.60; B's net cr. int., $565.75; C's net cr. int., $560.57.
A's net capital, $6363,12; B's net capital, $9738.44; C's net capital, $8026.44.
7. Gardner's cap., $10319.39; Rockwood's cap., $13319.39; Macdonald's cap., $9861.22.

BANKRUPTCY.

Page 344, Art. 852.

2. $ .68414 +; $1026.21.

Page 345.

3. $1236.49.
4. $7193.33.
5. Morrison, $3324-.98; Slyder, $4161.43; Bitner, $4640.31.
6. $ .56279 +.
7. $ .754226 +, $4336.80.
8. $ .38988 +.
9. $65\frac{122}{125}$ %. Horst, $12227.84; Stumbaugh, $9782.27; Reynolds, $8151-.89.

AVERAGE OF ACCOUNTS.

Page 347, Art. 861.

2. May 20.
3. Feb. 26.
4. May 2.

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5. July 30.
6. July 31.
7. Mar. 15.

Page 349, Art. 863.

2. Apr. 17.
3. June 23.

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4. July 8.
5. $501.85; May 16.
6. $736.44.

ACCOUNTS CURRENT.

Page 352, Art. 868.

2. $719.40.
3. $1151.06.
4. $1801.58.

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5. $1922.80.
6. $174.54.

AVERAGING ACCOUNT SALES.

Page 354, Art. 870.

1. $776.22; Feb. 25, 1900.
2. $1056.46; Nov. 16, 1899.

Page 355.

3. Aug. 12, 1900; $1174.01.
4. Mar. 8, 1904; $2233.15.

GENERAL REVIEW.

356, Art. 871.

$\frac{17}{19}$ %.
17.74.
587.50.
050.
80.63.
41.50.
$\frac{1}{2}$¢.
A.
36.14 + gal.
3.88.
$\frac{2}{3}$ %.
$\frac{11}{16}$ in.
$\frac{3}{8}$.

357.

ıin, $2405.
1.36.
4.06.
)30.88.
4.50.
$\frac{1}{5}$ bu.
$\frac{1}{5}$ bu.
).60.
83.
loss or gain.
ss, 746\frac{2}{3}$.
)00.

58.

'75.
'78.99.
).
ft.
$\frac{3}{4}$ ft.
7.61.
r. 9 min. 18$\frac{4}{5}$

3.89.
) bricks.
%.
)0.
).
l.48.
$\frac{5}{8}$.

59.

)4.44 ;
:.66.

41. 25.13 + ft.
42. $64\frac{9}{19}$ %.
43. $40.
44. $454.55.
45. 150.7968 sq. ft.
46. $632.97.
47. $472.73.
48. $24\frac{4}{9}$ %.
49. 25 shares ; $250.
50. $5437.50.
51. $47400.
52. $13368.75.

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53. 180 shares ; $111.25.
54. $372.24.
55. $7600.
56. $6\frac{12}{23}$ %.
57. $9\frac{3}{4}$ %.
58. $2532.52.
59. A's gain, $1531-.25 ;
B's gain, $1750 ;
C's gain, $1968.75.
60. $1696.97.
61. $945.40.
62. $152.72.
63. $12.75; $1487.25.
64. Lost, $1.62.
65. $53\frac{1}{3}$ yd.
66. 19235.62 + bar.

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67. 22¢.
68. 2120.58 sq. ft.
69. $8\frac{4}{87}$ da.
70. $93.75.
71. 17 %.
72. $43666.67.
73. $1020 ; $950.
74. 17280 bricks.
75. £172 11s. 5d. 2 + far.
76. $14.77.
77. 67054.0003287.
78. 9 ft.
79. 1800 ft.
80. 56.994 + bar.

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81. $ .4484 +.
82. $12500.
83. $7726.83.
84. $902.34.
85. $164.06.
86. $1933.62.
87. $6\frac{1}{3}$ mo.

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88. $1025 64.
89. 3 yr.
90. 3.18750.
91. 3.1875.
92. $33 55.
93. 3322.54 + lb.
94. $384.35.
95. Mar. 1, 1901.
96. $1210.
97. $192.50.
98. $1\frac{1}{2}$ %.
99. 60 %.
100. 1320 marks.
101. $6655.17.

Page 364.

102. $1324.90.
103. $1623.02.
104. 46.572 + ft.
105. 89.814 + ft.
106. $1115.37.
107. $112.38.
108. $4\frac{4}{15}$ da.
109. $522.66.
110. $98\frac{58}{125}$ %.
111. 91.44^{m}.
112. 4.97 + mi.
113. $2.63 + {}^{Ha}$.

Page 365.

114. $6.12.
115. $5.678 + {}^{l}$.
116. 77.16 + gr.
117. 27.216^{Kg}.
118. 120.6975^{Km}.
119. $12.95 + {}^{sq\ Km}$.
120. $152.9\ {}^{cu\ m}$.
121. $1833.92.
122. $23863.56.

123. $1365.
124. $1111.20.
125. $152.76.
126. 10000 rubles.
127. 1000 colons.
128. £244 17s. 6d.
129. £1225 18s. 6d.
130. 6250 marks.
131. 1012 bar.; 14¢ over.

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132. 74046 bricks.
133. $946.07.
134. $200.
135. $ .44$\frac{4}{9}$; $1200.
136. $12950.
137. $25149.25.
138. $4\frac{1}{50}+\%$.
139. 2700 sq. yd.
140. $14\frac{47}{50}+\%$.
141. 16¢.

Page 367.

142. $20.95; £869 2s. 5d. 2 far.
143. £82 7½d.; £3199 4s. 4½d.; $15669.64.
144. A, $5666.67; B, $2333.33.
145. A's cap., $7810.41 B's cap., $9602.59